Two Cards Are Drawn Without Replacement From An Ordinary Deck. Find The Probability That The Second Is

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Understanding probability in card games is fundamental for both enthusiasts and students of mathematics. When drawing two cards from a standard deck without replacement, the outcome of the second draw depends heavily on the first draw. In particular, calculating the probability that the second card has a certain characteristic—such as being a specific suit, rank, or color—requires careful analysis. This article provides a comprehensive explanation of how to determine the probability that the second card drawn from an ordinary 52-card deck possesses a particular property when two cards are drawn sequentially without replacement. We will explore different scenarios, methods of calculation, and practical examples to solidify your understanding of this important concept in probability theory.

Understanding the Basic Setup

What Is an Ordinary Deck?

- An ordinary deck contains 52 cards.
- It consists of 4 suits: Clubs, Diamonds, Hearts, and Spades.
- Each suit has 13 ranks: Ace, 2 through 10, Jack, Queen, King.
- The deck is well-shuffled, ensuring randomness before each draw.

Drawing Without Replacement

- When a card is drawn, it is not returned to the deck.
- The total number of cards decreases with each draw.
- The probability of subsequent events depends on previous outcomes, making the analysis slightly more complex than with replacement.

What Does "The Second Card" Mean?

- The focus is on the second card drawn.
- The probability that this card has a certain property (e.g., is a spade, a face card, red, etc.) depends on the outcome of the first draw.
- Calculations often involve considering all possible outcomes of the first draw.

Fundamental Principles of Probability in Sequential Draws

Conditional Probability

- The probability of an event depends on the outcome of a previous event.
- For example, the probability that the second card is a spade depends on whether the first card was a spade.

Multiplication Rule

- The overall probability of a sequence of events is the product of the probabilities of each event, considering the conditions at each step.
- When drawing without replacement, probabilities change after each draw.

Law of Total Probability

- To find the probability that the second card has a property, sum over all possible outcomes of the first draw, weighted by their probabilities.

Calculating the Probability That the Second Card Has a Specific Property

Let's consider the general approach to find the probability that the second card has a certain property, say property P, when drawing two cards without replacement.

Step 1: Identify the property P

- For example, P could be:
- Being a specific suit (e.g., Spades)
- Being a face card (Jack, Queen, King)
- Being a red card (Diamonds or Hearts)
- Having a specific rank (e.g., an Ace)

Step 2: Determine the possible scenarios based on the first draw

- The first card could either have property P or not.
- These scenarios influence the probability for the second card.

Step 3: Use total probability considering both scenarios

- Calculate the probability that the second card has property P, considering whether the first card had property P or not.

Step 4: Combine the scenarios to find the overall probability

- Sum the probabilities of each scenario weighted by their likelihood.

Specific Examples and Calculations

Example 1: Probability that the Second Card Is a Spade

Suppose you draw two cards from a standard deck without replacement. What is the probability that the second card is a spade?

Solution Approach:

- The total number of spades in the deck is 13.
- The total number of cards is 52.

Method 1: Direct reasoning

- The probability that the second card is a spade can be approached by considering all possible outcomes of the first draw.

Scenario A: First card is a spade

- Probability: 13/52 = 1/4
- After drawing a spade, remaining spades: 12
- Remaining cards: 51
- Probability that the second card is a spade given the first was a spade: 12/51

Scenario B: First card is not a spade

- Probability: (52 - 13)/52 = 39/52 = 3/4
- Remaining spades: still 13
- Remaining cards: 51
- Probability that the second card is a spade given the first was not a spade: 13/51

Calculate total probability:

$$
P(\text{second is spade}) = P(\text{first is spade}) \times P(\text{second is spade} | \text{first is spade}) + P(\text{first not spade}) \times P(\text{second is spade} | \text{first not spade})
$$

\[

= \frac{1}{4} \times \frac{12}{51} + \frac{3}{4} \times \frac{13}{51}
\]

\[
= \frac{1}{4} \times \frac{12}{51} + \frac{3}{4} \times \frac{13}{51}
\]

\[
= \frac{12}{204} + \frac{39}{204} = \frac{51}{204} = \frac{17}{68}
\]

Result:

\[
P(\text{second card is a spade}) = \frac{17}{68} \approx 0.25
\]

which aligns with the initial intuition that, on average, the probability remains close to the proportion of spades in the deck.

