Triangle ABC Is Translated To Image ABC. In This Translation, A(5, 1) Maps To A(6, 2). The Coordinates

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Understanding geometric transformations is fundamental in the study of geometry, especially translations. When a triangle undergoes a translation, every point of the triangle moves the same distance in the same direction, resulting in an image that is congruent to the original. This article explores the details of translating Triangle ABC to its image, focusing on the specific case where point A(5, 1) maps to A(6, 2). We will analyze the translation process, determine the translation vector, and understand how all points of the triangle are affected.

Introduction to Translations in Geometry

A translation is a type of rigid motion that slides every point of a figure along a straight line a certain distance and in a specified direction. Unlike rotations or reflections, translations do not alter the shape or size of the figure, only its position.

Key Concepts in Translation

- Translation Vector: Represents the distance and direction of the slide.
- Image of a Figure: The figure after translation.
- Corresponding Points: Points on the original figure and their images after translation.

Given Data and Problem Statement

In our specific scenario, we are given:

- The original triangle: ABC.
- The image of the triangle after translation: A'B'C'.
- The coordinates of point A before translation: A(5, 1).
- The coordinates of point A after translation: A'(6, 2).

The goal is to analyze the translation process, determine the translation vector, and understand how the other vertices of the triangle are affected.

Determining the Translation Vector

Since point A maps from (5, 1) to (6, 2), the translation vector can be calculated as the difference between the coordinates of A' and A.

Calculating the Translation Vector

The translation vector, often denoted as $\vec{T}$, is found by subtracting the original coordinates from the translated coordinates:

$$
\vec{T} = (x_{A'} - x_A, y_{A'} - y_A)
$$

Substituting the given values:

$$
\vec{T} = (6 - 5, 2 - 1) = (1, 1)
$$

This indicates that every point of triangle ABC moves 1 unit to the right and 1 unit upward during the translation.

Applying the Translation Vector to Find Other Image Coordinates

Since translation is uniform across all points, the same vector applies to the other vertices B and C of the original triangle.

Example: Coordinates of B and C

Suppose the original coordinates of B and C are known:

- $B(x_B, y_B)$
- $C(x_C, y_C)$

The images of B and C, denoted as B' and C', are computed as:

$$
B' = (x_B + 1, y_B + 1)
$$
$$
C' = (x_C + 1, y_C + 1)
$$

\]

If the specific coordinates of B and C are provided, their images can be calculated directly.

Understanding the Impact of the Translation

Since translation preserves lengths and angles, the shape and size of triangle ABC remain unchanged in its image, but its position is shifted.

Properties of Translated Figures

- Congruence: The original and translated figures are congruent.
- Orientation: The orientation remains the same; the figure is simply moved.
- Parallelism: Corresponding sides are parallel because the entire figure is shifted uniformly.

Visualizing the Translation Process

Graphical representation helps in understanding the translation process:

- Plot the original triangle with vertices A, B, and C.
- Draw the translation vector \(\vec{T} = (1, 1)\) as an arrow starting from A.
- From each vertex, draw an arrow of the same length and direction to locate the image points A', B', and C'.
- Confirm that the image triangle A'B'C' maintains the same shape and size as ABC, just shifted.

Mathematical Summary of the Translation

Original Point	Coordinates	Translated Point	Coordinates	Calculation
A	(5, 1)	A'	(6, 2)	(5 + 1, 1 + 1) = (6, 2)
B	(x_B, y_B)	B'	(x_B + 1, y_B + 1)	Apply same vector
C	(x_C, y_C)	C'	(x_C + 1, y_C + 1)	Apply same vector

This table emphasizes how each point shifts identically according to the translation vector.

Applications of Translations in Geometry

Understanding translations has practical implications in various fields:

- Computer Graphics: Moving images or objects across the screen.

- Robotics: Planning movements where parts are shifted without rotation.
- Engineering and Design: Positioning components precisely.
- Mathematics Education: Reinforcing concepts of congruence and geometric transformations.

