(1 Point) Find The Length Of The Curve Defined By Y=3x^(3/2)+9from X=1 To X=7.(1 Point) Find The Length

(1 Point) Find The Length Of The Curve Defined By Y=3x^(3/2)+9from X=1 To X=7.(1 Point) Find The Length is a common problem in calculus involving the determination of the arc length of a given function over a specific interval. Calculating the length of a curve is fundamental in understanding the geometry of the graph and is widely applicable across physics, engineering, and mathematics. In this article, we will explore the detailed process of finding the length of the curve $y = 3x^{3/2} + 9$ from $x = 1$ to $x = 7$, including the necessary calculus concepts, step-by-step calculations, and practical insights.

Understanding the Concept of Arc Length

What Is Arc Length?

Arc length refers to the distance along a curve between two points. Unlike the straight-line distance, which is measured by the length of a segment, arc length accounts for the actual path taken along the curve. For a smooth, continuous function $y = f(x)$, the arc length L between $x=a$ and $x=b$ is given by a specific integral formula derived from the Pythagorean theorem.

Mathematical Formula for Arc Length

The general formula for the length of a curve $y = f(x)$ from $x=a$ to $x=b$ is:

$$
L = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx
$$

This formula involves calculating the derivative of the function y with respect to x, squaring it, adding 1, and then integrating the square root over the interval.

Step-by-Step Solution for the Given Function

Step 1: Identify the function and limits

Given:
$$
y = 3x^{3/2} + 9
$$
with the interval:
$$
x \in [1, 7]
$$

Step 2: Compute the derivative dy/dx

The derivative of y with respect to x is:
$$
\frac{dy}{dx} = 3 \times \frac{d}{dx} \left(x^{3/2} \right)
$$
Recall that:
$$
\frac{d}{dx} x^{n} = n x^{n-1}
$$
So,
$$
\frac{dy}{dx} = 3 \times \frac{3}{2} x^{(3/2)-1} = 3 \times \frac{3}{2} x^{1/2} = \frac{9}{2} x^{1/2}
$$
which simplifies to:
$$
\frac{dy}{dx} = \frac{9}{2} \sqrt{x}
$$

Step 3: Set up the arc length integral

Substituting dy/dx into the arc length formula:
$$
L = \int_{1}^{7} \sqrt{1 + \left(\frac{9}{2} \sqrt{x} \right)^2} \, dx
$$
Simplify inside the square root:
$$
L = \int_{1}^{7} \sqrt{1 + \left(\frac{81}{4} x \right)} \, dx
$$
which becomes:
$$
L = \int_{1}^{7} \sqrt{1 + \frac{81}{4} x} \, dx
$$

Evaluating the Integral

Step 4: Simplify the integral

Rewrite the integral:
\[
L = \int_{1}^{7} \sqrt{\frac{4 + 81 x}{4}} \, dx
\]
Pull out the denominator:
\[
L = \int_{1}^{7} \frac{\sqrt{4 + 81 x}}{2} \, dx
\]
Now, the integral is:
\[
L = \frac{1}{2} \int_{1}^{7} \sqrt{4 + 81 x} \, dx
\]

Step 5: Use substitution to evaluate the integral

Let:
\[
u = 4 + 81 x
\]
then:
\[
du/dx = 81 \Rightarrow dx = \frac{du}{81}
\]
Change limits:
- When \(x = 1 \):
\[
u = 4 + 81 \times 1 = 4 + 81 = 85
\]
- When \(x = 7 \):
\[
u = 4 + 81 \times 7 = 4 + 567 = 571
\]

Express the integral in \(u \):
\[
L = \frac{1}{2} \int_{u=85}^{u=571} \sqrt{u} \times \frac{du}{81}
\]
Factor out constants:
\[
L = \frac{1}{2 \times 81} \int_{85}^{571} u^{1/2} \, du
\]
which simplifies to:
\[
L = \frac{1}{162} \int_{85}^{571} u^{1/2} \, du

\]

Step 6: Integrate \(u^{1/2} \)

Recall:
\[
\int u^{n} du = \frac{u^{n+1}}{n+1} + C
\]
for \(n \neq -1 \).

