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Introduction

Root locus analysis is a fundamental technique in control system design and stability analysis. It provides valuable insights into how the closed-loop poles of a system vary with changes in the system gain, K. By plotting the root locus of an open-loop transfer function, engineers can determine the stability, transient response, and steady-state behavior of control systems.

In this article, we will provide a comprehensive, step-by-step guide to obtaining the root locus plot for the given open-loop system:

$$ S + 3 G(s) = (s + 5)(s + 2)(s - 1) $$

where $G(s)$ represents the open-loop transfer function. We will explore the theoretical background, derive the necessary components, and demonstrate the plotting process, ensuring a thorough understanding suitable for students and professionals alike.

Understanding the System and Its Open-Loop Transfer Function

The Given System

The problem statement presents an open-loop transfer function that appears as:

$$ S + 3 G(s) = (s + 5)(s + 2)(s - 1) $$

However, in control systems, the typical notation involves an open-loop transfer function $G(s)$ and a gain parameter K. The standard form for root locus analysis is:

$$ 1 + K G(s) = 0 $$

Given the expression, it is likely that the system is described as:

$$ G(s) = \frac{(s + 5)(s + 2)(s - 1)}{S} $$

and the characteristic equation becomes:

$$ 1 + 3 G(s) = 0 $$

or equivalently,

$$ 1 + K G(s) = 0 $$

where $K = 3$. For clarity, we will assume the open-loop transfer function is:

$$ G(s) = \frac{(s + 5)(s + 2)(s - 1)}{S} $$

and the gain $K = 3$. Our goal is to analyze the root locus as K varies.

Clarifying the Transfer Function

The numerator of $G(s)$:

$$ N(s) = (s + 5)(s + 2)(s - 1) $$

expands to:

$$ N(s) = (s + 5)(s + 2)(s - 1) $$

which can be expanded stepwise:

- First, multiply $(s + 5)(s + 2)$:

$$ (s + 5)(s + 2) = s^2 + 2s + 5s + 10 = s^2 + 7s + 10 $$

- Then, multiply this by $(s - 1)$:

$$ (s^2 + 7s + 10)(s - 1) = s^3 - s^2 + 7s^2 - 7s + 10s - 10 $$

Simplify:

$$ s^3 + 6s^2 + 3s - 10 $$

Thus, the numerator:

$$ N(s) = s^3 + 6s^2 + 3s - 10 $$

The denominator needs clarification; it appears as 'S' in the original expression, but likely refers to the Laplace variable s. If the system is strictly a unity feedback with open-loop transfer function $G(s)$, then the overall open-loop transfer function is:

$$ G(s) = K \times \frac{N(s)}{D(s)} $$

where $D(s)$ is the denominator polynomial of the plant.

Assuming the open-loop transfer function is:

$$ G(s) = \frac{N(s)}{D(s)} $$

with

$$ N(s) = s^3 + 6s^2 + 3s - 10 $$

and for simplicity, assuming the denominator polynomial is 1 (i.e., pure numerator), then the open-loop transfer function is:

$$ G(s) = K \times N(s) $$

Alternatively, if there's a specific denominator, it should be specified. For the purpose of root locus, the typical form is:

$$ G(s) = \frac{N(s)}{D(s)} $$

and the characteristic equation:

$$ 1 + K \times G(s) = 0 $$

Step-by-Step Procedure to Obtain the Root Locus Plot

1. Identify the Poles and Zeros

The first step involves finding the poles and zeros of the open-loop transfer function $G(s)$.

- Zeros: Roots of numerator polynomial $N(s)$:

$$ N(s) = s^3 + 6s^2 + 3s - 10 $$

- Poles: Roots of denominator polynomial $D(s)$. If not specified, assume $D(s) = 1$, which implies no poles, but this is unlikely. Alternatively, if the system is a pure zero polynomial, the root locus is trivial.

Assuming the system's denominator is:

$$ D(s) = 1 $$

then the open-loop transfer function has zeros only at the roots of $N(s)$.

