1. Sketch The Function F(x) = X^2 - 1. Show That The Graph Crosses The I-axis At X = 1. Use Integration

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Introduction

Understanding the behavior of a quadratic function such as $F(x) = x^2 - 1$ is fundamental in algebra and calculus. In this article, we will explore how to sketch the graph of the function, analyze its key features, and demonstrate how it intersects the x-axis at a specific point ($x = 1$) using integration techniques. This comprehensive guide will facilitate a deeper understanding of the relationship between functions, their graphs, and the role of integration in analyzing their properties.

Overview of Quadratic Functions

Quadratic functions are polynomial functions of degree two, generally expressed as $F(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is a parabola, which opens upwards if $a > 0$ and downwards if $a < 0$.

For our specific function:

- $F(x) = x^2 - 1$

the parabola opens upwards because the coefficient of x^2 (which is 1) is positive. The function shifts the basic parabola $y = x^2$ downward by 1 unit due to the "-1" term.

Step 1: Sketching the Graph of $F(x) = x^2 - 1$

Identifying Key Features

Before sketching, it's essential to determine some key points and features of the graph:

- Vertex: The lowest point of the parabola, given by the vertex formula.

- Axis of symmetry: The vertical line passing through the vertex.

- Intercepts: Points where the graph crosses the axes.

Finding the Vertex

Since $F(x) = x^2 - 1$ is a simple quadratic, the vertex occurs at the minimum point of the parabola. The vertex formula for a quadratic $ax^2 + bx + c$ is at $x = -b/(2a)$. Here, $b = 0$, so:

$$ x_{vertex} = -\frac{0}{2 \times 1} = 0 $$

Plugging $x = 0$ into the function:

$$ F(0) = 0^2 - 1 = -1 $$

Vertex point: (0, -1)

Finding Intercepts

- Y-intercept: At $x = 0$:

$$ F(0) = -1 $$

- X-intercepts: Points where $F(x) = 0$:

$$ x^2 - 1 = 0 $$
$$ x^2 = 1 $$
$$ x = \pm 1 $$

So, the graph crosses the x-axis at $x = -1$ and $x = 1$.

Step 2: Drawing the Sketch

Based on the above features:

- Plot the vertex at (0, -1).

- Plot the x-intercepts at (-1, 0) and (1, 0).

- Draw a smooth parabola passing through these points, opening upwards.

- The axis of symmetry is $x = 0$.

This sketch visually represents the quadratic function, highlighting the key points and symmetry.

Step 3: Demonstrating the Cross at $x = 1$ Using Integration

While the previous steps involve algebraic methods, integration offers a powerful tool to analyze the function's properties, such as the area under the curve, and can help verify points of intersection with axes.

Understanding the Role of Integration

Integration allows us to calculate the area under a curve between two points, which relates to the definite integral:

$$ \int_a^b F(x) \, dx $$

In this context, we will utilize integration to verify the x-intercept at $x = 1$, by examining the accumulated area from a starting point to $x = 1$ and analyzing when this area reaches zero, indicating the crossing point.

Step 4: Using Integration to Show the Graph Crosses the x-axis at x = 1

The Definite Integral of F(x)

To analyze the behavior of $F(x)$ between specific points, we compute:

$$ \int_{a}^{b} (x^2 - 1) \, dx $$

This integral calculates the net area between the curve and the x-axis from $x = a$ to $x = b$.

Calculating the Indefinite Integral

First, find the indefinite integral of $F(x)$:

$$ \int (x^2 - 1) \, dx = \frac{x^3}{3} - x + C $$

where C is the constant of integration.

Applying Limits to Find the Area

Suppose we want to find the area from $x = 0$ to $x = 1$:

$$ \int_{0}^{1} (x^2 - 1) \, dx = \left[\frac{x^3}{3} - x \right]_0^1 $$

Calculating at the bounds:

$$ \left(\frac{1^3}{3} - 1 \right) - \left(\frac{0^3}{3} - 0 \right) = \left(\frac{1}{3} - 1 \right) - 0 = -\frac{2}{3} $$

This negative value indicates that, from $x=0$ to $x=1$, the curve is below the x-axis, and the area (in magnitude) is $2/3$.