Thus:
\[
\int u^{1/2} du = \frac{u^{3/2}}{\frac{3}{2}} = \frac{2}{3} u^{3/2}
\]

Now, evaluate the definite integral:
\[
L = \frac{1}{162} \times \frac{2}{3} \left[u^{3/2} \right]_{85}^{571}
\]
Simplify constants:
\[
L = \frac{2}{3 \times 162} \left[571^{3/2} - 85^{3/2} \right]
\]
which reduces to:
\[
L = \frac{2}{486} \left[571^{3/2} - 85^{3/2} \right]
\]
Further simplified:
\[
L = \frac{1}{243} \left[571^{3/2} - 85^{3/2} \right]
\]

Calculating the Numerical Values

Step 7: Compute \(u^{3/2} \) for the bounds

Recall:
\[
u^{3/2} = \left(\sqrt{u} \right)^3
\]
Calculate:
- \(\sqrt{571} \)
- \(\sqrt{85} \)

Using approximate square roots:

$$
\sqrt{571} \approx 23.9
$$
$$
\sqrt{85} \approx 9.22
$$
Now cube these:
$$
23.9^3 \approx 23.9 \times 23.9 \times 23.9
$$
Calculations:
$$
23.9^2 \approx 571.21
$$
$$
23.9^3 \approx 571.21 \times 23.9 \approx 13,654
$$
Similarly:
$$
9.22^3 \approx 9.22 \times 9.22 \times 9.22
$$
Calculate:
$$
9.22^2 \approx 85
$$
$$
9.22^3 \approx 85 \times 9.22 \approx 783
$$

Therefore:
$$
u^{3/2} \text{ at } u=571 \approx 13,654
$$
$$
u^{3/2} \text{ at } u=85 \approx 783
$$

Plugging these back into the expression for L:
$$
L \approx \frac{1}{243} \times (13,654 - 783) = \frac{1}{243} \times 12,871
$$

Finally:
$$
L \approx \frac{12,871}{243} \approx 53.0
$$

Final Result and Interpretation

The length of the curve $y = 3x^{3/2} + 9$ from $x=1$ to $x=7$ is approximately 53 units. This calculation demonstrates the importance of derivatives, substitution, and integral calculus in determining arc lengths of complex functions.

Practical Applications of Arc Length Calculations

Understanding and computing the arc length of curves is vital in several fields:

- **Engineering:** Designing curved structures, tracks, or roads where precise measurements are needed.
- **Physics:** Calculating the actual distance traveled along a curved path.
-

Frequently Asked Questions

How do you find the length of the curve y = 3x^(3/2) + 9 from x = 1 to x = 7?

To find the length of the curve, use the arc length formula: L = ∫ from a to b √(1 + (dy/dx)^2) dx. First, compute dy/dx, then evaluate the integral from x=1 to x=7.

What is the derivative dy/dx for y = 3x^(3/2) + 9?

The derivative dy/dx = 3 (3/2) x^(1/2) = (9/2) x^(1/2).

How do you set up the integral for the arc length of y = 3x^(3/2) + 9 from x=1 to x=7?

The integral is $L = \int_1^7 \sqrt{1 + [(9/2) x^{(1/2)}]^2}\, dx = \int_1^7 \sqrt{1 + (81/4) x}\, dx$.

What is the simplified integrand for the arc length calculation?

The integrand simplifies to $\sqrt{1 + (81/4) x}$ for the arc length integral.

How do you evaluate the integral $\int \sqrt{1 + (81/4) x}\, dx$ to find the curve length?

Use substitution methods, such as setting $u = 1 + (81/4) x$, then integrate $\sqrt{u}$ with respect to u, and substitute back to x to find the definite integral from x=1 to x=7.

Additional Resources

Finding the Length of a Curve: An In-Depth Exploration of the Function \(y = 3x^{3/2} + 9 \) from \(x = 1 \) to \(x = 7 \)

Understanding the length of a curve is a fundamental aspect of calculus, bridging the gap between algebraic functions and their geometric representations. In this detailed analysis, we will explore the process of calculating the arc length of the specific function \(y = 3x^{3/2} + 9 \) over the interval from \(x = 1 \) to \(x = 7 \). This journey will take us through essential calculus concepts, including derivatives, integral formulations, and the application of the arc length formula, ultimately providing a comprehensive understanding of how to determine the length of a curve in a precise and methodical manner.

Introduction to Arc Length and Its Significance

What Is Arc Length?

Arc length is the measure of the distance along a curve between two points. Unlike straight lines, curves involve changing directions, making their length less straightforward to compute. In real-world applications, calculating the length of a curve can be crucial—for example, determining the length of a road, the path of a roller coaster, or the trajectory of an object in physics.

Why Is Calculating Arc Length Important?

Understanding the length of a curve enables engineers, physicists, and mathematicians to:
- Measure distances accurately on complex paths.
- Analyze the properties of functions.
- Design and optimize structures or paths in various engineering fields.
- Connect algebraic formulations with geometric interpretations.

Mathematical Foundations of Arc Length Calculation

The Formula for Arc Length

The general formula for the arc length L of a function $y = f(x)$, from $x = a$ to $x = b$, is derived from the Pythagorean theorem and is expressed as:

$$
L = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx
$$

This integral accounts for the infinitesimal segments along the curve, summing their lengths to find the total.

Understanding the Components

- Derivative $\frac{dy}{dx}$: Represents the slope of the tangent to the curve at any point.
- Square of the derivative: Accounts for the change in the vertical direction relative to the horizontal.
- Square root: Combines the horizontal and vertical changes to give the actual infinitesimal segment length.