To proceed, find the zeros:

Solve:

$$ s^3 + 6s^2 + 3s - 10 = 0 $$

This cubic can be solved using methods like synthetic division or Cardano's formula. Let's approximate roots for illustrative purposes.

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2. Find the Zeros of the Numerator polynomial

Using Rational Root Theorem, possible rational roots are factors of constant term (-10) over factors of leading coefficient (1):

Possible roots: ±1, ±2, ±5, ±10

Test these:

- $s=1$:

$$ 1 + 6 + 3 - 10 = 0 $$

Yes! So, $s=1$ is a root.

Perform polynomial division to factor out $(s - 1)$:

Divide $s^3 + 6s^2 + 3s - 10$ by $(s - 1)$:

Coefficients: 1 | 6 | 3 | -10

Using synthetic division:

	1	6	3	-10
1		1	7	10

Bring down 1:

- Multiply 1 by 1, add to 6: 6 + 1 = 7

- Multiply 7 by 1, add to 3: 3 + 7 = 10

- Multiply 10 by 1, add to -10: -10 + 10 = 0

Remainder zero, so the quotient polynomial is:

$$ s^2 + 7s + 10 $$

Factor:

$$ s^2 + 7s + 10 = (s + 2)(s + 5) $$

Thus, zeros are at:

$$ s = 1, -2, -5 $$

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3. Poles of the System

In the absence of a specified denominator, the open-loop transfer function effectively has no poles, which would make the root locus trivial. However, in most practical systems, the plant has poles. Let's assume the plant has poles at:

$$ s = 0 \quad \text{(for example)} $$

or perhaps, for a more typical case, the denominator polynomial is:

$$ D(s) = (s + a)(s + b)(s + c) $$

Suppose the system's denominator is:

$$ D(s) = (s + 4)(s + 3)(s + 1) $$

This is common in control system analysis.

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4. Construct the Root Locus

Step 1: Number of Poles and Zeros

- Poles: 3 (from denominator)

- Zeros: 3 (from numerator zeros at $s = 1, -2, -5$)

Since the number of poles equals zeros, the root locus branches originate and terminate at zeros and poles, respectively, and the locus is symmetric.

Step 2: Plot Poles and Zeros

Plot the locations of the poles and zeros on the complex s-plane:

- Zeros at: $s=1, -2, -5$

- Poles at: $s=-4, -3, -1$

Step 3: Identify Asymptotes and Centroid

Number of asymptotes:

$$ N_{asymptotes} = \text{Number of poles} - \text{Number of zeros} = 3 - 3 = 0 $$

Since the poles and zeros are equal in number, there are no asymptotes, and the root locus branches connect poles to zeros directly.

Step 4: Breakaway and Break-in Points

Calculate points where the root locus branches break away from or break into the real axis, by solving:

$$ \frac{d}{ds} \left[1 + K G(s) \right] = 0 $$

or more practically by examining the real axis segments.

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5. Plotting the Root Locus

Using the above information, plot the root locus:

- Connect each pole to

Frequently Asked Questions

What are the steps to obtain the root locus plot for the open-loop transfer function G(s) = (s+5)(s+2)(s-1) / (s + 3)?

First, express the open-loop transfer function in standard form, identify poles and zeros, find the root locus segments based on the pole-zero configuration, determine asymptotes and centroid, and then plot the locus using these characteristics. This involves calculating angles and distances from poles and zeros to points on the real axis, and analyzing how the roots move as the gain varies.

How do the poles and zeros of G(s) = (s+5)(s+2)(s-1) / (s + 3) influence the shape of the root locus?

The poles at s = -3, -5, -2, and the zero at s = 1 determine the root locus branches. Poles are starting points for the locus, zeros are ending points, and the locus branches originate from poles and tend toward zeros or infinity. Their locations influence the curvature, the asymptotes, and the regions where the system stability changes.

What is the significance of the asymptotes in the root locus of this system?

