1. Use The Method Of Example 3 To Show That The Following Set Of Vectors Forms A Basis For R2. {(2, 1),

1. Use The Method Of Example 3 To Show That The Following Set Of Vectors Forms A Basis For R2. {(2, 1), is a fundamental question in linear algebra that helps us understand the structure of vector spaces. When working with vectors in R^2, one of the key concepts is determining whether a set of vectors forms a basis. A basis allows us to express any vector in R^2 uniquely as a linear combination of the basis vectors. In this article, we will explore how to use a systematic method, inspired by Example 3, to verify whether the set {(2, 1)} along with another vector forms a basis for R^2. This process involves checking linear independence and span, two crucial properties for basis vectors.

Understanding the Concept of a Basis in R2

What Is a Basis?

A basis of R^2 is a set of vectors that are both:

- **Linearly Independent**: No vector in the set can be written as a linear combination of the others.
- **Span R^2**: Any vector in R^2 can be expressed as a linear combination of the vectors in the set.

For a set of vectors to be a basis, it must satisfy both conditions simultaneously.

Why Is It Important?

Knowing whether a set of vectors forms a basis helps in:

- Simplifying vector representation.
- Understanding the structure of the vector space.
- Facilitating calculations in linear algebra, such as coordinate transformations.

Applying Example 3 Method to Verify a Basis

Step 1: Identify the Vectors

Suppose we are given the set of vectors:

- $v_1 = (2, 1)$
- $v_2 = (a, b)$

Our goal is to determine whether these vectors form a basis for R^2. To do this, we will use a method similar to that demonstrated in Example 3, which emphasizes checking linear independence via the calculation of determinants or solving linear equations.

Step 2: Check for Linear Independence

The standard approach involves setting up the vectors as columns in a matrix and calculating its determinant:

```
| 2  a |
| 1  b |
```

If the determinant of this matrix is non-zero, the vectors are linearly independent, and thus form a basis for R^2.

Step 3: Calculate the Determinant

The determinant is calculated as:

```
det = (2)(b) - (a)(1) = 2b - a
```

For the set to be linearly independent, the determinant must not be zero:

```
2b - a ≠ 0
```

This inequality provides the condition on the components of the second vector.

Step 4: Determine The Conditions for the Second Vector

Based on the previous step, the set {(2, 1), (a, b)} forms a basis for R^2 if:

- **$2b - a \neq 0$**

This condition ensures the vectors are linearly independent, and therefore, span R^2.

Example: Verifying a Specific Set of Vectors

Let’s consider a specific example with concrete values:

- $v_1 = (2, 1)$
- $v_2 = (3, 4)$

Applying the method:

$$\det = (2)(4) - (3)(1) = 8 - 3 = 5 \neq 0$$

Since the determinant is non-zero, the vectors {(2, 1), (3, 4)} are linearly independent and thus form a basis for R^2.

Verifying the Spanning Property

Having established linear independence, the next step is to confirm whether these vectors span R^2.

Expressing an Arbitrary Vector as a Linear Combination

Suppose we want to express an arbitrary vector (x, y) as a linear combination of v1 and v2:

$$(x, y) = c_1(2, 1) + c_2(a, b)$$

This leads to the system of equations:

- $2c_1 + a\,c_2 = x$
- $c_1 + b\,c_2 = y$

Solving the System

The system can be written in matrix form:

$$\begin{vmatrix} 2 & a \\ 1 & b \end{vmatrix} \begin{vmatrix} c_1 \\ c_2 \end{vmatrix} = \begin{vmatrix} x \\ y \end{vmatrix}$$

The solution exists and is unique if the coefficient matrix has a non-zero determinant, which we already confirmed.

Conclusion on Spanning

Since the determinant is non-zero, any (x, y) in R^2 can be expressed as a linear combination of v1 and v2. Therefore, the set {(2, 1), (a, b)} spans R^2, satisfying the second condition for being a basis.

Summary and Final Remarks

- Using the method inspired by Example 3, verifying whether a set of vectors forms a basis for R^2 involves checking linear independence via the determinant of a matrix formed by the vectors.
- If the determinant is non-zero, the vectors are linearly independent and span R^2, thus forming a basis.
- For the specific set {(2, 1), (3, 4)}, the calculation confirms they are a basis for R^2.
- This approach is systematic, straightforward, and fundamental in linear algebra for understanding vector spaces.

Additional Tips for Applying the Method

1. Always write the vectors as columns in a matrix before calculating the determinant.
2. Remember that the determinant of a 2x2 matrix (ad - bc) is a quick way to check linear independence in R^2.
3. If working with more vectors or higher dimensions, consider row reduction or other methods to check independence and span.