Practice Problems

1. If point B of triangle ABC is at (3, 4), what are the coordinates of B' after the translation?
2. Given C is at (7, 0), find C' after applying the translation vector.
3. If the original triangle ABC has vertices at A(2, 3), B(4, 5), and C(6, 3), what are the coordinates of the translated triangle?

Answers:

1. B' = (3 + 1, 4 + 1) = (4, 5)
2. C' = (7 + 1, 0 + 1) = (8, 1)
3. A' = (2 + 1, 3 + 1) = (3, 4), B' = (4 + 1, 5 + 1) = (5, 6), C' = (6 + 1, 3 + 1) = (7, 4)

Conclusion

The translation of triangle ABC to its image ABC' involves shifting every point by the same vector, which in this case is (1, 1). This process maintains the size and shape of the triangle but moves it to a new position. Understanding how to determine and apply translation vectors is fundamental in geometry and has broad applications across various scientific and technological fields. By mastering these concepts, students can develop a deeper comprehension of geometric transformations and their practical uses.

Summary

- The translation vector from A(5, 1) to A'(6, 2) is (1, 1).
- All points of triangle ABC are shifted by this vector.
- The shape and size of the triangle remain unchanged.
- Translations are essential tools in geometry, computer graphics, and engineering.

By analyzing the specific case where point A maps from (5, 1) to (6, 2), students and learners can better understand the mechanics of translation and the importance of coordinate geometry in visualizing and solving geometric problems.

Note: To fully visualize the translation, knowing the original coordinates of B and C is helpful. If these are provided, the process can be completed for the entire triangle.

Frequently Asked Questions

What is the translation vector used to move triangle ABC from point A(5, 1) to A'(6, 2)?

The translation vector is (1, 1), since A(5, 1) moves to A'(6, 2) by adding 1 to both the x and y coordinates.

If triangle ABC is translated so that point A(5, 1) maps to A'(6, 2), how do you find the new coordinates of point B after the translation?

You add the translation vector (1, 1) to point B's original coordinates. For example, if B is at (x, y), then the new point B' will be at (x+1, y+1).

Does translating triangle ABC by the vector (1, 1) change the size or shape of the triangle?

No, translation moves the entire figure without altering its size or shape; it only changes its position on the coordinate plane.

If point C of triangle ABC has coordinates (7, 4), what are its new coordinates after the translation?

Applying the translation vector (1, 1), point C(7, 4) moves to C'(8, 5).

How can you verify that the translation from A(5, 1) to A'(6, 2) is consistent for the entire triangle?

Check that all other points of the triangle are shifted by the same translation vector (1, 1). For each point, add 1 to both x and y coordinates and verify they match their corresponding translated points.

What is the general rule for translating a point (x, y) based on the given example?

To translate a point (x, y), add the same translation vector to both coordinates. In this case, (x, y) becomes (x+1, y+1).

Additional Resources

Triangle ABC Is Translated To Image ABC. In This Translation, A(5, 1) Maps To A(6, 2). The Coordinates

Understanding transformations in geometry is fundamental for grasping how figures move within the coordinate plane. Among these transformations, translation stands out as one of the most

straightforward and intuitive. When a triangle, such as Triangle ABC, undergoes a translation, it shifts from its original position to a new location without changing its size, shape, or orientation. This article explores the concept of translating a triangle, specifically focusing on how Triangle ABC is mapped onto its image through translation, with an emphasis on the given coordinate shift where point A(5, 1) maps to A(6, 2). We will analyze the details of this translation, its properties, and its implications for the entire triangle.

Understanding Translation in Geometry

Definition of a Translation

A translation in geometry involves sliding every point of a figure a fixed distance in a specified direction. This movement is characterized by a translation vector, which indicates how far and in which direction the figure is shifted. The key features of translation include:
- Preservation of size, shape, and orientation
- Movement without rotation or reflection
- Uniform displacement of all points in the figure

Mathematical Representation of Translation

Mathematically, a translation can be represented as:
$$ (x, y) \rightarrow (x + a, y + b) $$
where (a, b) is the translation vector indicating the horizontal and vertical shifts, respectively. For example, if the translation vector is (1, 1), every point moves one unit to the right and one unit up.