Asymptotes indicate the directions in which the root locus branches tend to infinity as the gain increases. For the given system, the number of asymptotes equals the number of poles minus zeros, and their angles and centroid help understand the system's stability boundaries and the behavior of roots at high gain.

How can the gain K be varied to visualize the root locus for the given system?

The gain K can be varied from zero to infinity, and for each value, the roots of 1 + K G(s) = 0 are computed. Plotting these roots in the complex plane creates the root locus, showing how the system poles move with changing gain, which helps in stability analysis and controller design.

What tools or software can be used to plot the root locus for G(s) = (s+5)(s+2)(s-1) / (s + 3)?

Popular tools include MATLAB with its 'rlocus' command, Python with control systems libraries like 'python-control', or online root locus plotters. These tools automate calculations and provide accurate, visual representations of the root locus for complex transfer functions.

Additional Resources

Root Locus Plot for the Open-Loop System: An In-Depth Analysis of S + 3 G(s) = (s + 5)(s + 2)(s - 1)

Introduction

Understanding the dynamic behavior of control systems is fundamental in engineering, especially when designing systems that are stable, responsive, and robust. Among various analytical tools, the Root Locus method stands out for its intuitive visualization of how system poles migrate in the complex plane as system gain varies. This article provides a comprehensive exploration of the process to obtain and interpret the root locus plot for a specific open-loop transfer function: S + 3 G(s) = (s + 5)(s + 2)(s - 1). Through detailed explanations, mathematical derivations, and practical insights, we aim to elucidate the significance of root locus analysis in control system design.

Understanding the System: Deriving the Open-Loop Transfer Function

The Given System Equation

The problem statement indicates an open-loop transfer function associated with the relationship:

$$ S + 3 G(s) = (s + 5)(s + 2)(s - 1) $$

At first glance, this expression appears to combine the Laplace variable s, a constant 3, and the transfer function $G(s)$. However, for control system analysis, it is customary to focus on the open-loop transfer function $L(s)$, which combines the plant $G(s)$ and the controller or gain factor.

Clarifying the Transfer Function

Given the form, it is reasonable to interpret the expression as:

$$ 1 + L(s) = 0 \quad \text{where} \quad L(s) = K \cdot G(s) $$

and the characteristic equation of the closed-loop system is:

$$ 1 + K G(s) = 0 $$

From the given, the numerator of $G(s)$ is $(s + 5)(s + 2)(s - 1)$, and the gain K is associated with the system via the constant 3.

Thus, the open-loop transfer function can be expressed as:

$$ L(s) = K \cdot \frac{(s + 5)(s + 2)(s - 1)}{1} $$

Given the presence of $S + 3 G(s)$, and the coefficient 3, it suggests that the overall open-loop transfer function is:

$$ L(s) = 3 \cdot \frac{(s + 5)(s + 2)(s - 1)}{1} $$

which simplifies to:

$$ L(s) = 3 (s + 5)(s + 2)(s - 1) $$

Final Open-Loop Transfer Function

Therefore, the key transfer function for root locus analysis is:

$$ L(s) = 3 (s + 5)(s + 2)(s - 1) $$

The characteristic equation for the closed-loop system becomes:

$$ 1 + L(s) = 0 $$

or

$$ 1 + 3 (s + 5)(s + 2)(s - 1) = 0 $$

which forms the basis for plotting the root locus.

Step-by-Step Approach to Obtain the Root Locus

1. Expressing the Open-Loop Transfer Function

The initial step involves expressing the transfer function in a standard form:

$$ L(s) = K \frac{\prod_{i=1}^{n} (s + z_i)}{\prod_{j=1}^{m} (s + p_j)} $$

In our case, the numerator is $(s + 5)(s + 2)(s - 1)$, and the denominator is unity (since it's a pure polynomial). The gain K is the variable parameter influencing pole locations.

2. Identifying Poles and Zeros

- Zeros: The roots of the numerator, where the transfer function equals zero.

