Two Similar Solids Have Edges Of 4 Feet In 24 Feet If The Smaller Saw That Has A Volume Of 16 Ft. Find

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Understanding the properties of similar solids is fundamental in geometry, especially when it comes to calculating unknown dimensions and volumes. When two solids are similar, their corresponding edges are proportional, and their volumes relate to the cube of the similarity ratio. This principle allows us to solve complex problems involving unknown measurements, given some known dimensions and volumes. In this article, we will explore a detailed approach to solving a problem involving two similar solids with specific edge lengths and volumes, guiding you step-by-step through the process.

Understanding the Problem Statement

Before diving into the solution, it's important to clarify what the problem entails. The problem states:

- Two solids are similar in shape.
- For these solids, a specific set of edges are given: 4 feet and 24 feet.
- The smaller solid has a volume of 16 cubic feet.
- The task is to find an unknown measurement or dimension related to the larger solid based on the given data.

The key points to focus on are:

1. Similarity of solids: The two shapes are geometrically similar, implying all corresponding linear dimensions are proportional.
2. Given edges: 4 ft (smaller solid) and 24 ft (larger solid).
3. Known volume of smaller solid: 16 ft^3.
4. Unknown to find: Typically, this involves the volume or a specific edge length of the larger solid.

The problem appears to involve understanding the ratio of corresponding edges and how that relates to the volume ratio.

Fundamental Concepts of Similar Solids

To solve problems involving similar solids, it's crucial to understand the core concepts:

1. Similar Solids and Corresponding Edges

- Corresponding edges of similar solids are proportional.
- The ratio of any pair of corresponding edges is constant.

2. Scale Factor (k)

- The scale factor between two similar solids is the ratio of their corresponding linear dimensions.
- If the smaller solid has an edge of length a, and the larger solid has an edge of length b, then:

$$
k = \frac{b}{a}
$$

3. Volume Ratio and Scale Factor

- The volumes of two similar solids relate to the cube of the scale factor:

$$
\frac{V_{larger}}{V_{smaller}} = k^3
$$

- Conversely, if the volume ratio is known, the scale factor can be determined as:

$$
k = \sqrt[3]{\frac{V_{larger}}{V_{smaller}}}
$$

Applying the Concepts to the Given Problem

Now, let's analyze the specific details:

- Smaller solid edge length: 4 ft
- Larger solid edge length: 24 ft
- Volume of smaller solid: 16 ft^3
- Unknown: Volume or dimension of the larger solid

Step 1: Find the scale factor k

Given the corresponding edges:

$$
k = \frac{\text{Larger edge}}{\text{Smaller edge}} = \frac{24}{4} = 6
$$

This means every linear dimension of the larger solid is 6 times that of the smaller solid.

Step 2: Determine the volume of the larger solid

Since the volume ratio is the cube of the scale factor:

$$
\frac{V_{larger}}{V_{smaller}} = k^3 = 6^3 = 216
$$

Given the volume of the smaller solid:

$$
V_{smaller} = 16 \text{ ft}^3
$$

The volume of the larger solid:

$$
V_{larger} = V_{smaller} \times 216 = 16 \times 216 = 3456 \text{ ft}^3
$$

Answer: The volume of the larger solid is 3456 ft^3.

Additional Considerations: Finding Unknown Dimensions

Suppose the problem asks for a specific dimension of the larger solid (for example, its height, width, or depth), given the initial measurements and volume.

Example Scenario:

If the smaller solid has an edge of 4 ft and volume 16 ft^3, and the larger solid has an edge of 24 ft, then:

- The ratio of linear dimensions: 6
- The volume of the larger solid: 3456 ft^3

If the solids are cubes, then:

- The volume of a cube is edge^3.
- For the larger cube:

$$
\text{Edge length} = 24 \text{ ft}
$$

which aligns with the given data.

Practical Applications and Real-World Examples

Understanding how to relate the dimensions and volumes of similar solids has numerous practical applications:

- Engineering and Manufacturing: Designing parts where scale models are used to predict the size and volume of actual objects.
- Architecture: Calculating the volume of scaled models or structures based on their dimensions.
- Science and Biology: Comparing similar biological structures at different scales.