Example 2: Probability that the Second Card Is an Ace

Using the same method, what is the probability that the second card is an Ace?

Solution:

- Number of Aces in the deck = 4

Scenario A: First card is an Ace

- Probability: 4/52 = 1/13
- Remaining Aces: 3
- Remaining cards: 51
- Probability second card is an Ace given first was an Ace: 3/51

Scenario B: First card is not an Ace

- Probability: 48/52 = 12/13
- Remaining Aces: 4
- Remaining cards: 51
- Probability second card is an Ace given first was not an Ace: 4/51

Total probability:

\[
P = \frac{1}{13} \times \frac{3}{51} + \frac{12}{13} \times \frac{4}{51}
\]

\[

$$
= \frac{3}{663} + \frac{48}{663} = \frac{51}{663} = \frac{17}{221}
$$

Result:

$$
P(\text{second card is an Ace}) = \frac{17}{221} \approx 0.0769
$$

Example 3: General Formula for the Probability that the Second Card Has Property P

Suppose:

- Total cards: $N = 52$
- Number of cards with property P: k

The probability that the second card has property P can be found using the law of total probability:

$$
P(\text{second has P}) = P(\text{first has P}) \times P(\text{second has P} | \text{first has P}) + P(\text{first not P}) \times P(\text{second has P} | \text{first not P})
$$

which simplifies to:

$$
P(\text{second has P}) = \frac{k}{N} \times \frac{k-1}{N-1} + \frac{N - k}{N} \times \frac{k}{N-1}
$$

This formula accounts for both scenarios and provides a quick way to compute the probability for any property P, given the counts.

Important Observations and Intuition

- Symmetry in probabilities: When drawing without replacement, the probability that the second card has a property similar to the first is influenced by the initial proportion of that property in the deck.
- Unchanged probabilities for certain properties: For properties where the count does not change significantly or where symmetry exists, the probability that the second card has that property often remains close to the proportion in the deck.
- Conditional probabilities matter: The outcome of the first draw influences the second, emphasizing the importance of considering different scenarios separately.

Practical Applications of These Calculations

Understanding the probability that the second card has a certain property has several real-world and academic applications:

- Card games: Determining odds in poker, bridge, and other card games.
- Statistical modeling: Simulating scenarios where items are drawn without replacement.
- Educational purposes: Teaching concepts of conditional probability and combinatorics.
- Gambling strategies: Developing strategies based on probability calculations.

Common Mistakes to Avoid

- Assuming replacement: Remember that probability calculations change when

Frequently Asked Questions

What is the probability that the second card drawn from a standard deck without replacement is an Ace?

The probability is ½, or 50%, because the probability that the first card is not an Ace is ½, and given that the first card was not an Ace, the probability that the second card is an Ace remains ¼. Combining these, the overall probability is ¼.

How do you calculate the probability that the second card drawn is a face card (Jack, Queen, King) without replacement?

Since there are 12 face cards in a deck of 52, the probability that the second card is a face card can be found by considering all possible scenarios. The probability that the second card is a face card is ¼, because the first card's identity affects the total count, but overall, the probability remains ¼.

What is the probability that the second card drawn is a red card from a standard deck?

The probability that the second card is red is ¼, because regardless of the first draw, the chance that the second card is red remains consistent at ¼ after accounting for the previous draw.

If the first card drawn is a specific card (e.g., Queen of Hearts), what is the probability that the second card is an Ace?