By mastering this method, students and practitioners can confidently determine whether a given set of vectors forms a basis for R^2, enabling more advanced exploration in linear algebra and its applications.

Frequently Asked Questions

How can I determine if the set {(2, 1)} forms a basis for R2 using Example 3?

In Example 3, the method involves checking whether the vectors are linearly independent and span the space. Since {(2, 1)} contains only one vector, it cannot span R2, so it does not form a basis unless additional vectors are included. To use the method, verify if the vector is non-zero (which it is), and then see if it can generate the entire R2, which it cannot alone.

What steps are involved in applying the method of Example 3

to verify a set of vectors as a basis for R2?

The steps include: 1) Confirm the vectors are linearly independent, 2) Show that they span R2, and 3) Use algebraic methods such as forming a matrix and checking its rank or determinants to verify independence and spanning.

Given the set {(2, 1)}, why does it fail to form a basis for R2?

Because a single vector cannot span the entire 2-dimensional space R2. The set {(2, 1)} is only one vector, so it cannot generate all vectors in R2, which requires at least two linearly independent vectors.

How do you use the determinant method to check if {(2, 1)} can be part of a basis for R2?

Since a single vector cannot form a basis for R2, the determinant method applies when considering two vectors. For the given set with only one vector, the determinant isn't applicable. To check basis, you'd need at least two vectors and verify that their 2x2 matrix has a non-zero determinant.

Can the set {(2, 1)} be extended to form a basis for R2? How?

Yes, by adding a second vector that is linearly independent from (2, 1), such as (0, 1). Then, the set {(2, 1), (0, 1)} can form a basis for R2 after verifying their linear independence and span.

What is the importance of linear independence in using Example 3 to verify a basis?

Linear independence ensures that the vectors do not lie along the same line, which is crucial for forming a basis in R2. In Example 3, checking that the vectors are linearly independent confirms that they can span R2 without redundancy.

How can I verify whether a set of vectors forms a basis using the method shown in Example 3?

You verify basis by checking linear independence (e.g., via determinants or row reduction) and span (e.g., whether the vectors can generate all vectors in R2). If both conditions are met, the set is a basis.

Why is it insufficient to only check if vectors are non-zero when using Example 3's method?

Being non-zero ensures the vector isn't trivial, but in R2, a single non-zero vector cannot span the entire space. Both linear independence and spanning are needed; hence, checking only for non-zero vectors is insufficient.

What role does the determinant play when applying Example 3 to verify the basis of a set of vectors?

The determinant helps determine whether two vectors are linearly independent. A non-zero determinant of the matrix formed by the vectors indicates they are linearly independent, which is necessary for forming a basis in R2.

If a set contains only one vector in R2, how do you determine if it forms a basis using Example 3?

A single vector can only form a basis if it is non-zero but cannot span R2. To use Example 3, you check linear independence (which is trivial here) and whether it spans R2 (which it does not). Therefore, it does not form a basis unless combined with another vector.

Additional Resources

Use The Method Of Example 3 To Show That The Following Set Of Vectors Forms A Basis For R^2: {(2, 1), ...}

When working with vectors in R^2, determining whether a set of vectors forms a basis is a fundamental task in linear algebra. A basis provides a minimal, complete set of vectors that can be used to represent any vector in the space through linear combinations. In this guide, we'll explore how to use the method of Example 3—a structured approach often introduced in textbooks—to verify whether the set of vectors {(2, 1), ...} indeed forms a basis for R^2. This process involves checking key properties such as linear independence and spanning the entire space, ensuring a thorough and rigorous verification.

Understanding the Concept of a Basis in R^2

Before diving into the method, it's essential to revisit what it means for a set of vectors to be a basis in R^2:

- Linear Independence: No vector in the set can be written as a linear combination of the others.
- Spanning R^2: The set of vectors covers the entire two-dimensional space, meaning any vector in R^2 can be expressed as a linear combination of the vectors in the set.

In essence, a basis must satisfy both conditions simultaneously. For R^2, this typically reduces to verifying that the vectors are not scalar multiples of each other (to ensure independence) and that they are not confined to a line (to ensure spanning).

Introducing the Method of Example 3

The Method of Example 3 is a systematic technique used to verify the basis property, often involving:

1. Forming a matrix with the vectors as columns.
2. Calculating the determinant of this matrix.
3. Interpreting the determinant to determine linear independence.
4. Concluding whether the set forms a basis based on the determinant's value.

This approach is straightforward, especially for small vectors in R^2, and provides clear, computational criteria for basis verification.

Step-by-Step Guide to Applying the Method

Let's now go through the detailed steps to verify whether the set of vectors {(2, 1), v} forms a basis for R^2.

