1)Use An Addition Or Subtraction Formula To Simplify The Equation.sin(3) Cos() Cos(3) Sin() = 0Find All

1)Use An Addition Or Subtraction Formula To Simplify The Equation.sin(3) Cos() Cos(3) Sin() = 0Find All

When approaching trigonometric equations, one of the most effective strategies is to utilize addition or subtraction formulas to simplify complex expressions. These formulas allow us to rewrite products of sine and cosine functions as sums or differences, making the equations more manageable and easier to solve. In this article, we will analyze the given equation:

$$ \sin(3) \cos() \cos(3) \sin() = 0 $$

and demonstrate how to apply addition and subtraction formulas to simplify it systematically. We will also explore the solutions (or roots) of the equation, considering all possible cases where the expression equals zero.

Understanding the Given Equation

Before diving into the solution process, let's carefully interpret the equation:

$$ \sin(3) \cos() \cos(3) \sin() = 0 $$

Note that the notation appears to include some missing or placeholder variables. Typically, in trigonometric equations, the variable of interest is denoted as x. So, for clarity, we will assume the equation is:

$$ \sin(3) \cos(x) \cos(3) \sin(x) = 0 $$

where:

- 3 is a constant angle (possibly in radians),
- x is the variable angle we want to solve for.

Alternatively, if the original problem intended to represent a product involving multiple sine and cosine functions with the same variable, this interpretation makes sense.

Assumption:

$$ \sin(3) \cos(x) \cos(3) \sin(x) = 0 $$

Our goal:

- Simplify this expression using addition or subtraction formulas.
- Find all solutions x that satisfy the equation.

Step 1: Factorization of the Expression

The given expression is a product of four factors:

$$ \sin(3) \times \cos(x) \times \cos(3) \times \sin(x) $$

Since the expression is a product equal to zero, the whole product equals zero if any factor is zero:

$$ \sin(3) = 0 \quad \text{or} \quad \cos(x) = 0 \quad \text{or} \quad \cos(3) = 0 \quad \text{or} \quad \sin(x) = 0 $$

However, simply setting each factor to zero overlooks potential simplifications that could combine terms. Especially, the factors involving $ \sin(3) $ and $ \cos(3) $ are constants, so their zeros are fixed, but $ \sin(x) $ and $ \cos(x) $ depend on $ x $.

Important: Since $ \sin(3) $ and $ \cos(3) $ are constants, their zeros are fixed values. The zeros of these constants are:

- $ \sin(3) = 0 $ when $ 3 = n \pi $, for $ n \in \mathbb{Z} $,
- $ \cos(3) = 0 $ when $ 3 = \frac{\pi}{2} + n \pi $, for $ n \in \mathbb{Z} $.

But unless these are specifically zero, the overall product could be zero only if $ \sin(x) $ or $ \cos(x) $ is zero.

Thus, the main focus is on simplifying the expression to understand its structure better.

Step 2: Rewriting the Expression Using Trigonometric Identities

Given the structure $ \sin(3) \cos(x) \cos(3) \sin(x) $, notice that the product involves pairs of sine and cosine functions. We can attempt to group terms:

$$ (\sin(3) \cos(3)) \times (\sin(x) \cos(x)) $$

since multiplication is associative and commutative.

Note:

$$ \sin(3) \cos(3) \quad \text{and} \quad \sin(x) \cos(x) $$

are separate products that can be simplified using product-to-sum formulas.

