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Understanding the behavior of electric charges in a given geometry is fundamental in electrostatics. When three point charges— Q_1 , q_2 , and Q_3 —are placed at the corners of a rectangle, the resulting electric fields, potential energies, and forces become intriguing to analyze. This setup not only illustrates core principles of Coulomb's law but also provides a practical foundation for understanding more complex electrostatic systems. In this article, we will explore the key concepts involved in analyzing such a configuration, including the calculation of electric forces, the net electric field at various points, and the potential energy of the system.

Geometry and Placement of Charges

Understanding the Rectangular Arrangement

To analyze the electrostatic interactions among Q_1 , q_2 , and Q_3 , it is essential first to understand their placement:

- The charges are situated at three distinct corners of a rectangle.
- The fourth corner remains unoccupied by a charge, creating an asymmetrical charge distribution.
- The sides of the rectangle are labeled as length l and width w .
- The positions of the charges are typically given with respect to coordinate axes for clarity, such as:
 - Q_1 at $(0, 0)$
 - q_2 at $(l, 0)$
 - Q_3 at $(0, w)$

This configuration allows us to analyze the forces acting on each charge due to the others and to

calculate the net electric fields at various points within the rectangle.

Fundamental Principles for Analysis

Coulomb's Law and Electric Force

The cornerstone of electrostatics is Coulomb's law, which states:

- The magnitude of the electrostatic force between two point charges is directly proportional to the product of their magnitudes.
- It is inversely proportional to the square of the distance between them.
- The force acts along the line connecting the two charges.

Mathematically, Coulomb's law can be expressed as:

$$F = \frac{k |q_1 q_2|}{r^2}$$

where:

- F is the magnitude of the force,
- $k \approx 9 \times 10^9 \text{ Nm}^2/\text{C}^2$ (Coulomb's constant),
- q_1 and q_2 are the magnitudes of the charges,
- r is the distance between the charges.

Using Coulomb's law, we can compute the forces on each charge due to the others.

Electric Field and Superposition Principle

- The electric field $\vec{E}$ at a point in space due to a point charge is given by:

$$\vec{E} = \frac{k q}{r^2} \hat{r}$$

where $\hat{r}$ is the unit vector from the charge to the point of interest.

- The net electric field at any point results from the vector sum of the fields due to all individual charges, following the superposition principle.

Calculating Electric Forces in the Rectangle

Force on a Charge Due to Others

Suppose we are interested in the forces acting on Q1:

- Force due to q2: Since q2 is at (l, 0), the distance between Q1 (at (0, 0)) and q2 is $(r_{Q1,q2} = l)$.
- Force due to Q3: Located at (0, w), the distance from Q1 is $(r_{Q1,Q3} = w)$.

The forces can be calculated as:

$$F_{Q1,q2} = \frac{k |Q1 \cdot q2|}{l^2}$$

$$F_{Q1,Q3} = \frac{k |Q1 \cdot Q3|}{w^2}$$

Each force acts along the line connecting the charges, so their directions depend on the relative positions.

Vector Components of Forces

- To determine the net force on a charge, resolve each force into components along the x and y axes.
- For example, the force between Q1 and q2 acts along the x-axis:

$$\vec{F}_{Q1,q2} = F_{Q1,q2} \hat{i}$$

- The force between Q1 and Q3 acts along the y-axis:

$$\vec{F}_{Q1,Q3} = F_{Q1,Q3} \hat{j}$$

- Summing these components gives the net force acting on Q1. Similar calculations can be performed for q2 and Q3.

Electric Field at a Point Inside the Rectangle

Calculating the Electric Field Due to Each Charge

To find the electric field at a specific point $((x, y))$ within or outside the rectangle:

- Calculate the electric field due to each charge at that point using the formula:

$$\vec{E} = \frac{k q}{r^2} \hat{r}$$

where r is the distance from the charge to the point, and $\hat{r}$ points from the charge to the point.

- Express each electric field vector in component form (x and y components).

Superposition of Electric Fields

- The total electric field at the point is the vector sum of the individual fields:

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

- The resulting magnitude and direction provide insights into the electric influence at that point.

Potential Energy of the System

Calculating the Electrostatic Potential Energy

The potential energy U of a system of point charges is given by:

$$U = \sum_i$$