Step 1: Identify the Vectors

Suppose the set is:

$$ S = \{ \mathbf{v}_1 = (2, 1),\, \mathbf{v}_2 = (a, b) \} $$

where (a, b) is an arbitrary vector in R^2 (or a specific vector provided in the problem). For demonstration, let's assume (a, b) = (3, 4) for concreteness.

Note: If the problem only provides one vector, such as (2, 1), then the set cannot be a basis for R^2 unless a second vector is specified. So, for the purpose of this guide, we'll assume the set includes (2, 1) and (a, b).

Step 2: Form the Matrix

Construct a matrix with the vectors as columns:

$$
A = \begin{bmatrix}
2 & a \\
1 & b
\end{bmatrix}
$$

This matrix compactly represents the linear transformation corresponding to the vectors.

Step 3: Compute the Determinant

Calculate the determinant of matrix A:

$$
\det(A) = (2)(b) - (a)(1) = 2b - a
$$

Interpretation:

- If $\det(A) \neq 0$, then the vectors are linearly independent, and they span R^2, hence form a basis.
- If $\det(A) = 0$, then the vectors are linearly dependent, and do not form a basis.

Step 4: Analyze the Determinant

- Case 1: $\det(A) \neq 0$

Conclusion: The set {(2, 1), (a, b)} forms a basis for R^2.

- Case 2: $\det(A) = 0$

Conclusion: The set is not a basis, as the vectors are linearly dependent.

Practical Example: Verifying a Specific Set

Let's verify whether the set {(2, 1), (3, 4)} forms a basis.

Step 1: Set vectors

$$
\mathbf{v}_1 = (2, 1), \quad \mathbf{v}_2 = (3, 4)
$$

Step 2: Form the matrix

$$
A = \begin{bmatrix}
2 & 3 \\
1 & 4
\end{bmatrix}
$$

Step 3: Compute the determinant

$$
\det(A) = (2)(4) - (3)(1) = 8 - 3 = 5
$$

Step 4: Conclusion

Since $\det(A) = 5 \neq 0$, the vectors are linearly independent and span R^2. Therefore, {(2, 1), (3, 4)} forms a basis for R^2.

Extending the Method: General Case

In many problems, the second vector may be arbitrary, or you may be asked to verify a set with arbitrary vectors. Here's a quick outline to handle such cases:

1. Write down the vectors explicitly.
2. Construct the matrix with these vectors as columns.
3. Calculate the determinant.
4. Determine whether it is zero or not.
5. Conclude the basis status.

Common pitfalls:

- Neglecting the order of vectors: The order matters; usually, vectors are placed as columns.
- Forgetting to check both linear independence and spanning: In R^2, the determinant check suffices.
- Using the wrong matrix: Ensure the vectors are correctly placed as columns.

Additional Considerations

When Vectors Are Not in Standard Form

Sometimes vectors are given in different formats or with parameters. The same method applies—form the matrix and compute the determinant with the general elements, then analyze for non-zero values.

Multiple Vectors and Higher Dimensions

For higher-dimensional spaces (R^3, R^n), the process is similar but involves larger matrices and determinants or rank calculations.

Using Row Reduction

Alternatively, especially for larger matrices, row-reduction techniques can verify independence without explicitly calculating determinants.

Summary and Key Takeaways

- The method of Example 3 involves forming a matrix with the vectors as columns and calculating its determinant.
- A non-zero determinant indicates linear independence and spanning of R^2, hence confirming the set as a basis.
- For vectors in R^2, this process is quick, reliable, and computationally straightforward.
- Always verify the determinants carefully, considering the specific vectors involved.

Final Thoughts

Verifying whether a set of vectors forms a basis for R^2 is a fundamental skill in linear algebra, reinforcing concepts of linear independence and spanning. The method of Example 3 provides a clear, computational approach that can be applied universally to finite-dimensional vector spaces. Mastering this method not only helps in solving textbook problems efficiently but also lays the groundwork for understanding more complex concepts like change of basis, vector space isomorphisms, and linear transformations.

By systematically forming matrices, calculating determinants, and interpreting the results, you develop a deeper intuition about the structure of R^2 and the nature of vector sets within it. Practice with various vectors enhances your confidence and prepares you for advanced topics in linear algebra and its applications across science and engineering.

1 Use The Method Of Example 3 To Show That The Following Set Of Vectors Forms A Basis For R2 2 1

1 Use The Method Of Example 3 To Show That The Following Set Of Vectors Forms A Basis For R2 2 1

Related Articles

- What Law Of Psychophysics States That Two Stimuli Must Differ By A Constant Minimum Percentage (rather
- 1) Which Enzyme Has The Highest Affinity For Substrate? How Do You Know?2) Which Enzyme Can Convert The
- What Is The Domain Of The Function Y = 2 E Superscript X Graphed Below? On A Coordinate Plane, A Curve

Back to Home