Applying the Product-to-Sum Formulas

Recall the following identities:

- Product-to-sum for sine and cosine:

$$
\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]
$$

- Product-to-sum for $\sin x \cos x$:

$$
\sin x \cos x = \frac{1}{2} \sin(2x)
$$

- Product-to-sum for $\sin A \cos A$:

$$
\sin A \cos A = \frac{1}{2} \sin(2A)
$$

Applying these identities:

1. Simplify $\sin(3) \cos(3)$:

$$
\sin(3) \cos(3) = \frac{1}{2} \sin(2 \times 3) = \frac{1}{2} \sin(6)
$$

2. Simplify $\sin(x) \cos(x)$:

$$
\sin(x) \cos(x) = \frac{1}{2} \sin(2x)
$$

Now, the original expression becomes:

$$
\left(\frac{1}{2} \sin(6) \right) \times \left(\frac{1}{2} \sin(2x) \right) = 0
$$

which simplifies to:

$$
\frac{1}{4} \sin(6) \sin(2x) = 0
$$

Multiplying both sides by 4:

$$
\sin(6) \sin(2x) = 0
$$

Step 3: Solving the Simplified Equation

The simplified equation:

$$
\sin(6) \sin(2x) = 0
$$

is satisfied when either:

- $\sin(6) = 0$, or
- $\sin(2x) = 0$

Let's analyze each case separately.

Case 1: $\sin(6) = 0$

Since $\sin(6)$ is a constant (note that 6 is in radians), this value is fixed:

$$
\sin(6) = 0 \quad \text{when} \quad 6 = n \pi, \quad n \in \mathbb{Z}
$$

which implies:

$$
6 = n \pi \quad \Rightarrow \quad n = \frac{6}{\pi}
$$

Since π is irrational, n is not an integer unless $6 / \pi$ is an integer, which it is not. Therefore,

$$
\sin(6) \neq 0
$$

unless 6 coincides with a multiple of π. Because 6 is approximately 6 radians, and $\pi \approx 3.1416$:

$$
6 \approx 1.9099 \pi
$$

which is not an integer multiple of π. So,

$$
\sin(6) \neq 0
$$

Therefore, this case does not yield solutions.

Case 2: $\sin(2x) = 0$

This is the key to solving for x:

$$
\sin(2x) = 0
$$

which occurs when:

$$
2x = n \pi, \quad n \in \mathbb{Z}
$$

Thus,

$$
x = \frac{n \pi}{2}
$$

for all integers n.

Final Solution Set

Given the analysis, the solutions to the original equation are:

$$
x = \frac{n \pi}{2}, \quad n \in \mathbb{Z}
$$

which satisfy the equation because:

$$
\sin(6) \neq 0
$$

and the entire product simplifies to zero precisely when $\sin(2x) = 0$.

Summary and Additional Insights

- The key step was expressing the product of sine and cosine functions as sums using product-to-sum identities.
- Simplification led to an easy-to-solve equation involving a single sine function.
- The solutions are all angles where $2x$ is an integer multiple of π, i.e.,

$$
x = \frac{n \pi}{2}
$$

- These solutions are valid for all integers n, representing an infinite set of solutions.

Additional Considerations

- Domain restrictions: Typically, for real solutions, x can be any real number, so the solution set is all x such that:

$$
x = \frac{n \pi}{2}, \quad n \in \mathbb{Z}
$$

- Special cases involving constants: If the problem had specific constraints or particular values for 3 (e.g., if \(

Frequently Asked Questions

How can I use addition or subtraction formulas to simplify the equation sin(3)cos() + cos(3)sin() = 0?

You can recognize that the expression resembles the sine addition formula: sin(A + B) = sinA cosB + cosA sinB. By matching terms, the equation simplifies to sin(3 +) = 0.

What is the next step after rewriting the equation as sin(3 +) = 0?

Solve for the angle by setting sin(3 +) = 0, which implies 3 + = nπ, where n is any integer. Then, solve for to find the solutions.

How do I find the values of ' ' from the equation sin(3 +) = 0?

Set 3 + = nπ, so ' ' = nπ - 3, where n is any integer. This gives all solutions in terms of n.

Are there restrictions on the values of ' ' in the solution?

Since sine is periodic with period 2π, the solutions for ' ' = nπ - 3 are valid for all integers n, covering all solutions.

How do I interpret the solutions for ' ' in the context of the original equation?

The solutions for ' ' correspond to the angles that satisfy the original equation when substituted back, based on the sine addition formula.

Can this method be applied to any similar equation involving sine and cosine products?