Given that the first card was the Queen of Hearts, which is not an Ace, the probability that the second card is an Ace is µ, since all four Aces are still in the deck, and one card has been removed.

Does the probability that the second card is a King depend on what the first card was?

No, because the probability that the second card is a King is ¼, regardless of the first card drawn, assuming draw without replacement and uniform chances.

What is the probability that the second card drawn is a club, given that the first card drawn was a spade?

If the first card is a spade, then there are 13 spades originally, and one has been removed, leaving 12 spades and 13 clubs in a 51-card deck. Therefore, the probability that the second card is a club is ½.

How does drawing cards without replacement affect the calculation of the probability of the second card being a specific suit?

Drawing without replacement changes the number of remaining cards and suits in the deck, so the probability depends on what was drawn first. The calculation involves updating the total and suit counts after the first draw to determine the probability accurately.

Additional Resources

Probability

Understanding the Context: Drawing Two Cards Without Replacement

Imagine you're at a classic card game table, and the dealer shuffles an ordinary 52-card deck—comprising four suits: hearts, diamonds, clubs, and spades, each with 13 cards. You’re about to engage in a simple yet fascinating experiment: drawing two cards sequentially without replacement. This means that once a card is drawn, it’s not returned to the deck before the second draw, influencing the probabilities involved.

The core question we explore today is: What is the probability that the second card drawn has a specific property or characteristic? For instance, what is the probability that the second card is a heart? Or a face card? Or perhaps an ace? Understanding these probabilities requires a detailed grasp of combinatorics and conditional probability, as well as an appreciation of how the first draw impacts the second.

Let’s delve into the mechanics, explore various scenarios, and analyze the probabilities step-by-step, as if we are reviewing a complex yet elegantly designed system—like a top-tier gadget that’s both reliable and predictable when you understand its inner workings.

Fundamental Concepts and Assumptions

Before diving into calculations, it’s crucial to clarify some foundational assumptions:

- Deck Composition: The deck contains 52 cards, with 4 suits (hearts, diamonds, clubs, spades), each with 13 cards.
- Drawing Process: Two cards are drawn sequentially without replacement—meaning the first card is not put back before the second draw.
- Event of Interest: The probability that the second card has a certain property (e.g., being a heart, a face card, etc.).

This setup is akin to a well-engineered product with consistent specifications, where each step impacts the subsequent ones, making the analysis both precise and insightful.

Breaking Down the Probability Problem

The question "Find the probability that the second card is..." requires us to understand that this is a conditional probability problem. Specifically, the probability depends on the possible outcomes of the first draw, which influence the second.

The General Approach

To analyze the probability that the second card has a property P, we use the law of total probability, considering all possible outcomes of the first draw:

$$
P(\text{second card has property } P) = \sum_{i} P(\text{first card outcome } i) \times P(\text{second card has } P \mid \text{first card outcome } i)
$$

However, because the deck is symmetric and properties are evenly distributed, we can simplify this analysis into more straightforward calculations.

Scenario 1: Probability that the Second Card Is a Heart

Let’s analyze a specific, classic case: What is the probability that the second card is a heart?

Step 1: Recognize Symmetry and Intuition

In an unconditioned setting, the chance that a randomly drawn card from a full deck is a heart is $\frac{13}{52} = \frac{1}{4}$. But since we are drawing two cards without replacement, the probability that the second card is a heart is not simply $\frac{1}{4}$.

Step 2: Formal Calculation

The total probability can be approached by considering two mutually exclusive scenarios:

- Scenario A: The first card is a heart.
- Scenario B: The first card is not a heart.

Because the process is sequential, the probability that the second card is a heart depends on which of these scenarios occurs.

Scenario A: First card is a heart.

- Probability that the first card is a heart: $\frac{13}{52} = \frac{1}{4}$.
- After drawing a heart, the remaining deck has 12 hearts left and 51 total cards.
- Probability that the second card is a heart in this case: $\frac{12}{51}$.

