(1 Point) The Planes 5x + 3y + 5z = -19 And 2z - 5y = 17 Are Not Parallel, So They Must Intersect Along

(1 Point) The Planes 5x + 3y + 5z = -19 And 2z - 5y = 17 Are Not Parallel, So They Must Intersect Along

Understanding the intersection of planes is a fundamental concept in three-dimensional geometry. When dealing with planes in space, determining whether they are parallel, intersecting, or coincident provides insight into their spatial relationships. In this article, we will analyze the specific planes given by the equations:

- Plane 1: $5x + 3y + 5z = -19$
- Plane 2: $2z - 5y = 17$

We will explore why these planes are not parallel and what their intersection looks like. The goal is to understand how to find the line of intersection between two non-parallel planes, interpret their geometric relationship, and apply this knowledge to related problems.

Understanding Planes in Three-Dimensional Space

The General Equation of a Plane

In three-dimensional Cartesian coordinates, a plane can be represented by a linear equation of the form:

$$ax + by + cz + d = 0$$

where
- a, b, and c are the coefficients representing the normal vector to the plane,
- d is the constant term.

The normal vector, $\vec{n} = (a, b, c)$, is perpendicular to the surface of the plane.

Conditions for Parallelism, Coincidence, and Intersection

- Parallel Planes: Two planes are parallel if their normal vectors are scalar multiples of each other, and they are not coincident.
- Coincident Planes: Two planes are coincident if their equations are scalar multiples of each other.
- Intersecting Planes: Two planes intersect along a line if their normal vectors are not scalar multiples;

in this case, they are neither parallel nor coincident.

In our case, the planes are given as:

- Plane 1: $5x + 3y + 5z = -19$
- Plane 2: $2z - 5y = 17$

We will verify that they are not parallel by examining their normal vectors.

Analyzing the Planes: Are They Parallel?

Normal Vectors of the Given Planes

- For Plane 1: $5x + 3y + 5z = -19$

The normal vector is:

$$
\vec{n}_1 = (5, 3, 5)
$$

- For Plane 2: $2z - 5y = 17$

Rewrite as:

$$
0x - 5y + 2z = 17
$$

The normal vector is:

$$
\vec{n}_2 = (0, -5, 2)
$$

Checking for Parallelism

Two vectors are scalar multiples if there exists a scalar k such that:

$$
\vec{n}_1 = k \cdot \vec{n}_2
$$

Check each component:

- $5 = k \times 0 \Rightarrow 5 = 0 \Rightarrow$ Contradiction

Since the first component of $\vec{n}_2$ is zero, and the first component of $\vec{n}_1$ is 5, they cannot be scalar multiples. Therefore, the planes are not parallel.

This confirms that the planes must intersect along a line, which is the common intersection line of the two planes.

Finding the Line of Intersection

To find the line along which the planes intersect, we need to:

1. Solve the system of equations representing the planes.
2. Express the solution as a parametric equation of a line.

Step 1: Write the system of equations

$$
\begin{cases}
5x + 3y + 5z = -19 \quad \text{(Equation 1)} \\
2z - 5y = 17 \quad \text{(Equation 2)}
\end{cases}
$$

Step 2: Express variables in terms of a parameter

Choose a variable to parameterize. Let’s choose y as the free parameter, say:

$$
y = t
$$

Now, solve Equation 2 for z:

$$
2z - 5t = 17
$$
$$
2z = 17 + 5t

$$
z = \frac{17 + 5t}{2}
$$

Next, substitute $y = t$ and z into Equation 1:

$$
5x + 3t + 5 \left(\frac{17 + 5t}{2} \right) = -19
$$

Multiply out:

$$
5x + 3t + \frac{5 \times (17 + 5t)}{2} = -19
$$

$$
5x + 3t + \frac{85 + 25t}{2} = -19
$$

Multiply both sides by 2 to clear the denominator:

$$
2 \times 5x + 2 \times 3t + 85 + 25t = -38
$$

$$
10x + 6t + 85 + 25t = -38
$$

Combine like terms:

$$
10x + (6t + 25t) + 85 = -38
$$

$$
10x + 31t + 85 = -38
$$

Solve for x:

$$
10x = -38 - 85 - 31t
$$

$$
10x = -123 - 31t
$$

$$
x = \frac{-123 - 31t}{10}
$$

Step 3: Write the parametric equations of the line

Using the parameter t, the parametric equations are:

$$
\boxed{
\begin{cases}
x = \frac{-123 - 31t}{10} \\
y = t \\
z = \frac{17 + 5t}{2}
\end{cases}
}
$$

or, more compactly, the line of intersection can be expressed as:

$$
\boxed{
\begin{aligned}
x &= -12.3 - 3.1 t \\
y &= t \\
z &= 8.5 + 2.5 t
\end{aligned}
}
$$

where t is any real number.

Geometric Interpretation of the Line of Intersection

The line of intersection between the two planes is an infinite line passing through space, defined by the parametric equations derived above. This line contains all points (x, y, z) that satisfy both plane equations simultaneously.

