02) A) Find The Period Of Ze Given By S LT 137 FindH) For Hin] =8) +26m-1)+28-2+6n-3) And Show That The

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Introduction

Understanding periodic functions is fundamental in the study of signals, vibrations, and many areas of engineering and physics. The problem at hand involves finding the period of a specific function, denoted as Ze, which is given by a complex sum involving variables m and n. The expression provided appears to be a sum of sinusoidal or exponential functions with different frequencies, and our goal is to determine the fundamental period of Ze, as well as demonstrate its periodicity properties.

In this comprehensive article, we will analyze the given function step-by-step, derive its period, and provide proofs to confirm the periodicity. We will also discuss the mathematical tools necessary for such analysis, such as properties of periodic functions, least common multiples (LCM), and concepts from Fourier analysis where applicable.

Understanding the Given Function

The Expression for Ze

The provided data appears to be:

> Ze is given by the sum:
> S_LT 137 FindH) For Hin] = 8) + 26m - 1) + 28 - 2 + 6n - 3)

While the expression seems somewhat fragmented, it suggests a sum over variables m and n involving terms like 8, 26m-1, 28-2, and 6n-3. For clarity, we interpret the function as:

Assumed Form of the Function

Let’s suppose the function Ze is expressed as:

$$
Ze(t) = \sum_{m} \sum_{n} A_{m,n} \cdot e^{j \omega_{m,n} t}
$$

where $A_{m,n}$ are amplitudes (possibly constants), and $\omega_{m,n}$ are angular frequencies depending on m and n.

Given the structure involving linear combinations of m and n, a typical form might be:

$$
Ze(t) = \sum_{m,n} C_{m,n} \cos(\alpha m + \beta n + \phi)
$$

or similar sinusoidal terms.

Step 1: Clarify the Mathematical Nature of Ze

Based on the data, it seems Ze involves sums over terms with frequencies related to linear combinations of m and n. The core idea is that Ze is composed of periodic signals with different fundamental frequencies.

Assumption:

Suppose the general form:

$$
Ze(t) = \sum_{m} \sum_{n} A_{m,n} \cos(\omega_{m,n} t + \phi_{m,n})
$$

with:

$$
\omega_{m,n} = a m + b n
$$

for some constants a, b.

Step 2: Identify the Frequencies

Frequencies Involved

From the expression, the terms involve:

- A constant term (possibly zero frequency)
- Terms proportional to m and n

Suppose the individual frequency components are:

$$
\omega_m = k_m \quad \text{and} \quad \omega_n = l_n
$$

or as linear combinations:

$$
\omega_{m,n} = p\, m + q\, n
$$

Given the expression:

$$
\text{Hin} = 8 + 26m - 1 + 28 - 2 + 6n - 3
$$

which simplifies to:

$$
H_{m,n} = (8 - 1 + 28 - 2 - 3) + 26m + 6n = (8 - 1 + 28 - 2 - 3) + 26m + 6n
$$

Calculating the constant:

$$
\begin{aligned}
8 - 1 &= 7 \\
7 + 28 &= 35 \\
35 - 2 &= 33 \\
33 - 3 &= 30
\end{aligned}
$$

Thus,

$$
H_{m,n} = 30 + 26m + 6n
$$

If these are frequencies (or related to frequencies), then the individual frequency components are proportional to $H_{m,n}$.

Step 3: Determine the Period of Ze

Fundamental Concept: Period of a Sum of Sinusoids

The sum of multiple sinusoids is periodic if and only if all individual sinusoids are periodic with a common period. The fundamental period T is related to the fundamental frequencies.

Suppose the angular frequencies involved are:

$$
\omega_{m,n} = \frac{2\pi}{T_{m,n}} = \text{some function of } m, n
$$

The period of each component:

$$
T_{m,n} = \frac{2\pi}{\omega_{m,n}}
$$

For Ze to be periodic, there must exist a common period T such that:

$$
\omega_{m,n} T = 2\pi \times \text{integer}
$$
for all m, n.

Step 4: Find the Fundamental Period

Given that the frequencies involve linear combinations of m and n, and constants, the key is to find the least common multiple (LCM) of the individual periods.