Scenario B: First card is not a heart.

- Probability that the first card is not a heart: $\frac{39}{52} = \frac{3}{4}$.
- The remaining deck has 13 hearts still, but only 51 cards total.
- Probability that the second card is a heart in this case: $\frac{13}{51}$.

Step 3: Combining the Scenarios

Using total probability:

$$
P(\text{second card is a heart}) = P(\text{first is heart}) \times P(\text{second is heart} \mid \text{first is heart}) + P(\text{first not heart}) \times P(\text{second is heart} \mid \text{first not heart})
$$

$$
= \left(\frac{1}{4} \times \frac{12}{51} \right) + \left(\frac{3}{4} \times \frac{13}{51} \right)
$$

Calculating each term:

$$
= \frac{1}{4} \times \frac{12}{51} + \frac{3}{4} \times \frac{13}{51} = \frac{12}{204} + \frac{39}{204} = \frac{51}{204}
$$

Simplifying:

$$
\frac{51}{204} = \frac{17}{68} \approx 0.25
$$

Result: The probability that the second card is a heart is exactly $\frac{1}{4}$, the same as the initial proportion of hearts in the deck.

Key Insight: Symmetry and Independence in Conditional Probability

This result reveals a fundamental symmetry: the probability that the second card is a heart remains $\frac{1}{4}$, regardless of the first draw, due to the uniform distribution of suits and the symmetry of the deck. This is a crucial insight that simplifies many similar probability calculations.

Scenario 2: Probability that the Second Card Is a Face Card

Let’s explore a different property: What is the probability that the second card is a face card?

Step 1: Deck Composition of Face Cards

- Total face cards in the deck: 12 (3 face cards per suit: Jack, Queen, King).
- Total cards: 52.

Step 2: Similar Approach as Before

Again, consider the two scenarios based on the first card:

- First card is a face card:
- Probability: $\frac{12}{52} = \frac{3}{13}$.
- Remaining face cards: 11.
- Remaining deck size: 51.
- Probability second card is a face card: $\frac{11}{51}$.

- First card is not a face card:
- Probability: $\frac{40}{52} = \frac{10}{13}$.
- Remaining face cards: still 12 (since the first wasn't a face card).
- Remaining deck size: 51.
- Probability second card is a face card: $\frac{12}{51}$.

Step 3: Total Probability Calculation

$$
P(\text{second is a face card}) = \frac{3}{13} \times \frac{11}{51} + \frac{10}{13} \times \frac{12}{51} = \frac{33}{663} + \frac{120}{663} = \frac{153}{663}
$$

Simplify numerator and denominator:

\[
\frac{153}{663} = \frac{17}{73}
\]

Result: The probability that the second card is a face card is \(\frac{17}{73} \approx 0.2466\), slightly less than \(\frac{12}{52} \approx 0.2308\).

This demonstrates that, unlike suits, the probability of drawing a face card on the second draw slightly exceeds the initial proportion, influenced by the conditional structure of the draw process.

Scenario 3: Probability that the Second Card Is an Ace

Consider now the probability that the second card is an ace, given that there are 4 aces in the deck.

Step 1: Basic Data

- Total aces: 4.
- Total cards: 52.

Step 2: Similar Conditional Analysis

Again, split into two scenarios:

- First card is an ace:
- Probability: \(\frac{4}{52} = \frac{1}{13}\).
- Remaining aces: 3.
- Remaining deck: 51 cards.
- Probability second card is an ace: \(\frac{3}{51}\).

- First card is not an ace:
- Probability: \(\frac{48}{52} = \frac{12}{13}\).
- Remaining aces: 4 (unchanged).
- Remaining deck: 51 cards.
- Probability second card is an ace: \(\frac{4}{51}\).

Step 3: Total Probability

\[
P(\text{second is an ace}) = \frac

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