Similarly, for $x = -1$ to $x = 0$:

$$ \int_{-1}^{0} (x^2 - 1) \, dx = \left[\frac{x^3}{3} - x \right]_{-1}^0 $$
$$ = \left(0 - 0 \right) - \left(\frac{(-1)^3}{3} - (-1) \right) = 0 - \left(-\frac{1}{3} + 1 \right) = -\left(-\frac{1}{3} + 1 \right) = -\left(\frac{2}{3} \right) = -\frac{2}{3} $$

Again, negative area indicates the curve is below the x-axis between $x = -1$ and 0.

Verifying the Cross at x = 1

The key insight is that at $x = 1$, the function $F(x)$ equals zero:

$$ F(1) = 1^2 - 1 = 0 $$

This algebraic result confirms the graph crosses the x-axis at $x = 1$. Using integration, we can see that the net area from $x = 0$ to $x = 1$ is negative, indicating the curve is below the x-axis and crossing it at $x = 1$.

Alternatively, by solving the integral for the area from $x = -1$ to $x = 1$:

$$ \int_{-1}^{1} (x^2 - 1) \, dx = \left[\frac{x^3}{3} - x \right]_{-1}^1 $$
$$ = \left(\frac{1}{3} - 1 \right) - \left(-\frac{1}{3} + 1 \right) = \left(-\frac{2}{3} \right) - \left(\frac{2}{3} \right) = -\frac{4}{3} $$

The negative total area confirms the curve remains below the x-axis between these bounds, crossing at $x = 1$.

Conclusion

In this detailed exploration, we've demonstrated how to sketch the quadratic function $F(x) = x^2 - 1$, identifying key features such as the vertex at $(0, -1)$, x-intercepts at $x = -1$ and $x = 1$, and the axis of symmetry at $x = 0$. Using algebraic methods, we've verified that the graph crosses the x-axis at $x = 1$.

Furthermore, by applying integration, we've analyzed the area under the curve between specific bounds and confirmed the point where the function intersects the x-axis. This approach not only reinforces the algebraic results but also showcases the power of calculus in understanding the behavior of functions.

Understanding these concepts provides a strong foundation for more advanced studies in calculus, such as analyzing the behavior of functions, finding areas, and solving real-world problems involving curves and their intersections.

Additional Tips for Students

- Always identify key points — vertex, intercepts, and axis of symmetry — before sketching.
- Use algebraic methods to verify points of intersection with axes.
- Apply integration to analyze the area under the curve, which relates to the function's behavior.
- Practice plotting functions with various features to develop intuition and accuracy.

References

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By mastering both the graphical and integral analysis of functions like $F(x) = x^2 - 1$, students can develop a comprehensive understanding of the relationship between algebraic expressions and their graphical representations.

Frequently Asked Questions

How do you verify that the graph of F(x) = x^2 - 1 crosses the x-axis at x = 1?

To verify, set $F(x) = 0$: $x^2 - 1 = 0$, which gives $x^2 = 1$, so $x = \pm 1$. Thus, the graph crosses the x-axis at $x = 1$ and $x = -1$.

What is the significance of showing that the graph crosses the x-axis at x = 1 using integration?

Using integration helps find the net area under the curve, and by analyzing the definite integral from a point to x, we can confirm where the function crosses the x-axis.

Can you demonstrate how to use integration to find the points where F(x) crosses the x-axis?

Yes. Since $F(x) = x^2 - 1$, integrate $F(x)$ over an interval:

$$\int F(x)\,dx = \int (x^2 - 1)\,dx = \frac{x^3}{3} - x + C.$$

Setting the definite integral from some point to x equals zero helps identify crossing points.

Why does the derivative approach confirm the crossing at x = 1, and how is this related to integration?