This intersection line is significant because:

- It represents the set of all points lying on both planes.
- It helps in understanding the spatial relationship between the planes.
- It can serve as a foundation for solving more complex geometric problems involving planes in three dimensions.

Applications of Plane Intersections in Real Life

Understanding the intersection of planes has numerous practical applications in various fields, including:

- Engineering and Architecture: Designing structures requires understanding how different surfaces intersect.
- Computer Graphics: Rendering 3D scenes involves calculating intersections of objects.
- Geology: Modeling fault lines and strata involves analyzing intersecting planes.
- Robotics: Navigating in 3D space often involves understanding plane intersections for obstacle avoidance.

Summary and Key Takeaways

- Two planes in space are not parallel if their normal vectors are not scalar multiples.
- When planes are not parallel, they intersect along a line.
- To find the line of intersection:
- Solve the system of plane equations.
- Express variables in terms of a free parameter.
- Write the parametric equations of the line.
- The normal vectors of the planes in our example confirm they are not parallel, and the derived parametric equations define their intersection line.

Conclusion

The analysis of the planes given by \(5x + 3y + 5z = -19\) and \(2z - 5y = 17\) demonstrates a fundamental concept in three-dimensional geometry: intersecting planes. By examining their normal vectors, confirming they are not scalar multiples, and solving their system of equations, we find that they intersect along a specific line described parametrically.

Understanding such intersections is crucial in various scientific and engineering disciplines. It allows professionals to visualize complex spatial relationships, solve real-world problems, and design systems with precision.

Whether you are a student learning about geometry or a professional applying these concepts in practical scenarios, mastering the process of analyzing plane intersections is essential for a comprehensive understanding of three-dimensional space.

Meta Description: Discover how to determine and find the line of intersection between two non-parallel planes given by \(5x + 3y + 5z = -19\) and \(2z - 5y = 17\). Learn the step-by-step process, parametric equations, and geometric interpretation in this detailed guide.

Frequently Asked Questions

What does it mean when two planes are not parallel?

When two planes are not parallel, it means they will intersect along a line rather than remaining separate or coinciding.

How can we determine the line of intersection between two planes?

By solving their equations simultaneously, we can find the line of intersection, which is the set of points satisfying both plane equations.

Given the planes $5x + 3y + 5z = -19$ and $2z - 5y = 17$, how do we find their intersection?

We solve the system of equations to find the values of x, y, and z that satisfy both equations, which together define their line of intersection.

Are the planes $5x + 3y + 5z = -19$ and $2z - 5y = 17$ parallel?

No, they are not parallel because their normal vectors are not scalar multiples; hence, they must intersect along a line.

What is the significance of the normal vectors when analyzing plane intersection?

Normal vectors help determine whether planes are parallel, coincident, or intersecting; if their vectors are not scalar multiples, the planes intersect along a line.

Can two planes intersect along more than one line?

No, two planes in three-dimensional space can either be parallel (no intersection), coincident (infinite intersection), or intersect along a single straight line.

What is the general form of the line of intersection between two planes?

The line can be expressed parametrically by solving the system of equations, often resulting in equations for x, y, and z in terms of a parameter t.

Why is it important to verify whether two planes are parallel before finding their intersection?

Because parallel planes do not intersect, verifying their orientation helps avoid unnecessary calculations and confirms whether an intersection line exists.

Additional Resources

(1 Point) The Planes $5x + 3y + 5z = -19$ And $2z - 5y = 17$ Are Not Parallel, So They Must Intersect Along

Understanding the geometry of planes in three-dimensional space is fundamental in fields ranging from engineering and architecture to computer graphics and physics. When examining two planes, one of the key questions is whether they are parallel, intersecting, or coincident. This article explores the intriguing case where two planes are not parallel, leading to an inevitable intersection along a line. Specifically, we analyze the planes defined by the equations $5x + 3y + 5z = -19$ and $2z - 5y = 17$, demonstrating why they must intersect and what that intersection looks like in space.

Foundations of Plane Equations in Three-Dimensional Space

What Is a Plane Equation?

A plane in three-dimensional space is a flat, two-dimensional surface extending infinitely. Its algebraic representation is typically given by a linear equation:

$$Ax + By + Cz + D = 0$$

where (A, B, C) is the normal vector to the plane, a vector perpendicular to every line lying on the plane, and D is a scalar offset.

In our case, the two planes are described by:

1. $5x + 3y + 5z = -19$
2. $2z - 5y = 17$

which can be rewritten in the standard form as:

- Plane 1: $5x + 3y + 5z + 19 = 0$
- Plane 2: $0x - 5y + 2z - 17 = 0$

Notice that the second plane lacks an x-term, indicating it is aligned in a specific orientation in space.

Normal Vectors and Their Significance

The normal vector ($\vec{n}$) of a plane is crucial because it indicates the plane’s orientation. For each plane:

- Plane 1: $\vec{n}_1 = (5, 3, 5)$
- Plane 2: $\vec{n}_2 = (0, -5, 2)$

These vectors are essential in determining whether the planes are parallel, coincident, or intersecting.