Common Mistakes to Avoid

While solving problems involving similar solids, be cautious of the following pitfalls:

- Mixing up the scale factor and volume ratio: Remember, the scale factor applies to linear dimensions, while the volume ratio involves the cube of the scale factor.
- Assuming non-similar shapes are related: The similarity condition is crucial; it guarantees proportionality.
- Incorrectly calculating the cube root: When reversing the volume ratio to find the scale factor, ensure you take the cube root correctly.

Summary of the Solution Steps

To summarize the process:

1. Identify the corresponding edges and calculate the scale factor k.
2. Use the scale factor to find the volume ratio as k^3.
3. Multiply the known volume of the smaller solid by the volume ratio to find the larger solid's volume.
4. Confirm the calculations with the context of the problem.

Final Thoughts

Understanding the relationship between linear dimensions and volume in similar solids is essential for accurate problem-solving in geometry. By applying the principles of proportionality and volume ratios, you can confidently analyze and solve a wide range of real-world problems involving similar three-dimensional shapes.

Whether you're a student preparing for exams or a professional dealing with design and modeling, mastering these concepts will enhance your spatial reasoning and analytical skills.

Frequently Asked Questions

What is the similarity ratio between the two solids if their edges are 4 feet and 24 feet?

The similarity ratio is the ratio of corresponding linear dimensions, so it is 4:24, which simplifies to 1:6.

Given the smaller solid has a volume of 16 cubic feet, what is the volume of the larger solid?

Since the solids are similar, the volume ratio is the cube of the linear scale ratio. Therefore, $(6)^3 = 216$. The larger solid's volume is 16 ft^3 × 216 = 3456 ft^3.

How do the edges relate to the volumes of two similar solids?

The ratio of their volumes is the cube of the ratio of their corresponding edges. If edges are in ratio a:b, then volumes are in ratio $a^3:b^3$.

If the smaller solid has an edge of 4 feet and volume of 16 ft^3, what is the scale factor for the edges?

The scale factor for edges is the ratio of their edges: 4 feet to 24 feet, which is 1:6.

What is the volume of the larger solid with edges of 24 feet, given the smaller's volume is 16 ft^3?

Using the ratio 1:6 for edges, the volume ratio is $1^3:6^3 = 1:216$. Therefore, the larger volume is 16 ft^3 × 216 = 3456 ft^3.

Why do the volumes of similar solids scale as the cube of their linear dimensions?

Because volume is a three-dimensional measure, and scaling each linear dimension by a factor k scales the volume by k^3, reflecting the three-dimensional nature of volume.

Additional Resources

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The realm of geometry often presents intriguing puzzles that challenge our understanding of spatial relationships and proportional reasoning. Among these, problems involving similar solids stand out for their capacity to test our grasp of scale factors, volume ratios, and dimensional analysis. One such problem, frequently encountered in academic settings, states: "Two similar solids have edges of 4 feet and 24 feet. If the smaller has a volume of 16 ft^3, find the volume of the larger." This question not only probes the core principles of similarity but also offers a compelling exploration into how linear dimensions relate to volumetric measures.

In this investigative article, we delve into the intricacies of this problem, dissecting the mathematical principles involved, exploring the geometric relationships, and illustrating the step-by-step process to arrive at the solution. Our goal is to provide a comprehensive understanding that will benefit students, educators, and enthusiasts eager to deepen their grasp of similar solids.

Understanding the Problem Context

Before diving into calculations, it’s crucial to interpret the problem correctly:

- Two similar solids: These are shapes that are identical in form but differ in size. Their corresponding angles are equal, and their corresponding sides are proportional.

- Edges of 4 feet and 24 feet: These are likely the lengths of corresponding edges of the two solids, serving as the linear scale factors.

- Smaller solid has a volume of 16 ft^3: The known volume of the smaller solid.

- Find the volume of the larger solid: The primary goal.

The key to solving this problem lies in understanding how the scale factor between similar solids influences their volumes.