The derivative $F'(x) = 2x$ indicates the slope at any point. At $x = 1$, $F(1) = 0$, confirming a root. Integration relates to the antiderivative, which helps determine the original function's behavior and confirms the crossing point.

How do the concepts of integration and the Fundamental Theorem of Calculus relate to sketching F(x) = x^2 - 1?

The Fundamental Theorem links differentiation and integration, allowing us to understand the area under the curve and the function's zeros. Integrating F(x) helps validate the crossing points and sketch the graph accurately.

Is it necessary to perform integration to show that the graph crosses the x-axis at x=1, or is setting F(x)=0 sufficient?

Setting $F(x)=0$ is sufficient to find crossing points algebraically. Integration provides additional insight into the area and the behavior of the function but isn't strictly necessary to identify x-intercepts.

What is the overall approach to sketching the graph of F(x) = x^2 - 1 and confirming it crosses the x-axis at x=1 using integration?

First, find where $F(x)=0$ to identify roots at $x=\pm1$. Then, calculate the definite integral to understand the area and behavior of the function around these points. Use these insights to sketch the parabola, noting it crosses the x-axis at $x=\pm1$, with the vertex at (0, -1).

Additional Resources

Sketch The Function F(x) = x^2 - 1. Show That The Graph Crosses The I-axis At X = 1. Use Integration

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Introduction

In the realm of calculus and analytic geometry, understanding the behavior of functions and their graphical representations is fundamental. Among the myriad of functions, quadratic functions hold special significance due to their simple yet profound properties. One such function, $F(x) = x^2 - 1$, exemplifies the core principles of polynomial behavior, roots, and how integration can serve as a powerful tool to analyze and confirm the features of its graph.

This article embarks on a detailed exploration of the quadratic function $F(x) = x^2 - 1$, focusing on sketching its graph, verifying where it crosses the x-axis (also known as the I-axis), and employing integration techniques to reinforce our understanding. Through a systematic approach, we aim to demonstrate not only the graphical behavior but also the underlying mathematical reasoning that confirms the intersection point at $x = 1$.

Theoretical Foundations of Quadratic Functions

Understanding $F(x) = x^2 - 1$

The function $F(x) = x^2 - 1$ is a quadratic polynomial, characterized by a parabola opening upwards, with its vertex at the point where the quadratic attains its minimum value. Its general form is:

$$ F(x) = ax^2 + bx + c $$

where, in our case:

- $a = 1$,
- $b = 0$,
- $c = -1$.

The basic properties include:

- Symmetry about the y-axis (since $b=0$),
- The vertex at the minimum point,
- Roots at points where $F(x) = 0$.

Graphical Sketching of $F(x) = x^2 - 1$

Step 1: Identify Key Points

To sketch the graph, determine critical points, such as roots, vertex, and additional points for accuracy.

- Roots: Solve $F(x) = 0$:

$$
x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1
$$

So, the parabola crosses the x-axis at $x = -1$ and $x = 1$.

- Vertex: For a quadratic $ax^2 + bx + c$, vertex at $x = -\frac{b}{2a}$. Here:

$$
x_{v} = -\frac{0}{2 \times 1} = 0
$$

Evaluate $F(0)$:

$$
F(0) = 0^2 - 1 = -1
$$

So, the vertex is at $(0, -1)$.

Step 2: Plot Points

- At $x = -2$:

$$
F(-2) = 4 - 1 = 3
$$

- At $x = 0$:

$$
F(0) = -1
$$

- At $x = 2$:

$$
F(2) = 4 - 1 = 3
$$

- At $x = 1.5$:

$$
F(1.5) = 2.25 - 1 = 1.25
$$

This set of points allows us to sketch a smooth parabola opening upward, crossing the x-axis at $x = -1$ and $x = 1$, with a minimum at $(0, -1)$.

Confirming the x-Intercepts Using Integration

While solving the quadratic equation directly yields the roots, integrating the function over certain intervals can provide an alternative, analytical confirmation of where the function crosses the x-axis.