Assuming the frequencies are proportional to:

$$
\omega_m = 26m \quad \text{and} \quad \omega_n = 6n
$$

then, the periods associated are:

$$
T_m = \frac{2\pi}{26m} \\
T_n = \frac{2\pi}{6n}
$$

But since these depend on m, n, the overall period depends on the least common multiple of the fundamental periods corresponding to the smallest frequencies.

Step 5: Applying Number Theory to Find the Period

Find the Least Common Multiple (LCM)

- The basic frequencies are proportional to 26 and 6.
- The fundamental periods are:

$$
T_{26} = \frac{2\pi}{26} \\
T_6 = \frac{2\pi}{6}
$$

- To find the overall fundamental period, find the LCM of these two periods.

Since:

$$
$$

$$
T_{26} = \frac{2\pi}{26} = \frac{\pi}{13} \\
T_6 = \frac{2\pi}{6} = \frac{\pi}{3}
$$

The LCM of $\frac{\pi}{13}$ and $\frac{\pi}{3}$ is:

$$
T_{Ze} = \text{LCM}\left(\frac{\pi}{13}, \frac{\pi}{3}\right)
$$

which is:

$$
T_{Ze} = \pi \times \text{LCM}\left(\frac{1}{13}, \frac{1}{3}\right)
$$

The least common multiple of $\frac{1}{13}$ and $\frac{1}{3}$ is:

$$
\text{LCM}\left(\frac{1}{13}, \frac{1}{3}\right) = \frac{1}{\gcd(13,3)} \times \text{lcm}(13, 3) \quad \text{(since reciprocal relationship)}
$$

But 13 and 3 are co-prime, so:

$$
\text{lcm}(13,3) = 13 \times 3 = 39
$$

Thus, the common period:

$$
T_{Ze} = \pi \times \frac{39}{1} = 39 \pi
$$

---

6. Conclusion: The Period of Ze

Final Result

$$
\boxed{
\textbf{The fundamental period of Ze is } \boxed{T_{Ze} = 39 \pi}
}
$$

This means that the function Ze repeats itself every 39π units of time.

---

7. Demonstrating Periodicity

Proof of Periodicity

To confirm the periodicity, we verify that shifting Ze by $T_{Ze} = 39 \pi$ leaves it unchanged:

$$
Ze(t + T_{Ze}) = Ze(t)
$$

since all sinusoidal components satisfy:

$$
\cos(\omega_{m,n}(t + T_{Ze}) + \phi) = \cos(\omega_{m,n} t + \omega_{m,n} T_{Ze} + \phi)
$$

and:

$$
\omega_{m,n} T_{Ze} = (26m + 6n) \times 39 \pi
$$

Given that:

$$
26m \times 39 \pi = 26 \times 39 \pi \times m
$$

and similarly for $6n$.

Since:

$$
26 \times 39 = 1014 \\
6 \times 39 = 234
$$

then:

$$
\omega_{m,n} T_{Ze} = 1014 \pi m + 234 \pi n
$$

which are integer multiples of 2π, because:

$$
\frac{1014 \pi m}{2\pi} = 507 m \quad \text{(integer)} \\
\frac{234 \pi n}{2\pi} = 117 n \quad \text{(integer)}
$$

Therefore,

\[

Frequently Asked Questions

What is the significance of finding the period of the Ze function in the given problem?

Determining the period of the Ze function helps understand its repetitive behavior over intervals, which is essential in analyzing its properties and applications in periodic systems.

How do you interpret the given function H[n] = 8) + 26m - 1) + 28 - 2 + 6n - 3)?

The function H[n] appears to be a combination of terms involving variables m and n, possibly representing a sequence or a discrete function. Clarifying the notation is important for analysis.

What methods are commonly used to find the period of a discrete function like Ze?

Methods include analyzing the function's algebraic form, using Fourier analysis for periodic components, or applying difference equations and their solutions to identify repeating intervals.

Can the periodicity of Ze be shown mathematically from the given expression?

Yes, by expressing Ze in terms of known periodic functions or identifying a common difference that repeats after a certain interval, one can mathematically prove its periodicity.

What role does the function H[n] play in demonstrating the periodicity of Ze?

H[n] may serve as an auxiliary function or part of the analysis to relate Ze to known periodic functions, aiding in establishing the period through algebraic or functional relations.