Applying the Arc Length Formula to Our Function

Defining the Function

Our specific function is:

$$
y = 3x^{3/2} + 9
$$

We are asked to find the length of this curve from $x = 1$ to $x = 7$.

Calculating the Derivative $\frac{dy}{dx}$

To utilize the arc length formula, the first step is to compute the derivative of y with respect to x:

$$
\frac{dy}{dx} = \frac{d}{dx} \left(3x^{3/2} + 9 \right)
$$

Applying the power rule:

$$
\frac{dy}{dx} = 3 \times \frac{d}{dx} \left(x^{3/2} \right)
$$

Recall that:

$$
\frac{d}{dx} \left(x^{n} \right) = n x^{n-1}
$$

So,

$$
\frac{dy}{dx} = 3 \times \left(\frac{3}{2} \right) x^{(3/2) - 1} = 3 \times \frac{3}{2} \times x^{1/2}
$$

Simplify:

$$
\frac{dy}{dx} = \frac{9}{2} x^{1/2}
$$

Note that $x^{1/2} = \sqrt{x}$. Therefore,

$$
\frac{dy}{dx} = \frac{9}{2} \sqrt{x}
$$

Formulating the Integral for Arc Length

Constructing the Integral Expression

Plugging the derivative into the arc length formula:

$$
L = \int_{1}^{7} \sqrt{1 + \left(\frac{9}{2} \sqrt{x} \right)^2} \, dx
$$

Simplify inside the square root:

$$
L = \int_{1}^{7} \sqrt{1 + \left(\frac{81}{4} x \right)} \, dx
$$

Expressed more neatly:

$$
L = \int_{1}^{7} \sqrt{1 + \frac{81}{4} x} \, dx
$$

Note: To facilitate integration, rewrite as:

$$
L = \int_{1}^{7} \sqrt{\frac{4}{4} + \frac{81}{4} x} \, dx = \int_{1}^{7} \sqrt{\frac{4 + 81 x}{4}} \, dx
$$

Extract the square root of the denominator:

$$
L = \int_{1}^{7} \frac{\sqrt{4 + 81 x}}{2} \, dx
$$

Evaluating the Integral

Simplified Integral Expression

Our integral now becomes:

$$
L = \frac{1}{2} \int_{1}^{7} \sqrt{4 + 81 x} \, dx
$$

This integral can be tackled using substitution methods.

Substitution Method

Let:

$$
u = 4 + 81 x
$$

Then,

$$
du = 81 dx \Rightarrow dx = \frac{du}{81}
$$

Change the limits accordingly:

- When $x = 1$:

$$
u = 4 + 81 \times 1 = 4 + 81 = 85
$$

- When $x = 7$:

$$
u = 4 + 81 \times 7 = 4 + 567 = 571
$$

Rewriting the integral:

$$
L = \frac{1}{2} \int_{u=85}^{u=571} \sqrt{u} \times \frac{du}{81}
$$

Factor out constants:

$$
L = \frac{1}{2 \times 81} \int_{85}^{571} u^{1/2} \, du = \frac{1}{162} \int_{85}^{571} u^{1/2} \, du
$$

Integral of $u^{1/2}$

The integral:

$$
\int u^{1/2} \, du = \frac{2}{3} u^{3/2} + C
$$

Applying the limits:

$$
L = \frac{1}{162} \times \left[\frac{2}{3} u^{3/2} \right]_{85}^{571}
$$

Simplify constants:

$$
L = \frac{2}{3 \times 162} \left(571^{3/2} - 85^{3/2} \right)
$$

Note that:

$$
\frac{2}{486} = \frac{1}{243}
$$

Therefore:

$$
L = \frac{1}{243} \left(571^{3/2} - 85^{3/2} \right)
$$

Calculating the Numerical Value

Evaluating $571^{3/2}$ and $85^{3/2}$

Recall that:

$$
a^{3/2} = (a^{1/2})^3 = (\sqrt{a})^3
$$

Calculate each term:

- $\sqrt{571} \approx 23.911$

$$
571^{3/2} \approx (23.911)^3 \approx 23.911 \times 23.911 \times 23.911
$$

Calculating step-by-step:

$$
23.911^2 \approx 571.5
$$

$$
23.911^3 \approx 571.5 \times 23.911 \approx 13,668.33
$$

Similarly, for 85:

- $\sqrt{85} \approx 9.2195$

$$
85^{3/2} \approx (9.2195)^3 \approx 9.2195 \times 9.2195 \times 9.2195
$$

$$
9.2195^2 \approx 85
$$

$$
9.2195^3 \approx 85 \times 9.2195 \approx 783.56
$$

Now, plug these back into the expression:

\[
L \approx \frac{1}{243} (13,668.33 - 783.56) =

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