Step 1: Use of Definite Integrals to Identify Sign Changes

The integral of $F(x)$ over an interval reflects the net area between the graph and the x-axis. When the integral over an interval changes sign, it indicates the function crosses the x-axis within that interval.

Suppose we evaluate the definite integral of $F(x)$ from $x = -2$ to $x = 2$:

$$
I = \int_{-2}^{2} (x^2 - 1) \, dx
$$

Calculating:

$$
I = \left[\frac{x^3}{3} - x \right]_{-2}^{2}
$$

Plugging in the limits:

$$
I = \left(\frac{(2)^3}{3} - 2 \right) - \left(\frac{(-2)^3}{3} - (-2) \right)
$$

$$
= \left(\frac{8}{3} - 2 \right) - \left(\frac{-8}{3} + 2 \right)
$$

$$
= \left(\frac{8}{3} - \frac{6}{3} \right) - \left(- \frac{8}{3} + 2 \right)
$$

$$
= \frac{2}{3} - \left(- \frac{8}{3} + 2 \right)
$$

Express 2 as $\frac{6}{3}$:

$$
= \frac{2}{3} - \left(- \frac{8}{3} + \frac{6}{3} \right)
$$

$$
= \frac{2}{3} - \left(- \frac{2}{3} \right) = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}
$$

Since the integral over $[-2, 2]$ is positive, it indicates the net area is above the x-axis, but this does not directly identify the roots.

Step 2: Sign of $F(x)$ at Sample Points

- At $x = 0$: $F(0) = -1$ (below x-axis)
- At $x = 1$: $F(1) = 0$
- At $x = 1.5$: $F(1.5) = 1.25$ (above x-axis)

The change in sign from negative at $x = 0$ to zero at $x=1$ confirms the crossing at $x=1$. Similarly, the crossing at $x=-1$ can be confirmed by evaluating $F(-1)$:

$$
F(-1) = 1 - 1 = 0
$$

So, the function crosses the x-axis at $x = -1$ and $x=1$. Integration helps confirm the overall behavior but solving directly for roots remains the most straightforward method for precise points.

Deeper Analytical Validation via Integration

While the roots are directly obtainable through algebra, integrating the function over intervals that contain the roots offers a nuanced view of the function's behavior and can serve as a validation technique in more complex functions where algebraic solutions are not straightforward.

Step 1: Integrate from -1.5 to -0.5

Calculate:

$$
J = \int_{-1.5}^{-0.5} (x^2 - 1) \, dx
$$

Evaluating:

$$
J = \left[\frac{x^3}{3} - x \right]_{-1.5}^{-0.5}
$$

Compute at $x = -0.5$:

$$
\frac{(-0.5)^3}{3} - (-0.5) = \frac{-0.125}{3} + 0.5 = -\frac{0.125}{3} + 0.5 \approx -0.0417 + 0.5 = 0.4583
$$

At $x = -1.5$:

$$
\frac{(-1.5)^3}{3} - (-1.5) = \frac{-3.375}{3} + 1.5 = -1.125 + 1.5 = 0.375
$$

Now, subtract:

$$
J = 0.4583 - 0.375 = 0.0833
$$

Positive value indicates the net area is above the x-axis in this interval, which aligns with the graph crossing at $x = -1$.

Similarly, integrating over $[0.5, 1.5]$:

$$
K = \left[\frac{x^3}{3} - x \right]_{0.5}^{1.5}
$$

At $x=1.5$:

$$
\frac{3.375}{3} - 1.5 = 1.125 - 1.5 = -0.375
$$

At $x=0.5$:

$$
\frac{0.125}{3} - 0.5 = 0.0417 - 0.5 = -0.4583
$$

Subtract:

$$
K = -0.375 - (-0.4583) = 0.0833
$$

Again, positive

1 Sketch The Function F X X 2 1 Show That The Graph Crosses The I Axis At X 1 Use Integration

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