Analyzing the Given Translation: A(5, 1) Maps To A(6, 2)

Determining the Translation Vector

Given the initial point A(5, 1) and its image A'(6, 2), we can find the translation vector by subtracting the coordinates:
- Horizontal shift: $6 - 5 = 1$
- Vertical shift: $2 - 1 = 1$

Thus, the translation vector is $\vec{v} = (1, 1)$. This means that every point of Triangle ABC is shifted one unit to the right and one unit upward to produce its image.

Implications for the Entire Triangle

Since translation moves all points equally, the coordinates of points B and C will be shifted using the same vector:
$$ B(x_b, y_b) \rightarrow B'(x_b + 1, y_b + 1) $$
$$ C(x_c, y_c) \rightarrow C'(x_c + 1, y_c + 1) $$

This consistent movement preserves the size, shape, and orientation of the triangle, ensuring that Triangle ABC and its image are congruent.

Coordinates of Triangle ABC and Its Image

Original Coordinates of Triangle ABC

Suppose Triangle ABC has the following coordinates:
- $A(5, 1)$ (given)
- $B(x_b, y_b)$ (unknown)
- $C(x_c, y_c)$ (unknown)

Without loss of generality, let's consider specific coordinates for B and C to illustrate the translation. For example:
- $B(3, 4)$
- $C(7, 3)$

Coordinates of the Translated Triangle ABC'

Applying the translation vector $(1, 1)$:
- $A' = (5 + 1, 1 + 1) = (6, 2)$
- $B' = (3 + 1, 4 + 1) = (4, 5)$
- $C' = (7 + 1, 3 + 1) = (8, 4)$

This set of points defines the image triangle after translation.

Properties of the Translation

Key Features

- Congruence: The original triangle and its image are congruent because translation preserves distances and angles.
- Orientation: The orientation (clockwise or counterclockwise) remains unchanged.

- Parallelism: Corresponding sides are parallel, maintaining the shape's overall structure.

Advantages of Using Translation

- Simplifies the process of moving figures within the coordinate plane.
- Keeps the figure's properties intact, aiding in proofs and problem-solving.
- Useful in computer graphics and geometric modeling where movement without distortion is required.

Visualizing the Translation

Graphical Representation

Plotting the original and the translated triangle on a coordinate plane provides visual clarity. The initial triangle ABC with points at (5, 1), (3, 4), and (7, 3) can be sketched, followed by its image A'B'C' after shifting all points by (1, 1). The resulting figure should look identical in shape and size, merely shifted diagonally upward and to the right.

Tools and Techniques

- Use graph paper or graphing software for precise plotting.
- Draw the original triangle and then translate each vertex using the vector.
- Connect corresponding points to visualize the translation.

Applications of Translation in Various Fields

Mathematics Education

Teaching students about congruence and transformations. Visualizing translations helps develop spatial reasoning skills.

Computer Graphics and Animation

Objects are moved across scenes using translation vectors, ensuring smooth movement without distortion.

Engineering and Design

Part placement and movement within CAD (Computer-Aided Design) models often involve translation operations.

Robotics

Movement algorithms frequently incorporate translation to navigate robots in a coordinate space.

Summary of Key Takeaways

- The translation of Triangle ABC involves shifting every point by a fixed vector, in this case, $\vec{v} = (1, 1)$.
- The transformation preserves all geometric properties, making the image congruent to the original.
- Coordinates of the points after translation can be calculated simply by adding the components of the translation vector to each original coordinate.
- Visual and practical applications of translation are widespread across mathematics and technology.

Conclusion

The translation of Triangle ABC to its image demonstrates a fundamental and elegant concept in geometry: moving figures without altering their intrinsic properties. By understanding how a single point's movement (A from (5, 1) to (6, 2)) determines the entire transformation, students and practitioners can better grasp the nature of congruence and geometric transformations. Whether in theoretical mathematics, computer graphics, or engineering, translation remains an essential tool for manipulating figures within the coordinate plane. The example of shifting point A by (1, 1) to produce the new image underscores the simplicity and power of this transformation, emphasizing its importance in both academic learning and practical applications.

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