Are there specific properties of the sequence components (like 8, 26m, 1, 28, 2, 6n, 3) that influence the period of Ze?

Yes, properties such as linearity, common differences, or divisibility among these components can determine the overall periodicity and help identify the fundamental period.

What is the significance of showing that Ze is periodic in the

context of signal processing?

Establishing the periodicity of Ze is crucial in signal processing for analyzing signals, designing filters, and understanding harmonic content and spectral properties.

How does the notation H[H] = 8 + 26m - 1 + 28 - 2 + 6n - 3 relate to finding Ze's period?

This notation suggests a recursive or layered approach to defining H, which might be used to derive the periodicity of Ze by examining how these terms repeat or relate over intervals.

What are the typical steps to show that a function like Ze has a specific period?

Steps include expressing Ze explicitly, identifying the fundamental components, testing for periodicity by shifting the variable, and verifying if the function repeats over that interval, thus establishing the period.

Additional Resources

02) A) Find The Period Of Ze Given By S LT 137 FindH) For Hin] =8) +26m-1)+28-2+6n-3) And Show That The

In the realm of signal processing and mathematical analysis, understanding the periodicity of a given function is fundamental. It provides insights into the behavior of signals over time, their repetitive nature, and their potential applications across communications, control systems, and electronics. Today, we delve into a specific problem that involves determining the period of a function Ze, which is defined through a mathematical expression involving variables m and n. The task is to identify the period of Ze given by the expression S LT 137 FindH), with the function H(n) specified as H(n) = 8 + 26m - 1 + 28 - 2 + 6n - 3. Additionally, we aim to demonstrate the properties that establish this periodicity.

This analysis is not only an exercise in mathematical rigor but also a practical guide for engineers and scientists working with periodic signals. By the end of this discussion, readers will understand the step-by-step approach to identifying periods in complex functions and appreciate the underlying principles that govern periodicity in discrete and continuous signals alike.

Understanding the Problem Statement

The initial step involves clarifying what is given and what needs to be found:

- Given: A function Ze, which appears to be associated with the expression S LT 137 FindH) — likely a notation or shorthand for a specific function or signal, possibly involving the variable n or m.
- Function H(n): Defined as H(n) = 8 + 26m - 1 + 28 - 2 + 6n - 3.
- Objective: To find the period of Ze, which might be influenced by H(n), and then demonstrate the properties that establish this periodicity.

Given the complexity and the notation, it's essential to interpret the components correctly:

- The expression H(n) seems to be a linear combination involving variables m and n.
- The constants and coefficients indicate that the function may be periodic if certain conditions are met, especially in the context of integer-valued functions or signals.

Dissecting the Function H(n)

Let's analyze H(n) carefully:

H(n) = 8 + 26m - 1 + 28 - 2 + 6n - 3

To simplify and understand its structure, combine like terms:

1. Combine constant terms:
- 8 - 1 + 28 - 2 - 3
- (8 - 1) = 7
- 7 + 28 = 35
- 35 - 2 = 33
- 33 - 3 = 30

2. The terms involving m and n are:
- 26m (assuming m is an integer or parameter)
- 6n

Thus, the simplified form of H(n) is:

H(n) = 30 + 26m + 6n

This form indicates that H(n) is a linear function involving m and n, with coefficients 26 and 6, respectively.

Establishing the Periodicity: Key Concepts

To find the period of Ze (which is related to H(n)), it is essential to revisit the concepts of periodic functions:

- A periodic function f(x) satisfies:

f(x + T) = f(x) for all x in its domain, where T ≠ 0 is the period.

- For functions involving discrete variables like n and m, the periodicity often relates to the least common multiple (LCM) of the individual periods of the constituent terms.

- The periodicity of a linear function involving modular components depends on the coefficients and whether these coefficients are rationally related.

In the context of such functions, the key is to determine the smallest positive integer T such that:

f(n + T) = f(n) for all n, and similarly for m if m varies.

Determining the Period of Ze

Given that Ze is associated with the function H(n) and potentially with the variable n, the core question reduces to:

What is the smallest positive T such that H(n + T) ≡ H(n) (modulo some periodicity criteria)?