Determining the Relationship Between the Planes

Are the Planes Parallel?

Two planes are parallel if their normal vectors are scalar multiples of each other, i.e.,

$$
\vec{n}_1 = k \vec{n}_2
$$

for some scalar k.

Checking the components:

$$
(5, 3, 5) \quad \text{and} \quad (0, -5, 2)
$$

- The x-components: 5 and 0 — not proportional.
- The y-components: 3 and -5 — not proportional.
- The z-components: 5 and 2 — not proportional.

Since no scalar k satisfies all three component ratios simultaneously, the planes are not parallel.

Are the Planes Coincident?

For planes to be coincident (the same plane), their equations must differ only by a scalar multiple. Since the normal vectors are not scalar multiples, the planes are not coincident.

Conclusion: The Planes Must Intersect

Given that the planes are neither parallel nor coincident, the geometric implication is clear: they must intersect along a line.

This intersection line is the common set of points satisfying both equations simultaneously.

Finding the Line of Intersection

To understand the nature of the intersection, we can find the parametric equations describing the line where the two planes meet.

Methodology

1. Express one variable in terms of a parameter.
2. Substitute into the other equations.
3. Solve systematically to find parametric equations for x, y, z.

Step-by-Step Solution

Step 1: Express z in terms of y from the second plane

From plane 2:

$$
2z - 5y = 17 \implies 2z = 17 + 5y \implies z = \frac{17 + 5y}{2}
$$

Step 2: Substitute z into the first plane

Plane 1:

$$
5x + 3y + 5z = -19
$$

Substituting z:

$$
5x + 3y + 5 \left(\frac{17 + 5y}{2} \right) = -19
$$

Multiply through by 2 to clear denominators:

$$
2 \times 5x + 2 \times 3y + 5 \times (17 + 5y) = 2 \times (-19)
$$
$$
10x + 6y + 85 + 25y = -38
$$

Combine like terms:

$$

10x + (6y + 25y) + 85 = -38
\]
\[
10x + 31y + 85 = -38
\]

Solve for \(x\):

\[
10x = -38 - 85 - 31y
\]
\[
10x = -123 - 31y
\]
\[
x = \frac{-123 - 31y}{10}
\]

Step 3: Parametrize using a free parameter

Let \(y = t \), where \(t \in \mathbb{R} \).

Then:

\[
x = \frac{-123 - 31t}{10}
\]
\[
z = \frac{17 + 5t}{2}
\]

Thus, the parametric equations of the line are:

\[
\boxed{
\begin{cases}
x = \displaystyle \frac{-123 - 31t}{10} \\
y = t \\
z = \displaystyle \frac{17 + 5t}{2}
\end{cases}
}
\]

for all real values of \(t \).

Step 4: Express as a vector form

Let’s write the line in vector form:

\[
\vec{r}(t) = \vec{r}_0 + t \vec{d}

\]

where \(\vec{r}_0\) is a point on the line, and \(\vec{d}\) is the direction vector.

Choose \(t = 0 \):

\[
x_0 = \frac{-123}{10} = -12.3, \quad y_0 = 0, \quad z_0 = \frac{17}{2} = 8.5
\]

Direction vector \(\vec{d}\):

\(\displaystyle \left(-\frac{31}{10}, 1, \frac{5}{2} \right) \)

or simplified:

\[
\vec{d} = \left(-3.1, 1, 2.5 \right)
\]

Therefore, the parametric form:

\[
\boxed{
\vec{r}(t) = \left(-12.3, 0, 8.5 \right) + t \left(-3.1, 1, 2.5 \right)
}
\]

This line is the intersection of the two planes, extending infinitely in both directions along the vector \(\vec{d}\).

Geometric Significance of the Intersection Line

Understanding this line's properties illuminates the spatial relationship between the planes.

Direction Vector and Orientation

The direction vector \(\vec{d}\) is perpendicular to both normal vectors \(\vec{n}_1\) and \(\vec{n}_2\), as expected for the intersection line. This is verified by their cross product.

Calculate:

\[
\vec{d} = \vec{n}_1 \times \vec{n}_2
\]

\[
\vec{d} =

\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
5 & 3 & 5 \\
0 & -5 & 2
\end{vmatrix}
\]

Compute determinants:

\[
\mathbf{i} \quad: (3)(2) - (5)(-5) = 6 + 25 = 31
\]
\[
\mathbf{j} \quad: - \left((5)(2) - (5)(0) \right) = - (10 - 0) = -10
\]
\[
\mathbf{k} \quad: (5)(-5) - (3)(0) = -25 - 0 = -25
\]

So,

\[
\vec{d} = (31, -10, -25)
\]

which differs from the earlier parametrization but is proportional, confirming the intersection line's direction aligns with the cross product of the normals.

Implications and Applications

Understanding how two planes intersect has practical implications across various domains.

Engineering and Structural Design

In architectural design, intersecting planes can represent the edges of complex structures, such as roofs, facades

[1 Point The Planes 5x 3y 5z 19 And 2z 5y 17 Are Not Parallel So They Must Intersect Along]

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