Fundamental Principles of Similar Solids

Linearly Similar Dimensions and Scale Factors

When two solids are similar, their corresponding linear dimensions (edges, heights, widths) are proportional. If:

- The smaller solid has a characteristic length l_s,
- The larger solid has a characteristic length l_l,

then:

$$
\frac{l_l}{l_s} = \text{scale factor for lengths}
$$

Volume Ratios and Scale Factors

The volume of similar solids scales with the cube of their linear scale factor:

$$
\frac{V_l}{V_s} = \left(\frac{l_l}{l_s} \right)^3
$$

where:

- V_s = volume of the smaller solid,
- V_l = volume of the larger solid.

This relation is fundamental, as it directly connects linear dimensions with volumetric measures.

Dissecting the Given Data

Step 1: Determine the Scale Factor for Edges

Given the edges:

- Smaller solid: 4 ft
- Larger solid: 24 ft

The scale factor for linear dimensions:

$$
\text{Scale factor} = \frac{24\, \text{ft}}{4\, \text{ft}} = 6
$$

Step 2: Calculate the Volume Ratio

Since volume scales with the cube of the linear scale factor:

$$
\frac{V_l}{V_s} = 6^3 = 216

$$
\]

Step 3: Find the Volume of the Larger Solid

Given the smaller solid's volume:

$$
V_s = 16\, \text{ft}^3
$$

The larger solid's volume:

$$
V_l = V_s \times 216 = 16 \times 216 = 3456\, \text{ft}^3
$$

Verifying the Logic and Assumptions

The above approach hinges on the assumption that:

- The characteristic edges provided (4 ft and 24 ft) correspond to the same dimension (e.g., height, width, or length) of each solid.
- The solids are similar in shape, which ensures proportionality in all dimensions.

Potential Variations and Clarifications

- If the problem implied that the edges are not corresponding sides, the calculation would need to adapt.
- The problem mentions "edges of 4 feet in 24 feet," which suggests the edges are corresponding and proportional.

Implications

Given the linear scale factor is 6, the volume increases by $6^3 = 216$, which aligns with the principles of geometric similarity.

Broader Implications and Applications

Understanding how linear dimensions relate to volumetric measures in similar solids has practical applications across various fields:

- Engineering and Manufacturing: Scaling prototype models to real-world sizes.
- Architecture: Designing scaled models of structures.
- Biology: Understanding how size differences affect organism volumes.
- Mathematics Education: Teaching proportional reasoning and the properties

of similar figures.

Common Mistakes and Pitfalls

While the problem seems straightforward, students often stumble over:

- Misidentifying which dimensions are corresponding.
- Forgetting that volume scales with the cube of the scale factor.
- Confusing linear scale factors with surface area or volume ratios.

To avoid these, always verify the correspondence of dimensions and recall the fundamental relations of similarity.

Summary of the Solution

Step	Description	Calculation	Result
1	Find the linear scale factor	$\frac{24}{4}$	6
2	Determine volume scale factor	6^3	216
3	Calculate larger volume	16×216	3456 ft^3

Final Answer: The volume of the larger solid is 3456 cubic feet.

Conclusion

The problem of two similar solids with given edge lengths and a known smaller volume exemplifies the elegance of geometric similarity principles. By recognizing that linear dimensions scale directly, and volumes scale with the cube of that factor, we can confidently solve such problems with clarity and precision.

This exploration emphasizes the importance of understanding fundamental geometric relationships, which serve as powerful tools across both theoretical and practical domains. Whether in academic contexts, engineering design, or everyday problem-solving, mastering these concepts enhances our ability to analyze and interpret the spatial world around us.

Additional Resources

- Textbooks: Geometry textbooks covering similar figures and volume ratios.
- Online Tutorials: Interactive lessons on scale factors and volume calculations.
- Mathematical Software: Geometric visualization tools to model similar

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solids and observe scale effects.

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Disclaimer: This article provides a detailed explanation based on the typical
interpretation of the problem statement. Variations in the problem’s wording
or specific context may require adjusted approaches.
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