Yes, whenever the equation matches the form of sine or cosine addition formulas, you can use these identities to simplify and solve the equation.

What is the significance of using addition formulas in simplifying trigonometric equations?

Addition formulas help rewrite products of sine and cosine as sums or differences, making it easier to solve equations involving trigonometric functions.

How do I verify my solutions after solving for ' '?

Substitute the solutions back into the original equation to check if the equality holds, confirming their validity.

Are there special cases or solutions when the equation reduces to identities?

If the simplified form results in identities like sin(0) = 0 or cos(0) = 1, those can lead to specific solutions, but in this case, the general solution from sin(3 +) = 0 applies.

What is the general approach to solving equations involving products of sine and cosine?

Use trigonometric identities, such as addition and subtraction formulas, to rewrite the product as a sum or difference, then solve the resulting simple equation.

Additional Resources

Use of Addition or Subtraction Formulas to Simplify the Equation: sin(3) cos() cos(3) sin() = 0 — Find All

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Introduction

Trigonometry is a foundational branch of mathematics that explores the relationships between angles and sides within triangles. A crucial aspect of trigonometry involves transforming complex expressions into simpler forms using fundamental identities. Among these identities, the addition and subtraction formulas play a pivotal role in simplifying expressions involving sine and cosine functions.

In this comprehensive review, we focus on the specific task of simplifying and solving the trigonometric equation:

\[
\sin(3) \cos() \cos(3) \sin() = 0
\]

Note: The given expression appears to have some missing or ambiguous parts, especially within the parentheses. For clarity and meaningful analysis, we will interpret the expression as:

$$
\sin(3) \cos(\theta) \cos(3) \sin(\theta) = 0
$$

where θ is the variable angle, and 3 is a constant angle (likely in radians). This assumption aligns with standard trigonometric practices and allows us to proceed with applying addition/subtraction formulas.

Understanding the Equation

The Equation at a Glance

The equation:

$$
\sin(3) \cos(\theta) \cos(3) \sin(\theta) = 0
$$

is a product of four factors:

- $\sin(3)$, a constant sine value,
- $\cos(\theta)$, a variable cosine function,
- $\cos(3)$, another constant cosine value,
- $\sin(\theta)$, a variable sine function.

The entire product equals zero, which implies that at least one of these factors must be zero for the whole expression to be zero.

Goal of the Problem

- Simplify the expression using addition or subtraction formulas.
- Identify all solutions for θ that satisfy the equation.

Step 1: Factor Analysis and Simplification

Recognizing Constant Factors

Since $\sin(3)$ and $\cos(3)$ are constants (evaluated at the fixed angle 3 radians), we can consider their numerical values:

$$
\sin(3) \approx 0.14112, \quad \cos(3) \approx -0.98999
$$

Their product:

$$
\sin(3) \times \cos(3) \approx 0.14112 \times (-0.98999) \approx -0.1399
$$

But since the overall expression involves multiplication, and the constant factors cannot be zero unless explicitly specified, the zeros of the expression depend primarily on $\cos(\theta)$ and $\sin(\theta)$.

Rewriting the Equation

The initial expression simplifies to:

$$
(\sin(3) \times \cos(3)) \times \cos(\theta) \times \sin(\theta) = 0
$$

or equivalently,

$$
K \times \cos(\theta) \times \sin(\theta) = 0
$$

where $K = \sin(3) \times \cos(3)$.

Since $K \neq 0$, unless at specific angles, the zeroes are dictated by the factors $\cos(\theta)$ and $\sin(\theta)$.

Final Simplified Form

$$
\boxed{
\sin(3) \cos(3) \times \sin(\theta) \cos(\theta) = 0
}
$$

which reduces to:

$$
\text{Constant} \times \sin(\theta) \cos(\theta) = 0
$$

Step 2: Applying Trigonometric Identities

Using the Product-to-Sum Identity

One of the most useful identities involving products of sine and cosine functions is the product-to-sum formula:

$$
\sin(\alpha) \cos(\beta) = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]
$$

For our case, when $\alpha = \theta$ and $\beta = \theta$:

$$
\sin(\theta) \cos(\theta) = \frac{1}{2} [\sin(2\theta) + \sin(0)] = \frac{1}{2} \sin(2\theta)
$$

since $\sin(0) = 0$.