Given H(n) = 30 + 26m + 6n, consider the change when n increases by T:

H(n + T) = 30 + 26m + 6(n + T) = 30 + 26m + 6n + 6T

For H(n + T) to be equivalent to H(n) modulo some integer (say, in the context of signals modulated over a cycle), the difference:

H(n + T) - H(n) = 6T

must be congruent to 0 modulo the fundamental period. Typically, in discrete signals, the period T is the smallest positive integer satisfying:

6T ≡ 0 mod P

where P is the modulus, often the fundamental period of the signal.

Assumption: If the function is considered over the integers without a specified modulus, then the periodicity hinges on the coefficients' divisibility.

In particular, for the function to be periodic in n:

- The change in H(n) when n increases by T should be zero modulo the domain's periodicity constraints.

Since 6T determines the change:

- The minimal T such that 6T is an integer multiple of the underlying period is T = P/ gcd(6, P), where gcd is the greatest common divisor.

If the function is considered over the integers (no modulus specified), the period is generally infinite unless additional periodicity constraints are provided.

Connecting to Ze and Its Period

Given the initial problem statement, the function Ze might be a component of a more extensive signal or function that incorporates H(n). If Ze is a discrete-time signal defined as:

Ze(n) = some function involving H(n)

then, to find its period, we analyze the fundamental periodicity introduced by H(n).

Suppose Ze(n) includes a term like cos(ω n + φ), where ω is related to coefficients like 6 in H(n). The period T of such a cosine function is:

$T = 2\pi / \omega$

If the signal is discrete and ω is a rational multiple of π, then T becomes an integer.

Summarizing the Period Determination

Based on the linear form H(n) = 30 + 26m + 6n:

- The periodicity in n is determined by the coefficient of n, which is 6.
- The minimal period T satisfies:

T = (constant) / gcd(constant, coefficient)

- Since the coefficient of n is 6, the fundamental period T in integer terms is:

$T = 2\pi / (6)$ (if relating to frequency)

or, in discrete terms, the smallest T such that:

$6T \equiv 0 \mod P$

- Without an explicit modulus P, the function's periodicity depends on the context of the signal.

Therefore, the period of Ze, as derived from the analysis, is primarily influenced by the coefficient 6 in H(n), leading to a fundamental period $T = 2\pi / 6 = \pi / 3$ in the continuous case, or T = 6 in the discrete case once scaled appropriately.

Demonstrating the Periodicity: Formal Proof

To demonstrate that Ze exhibits periodic behavior, consider the following:

1. Invariance under shifts:

Ze(n + T) = Ze(n) implies:

H(n + T) ≡ H(n) mod some integer (depending on the context).

2. Substituting H(n + T):

H(n + T) = 30 + 26m + 6(n + T) = H(n) + 6T

3. Condition for periodicity:

For Ze(n + T) = Ze(n):

6T must be an integer multiple of the fundamental period (say, P).

- If P is large enough to encompass all possible values, then T must satisfy:

$6T \equiv 0 \mod P$

- In the simplest case, for functions over integers with no modulus, T must be such that 6T = 0, which implies T = 0.

4. Conclusion:

To have a meaningful period, the function Ze must be considered in a context where the coefficients induce periodicity—such as in signals with inherent periodic components.

Hence, the period T is primarily determined by the coefficient 6 in H(n), leading to a fundamental period of T = 6 in discrete units, or T = π/3 in continuous units, depending on the application.

Practical Implications and Applications

Understanding the period of Ze is not merely an academic exercise; it has tangible implications in various fields:

- Signal Design: Knowledge of the period helps in designing signals that are periodic over desired intervals, crucial in communication systems.
- Fourier Analysis: Periodicity enables decomposition into sinusoidal components, facilitating spectral analysis.
- System Stability: Recognizing periodic behavior assists in analyzing system responses and stability criteria.

Furthermore, the approach taken here—analyzing coefficient relationships and their influence on periodicity—is a cornerstone in the analysis of digital signals, control systems, and oscillatory phenomena.

Final Remarks

The process of determining the period of Ze, as derived from the function H(n) = 30 + 26m + 6n, underscores the importance of understanding linear relationships in signals. By

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26m 1 28 2 6n 3 And Show That The

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