- Zeros at: $s = -5, -2, 1$

- Poles: The roots of the denominator. Since the transfer function is a polynomial, it has no poles in the finite plane (or poles at infinity). But for system stability analysis, we consider the roots of the characteristic equation:

$$ 1 + L(s) = 0 $$

which introduce poles at:

$$ 1 + 3(s + 5)(s + 2)(s - 1) = 0 $$

So, the system's poles depend on the roots of this characteristic polynomial.

3. Constructing the Characteristic Equation

The characteristic equation becomes:

$$ 1 + 3(s + 5)(s + 2)(s - 1) = 0 $$

Expanding the polynomial:

\[
\begin{aligned}
(s + 5)(s + 2)(s - 1) &= (s + 5)((s + 2)(s - 1)) \\
&= (s + 5)(s^2 + s - 2) \\
&= s(s^2 + s - 2) + 5(s^2 + s - 2) \\

&= s^3 + s^2 - 2s + 5s^2 + 5s - 10 \\
&= s^3 + (s^2 + 5s^2) + (-2s + 5s) - 10 \\
&= s^3 + 6s^2 + 3s - 10
\end{aligned}
\]

Multiplying by 3:

\[3(s^3 + 6s^2 + 3s - 10) = 3s^3 + 18s^2 + 9s - 30 \]

Adding 1:

\[1 + 3(s + 5)(s + 2)(s - 1) = 1 + 3s^3 + 18s^2 + 9s - 30 = 0 \]

Simplify:

\[3s^3 + 18s^2 + 9s - 29 = 0 \]

This is the characteristic polynomial governing the pole locations.

4. Plotting the Root Locus

The root locus plot is constructed based on the locations of the roots of the characteristic polynomial as the gain \(K\) varies from zero to infinity.

Analytical Construction of the Root Locus

1. Number of Poles and Zeros

- Poles: Roots of the characteristic polynomial; here, a cubic polynomial indicating 3 poles.

- Zeros: Roots of the numerator polynomial \(L(s)\), at \(s = -5, -2, 1\).

The root locus will have:

- 3 branches originating from the poles at the roots of the characteristic polynomial.
- The zeros at \(s = -5, -2, 1\).

2. Asymptotes and Their Angles

Since the number of poles is 3, and zeros are 3, the root locus branches tend to finite zeros or infinity.

- Number of asymptotes:

\[
\text{Number of asymptotes} = |n - m| = 3 - 3 = 0

\]

indicating that the branches tend toward finite zeros or infinity, but since they are equal, the asymptotes are not prominent in this case.

3. Breakaway and Break-in Points

These are points on the real axis where branches of the root locus either diverge or converge, calculable via the derivative of the characteristic equation.

4. Real-Axis Segments

Segments of the real axis where the root locus exists are determined by the rule:

- The locus exists on the real axis to the left of an odd number of poles and zeros.

Practical Tools and Software for Plotting

While analytical techniques give insights, plotting the root locus accurately often involves computational tools like MATLAB's `rlocus` function or Python libraries such as `control` or `matplotlib`. These tools facilitate visualizing the pole migration as K varies, helping engineers validate system stability and transient response characteristics.

Interpretation of the Root Locus

Stability Analysis

- The system's stability hinges on whether all closed-loop poles reside in the left-half of the complex plane.
- As gain K increases, pole locations shift along the root locus branches.
- The crossing of poles into the right-half plane indicates potential instability at certain gain levels.

Transient Response

- The proximity of poles to the imaginary axis influences overshoot and settling time.
- Poles closer to the imaginary axis produce slower, more oscillatory responses.

Design Implications

- By adjusting the gain K, control engineers can place poles to achieve desired transient performance.
- The root locus guides the selection of K to ensure system stability while meeting performance specifications.

Conclusion

The process of obtaining and analyzing the root locus for the system $L(s) = 3 (s + 5)(s + 2)(s - 1)$ involves a systematic approach: deriving the characteristic polynomial, identifying poles and zeros, and understanding the locus' geometric behavior as gain varies. This analysis provides vital insights into the stability margins and transient response of the control system, serving as an indispensable tool for control engineers.

In practice, the combination of analytical

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