Simplified Equation in Terms of $\sin(2\theta)$

Using this identity, the original equation becomes:

$$
K \times \frac{1}{2} \sin(2\theta) = 0
$$

or

$$
\frac{K}{2} \sin(2\theta) = 0
$$

Given that $K \neq 0$ (as established earlier), the equation reduces to:

$$
\sin(2\theta) = 0
$$

Step 3: Solving the Simplified Equation

Solving $\sin(2\theta) = 0$

The general solutions for:

$$
\sin(2\theta) = 0
$$

are:

$$
2\theta = n\pi, \quad n \in \mathbb{Z}
$$

Thus,

$$
\theta = \frac{n\pi}{2}
$$

where n is any integer.

All Solutions for θ

Expressed explicitly, the solutions are:

$$
\boxed{
\theta = \frac{n\pi}{2}, \quad n \in \mathbb{Z}
}
$$

This encompasses:

- $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$

Interpreting the Results

- These solutions cover all angles where $\sin(2\theta) = 0$, which correspond to points where $\sin(\theta) \cos(\theta) = 0$, matching the original factors.

Step 4: Confirming No Other Solutions

Given our initial assumptions and transformations, the only solutions are those where $\sin(2\theta) = 0$.

Note: If the original expression was slightly different or involved other functions, the solutions might vary. However, based on the typical interpretation, the solutions derived here are complete.

Additional Insights and Considerations

1. Special Cases and Domain Restrictions

- If the problem specifies a domain for θ, such as $0 \leq \theta < 2\pi$, then the solutions are explicitly:

$$
\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}
$$

- For unrestricted domain, the solutions extend infinitely with period π.

2. Impact of Constants $\sin(3)$ and $\cos(3)$

- Since these are non-zero constants, they do not influence the zeros of the expression apart from scaling.
- If either $\sin(3)$ or $\cos(3)$ were zero, the entire product would be zero regardless of θ, making every θ a solution.

3. Graphical Interpretation

- Plotting the functions $\sin(3)$, $\cos(3)$, $\sin(\theta)$, and $\cos(\theta)$ can provide visual confirmation that the zeros occur at the specified angles.
- The product's zero crossings align with the zeros of $\sin(2\theta)$, i.e., at $\theta = n \pi/2$.

4. Extension to Other Angles

- If the problem involves degrees, conversions between radians and degrees are necessary:

$$
\theta = \frac{n\pi}{2} \quad \Rightarrow \quad \theta = n \times 90^\circ
$$

Practical Applications and Related Problems

a) Solving Trigonometric Equations in Engineering

- Such equations often appear in signal processing, wave analysis, or oscillatory systems where understanding the zeros of combined sine and cosine functions is essential.

b) Simplification Techniques

- Mastery of addition and subtraction formulas allows for transforming complex products into sums, simplifying the process of solving equations.

c) Extending to More Complex Expressions

- The same principles apply when dealing with more intricate trigonometric identities, such as multiple-angle formulas or sum/difference formulas.

Summary and Key Takeaways

- The initial expression was simplified using the product-to-sum identity:

$$
\sin(\theta) \cos(\theta) = \frac{1}{2} \sin(2\theta)
$$

- The equation reduces to solving $\sin(2\theta) = 0$.

- The solutions are:

$$
\boxed{
\theta = \frac{n\pi}{2}, \quad n \in \mathbb{Z}
}
$$

- Constants $\sin(3)$ and $\cos(3)$ affect the amplitude but not the zeros unless they are zero themselves.

- Understanding and applying addition/subtraction formulas is crucial in simplifying and solving trigonometric equations.

Final Remarks

The process of simplifying

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