1. Write Out The Law Of Cosines And Find The Hypotenuse Of Triangle With 2 Sides Lengths Equal To Three

1. Write Out The Law Of Cosines And Find The Hypotenuse Of Triangle With 2 Sides Lengths Equal To Three

In this article, we will explore the Law of Cosines, a fundamental theorem in trigonometry, and apply it to a specific problem involving a triangle with two sides measuring three units each. The goal is to understand how to formulate the Law of Cosines and use it to find the length of the hypotenuse, which is the side opposite the included angle. This process involves understanding the theorem's statement, its derivation, and the steps to solve for the unknown side when given certain side lengths and angles.

Understanding the Law of Cosines

What Is the Law of Cosines?

The Law of Cosines is a mathematical rule that relates the lengths of the sides of a triangle to the cosine of one of its angles. It is particularly useful for non-right triangles where the Pythagorean theorem does not apply directly. The law states that for any triangle with sides a, b, and c, and the angle C opposite side c:

- **Law of Cosines Formula:** $c^2 = a^2 + b^2 - 2ab \cos C$

Similarly, the formulas for the other sides are:

- $a^2 = b^2 + c^2 - 2bc \cos A$
- $b^2 = a^2 + c^2 - 2ac \cos B$

This law generalizes the Pythagorean theorem, which is a special case when the angle is 90 degrees (since $\cos 90^\circ = 0$).

Derivation of the Law of Cosines

While a detailed geometric proof involves dropping perpendiculars and using coordinate geometry or trigonometric identities, a common derivation involves:

1. Setting up coordinates: Place the triangle in the coordinate plane with points A at the origin, B along the x-axis, and C somewhere in the plane.
2. Expressing side lengths: Use the distance formula to write the lengths of sides a, b, and c in terms of coordinates.
3. Applying the Law of Cosines: Simplify the resulting equations to arrive at the formula relating side lengths and angles.

This derivation emphasizes that the Law of Cosines connects side lengths and angles in any triangle, making it invaluable for solving oblique triangles where at least two sides and an included angle are known or when two sides and a non-included angle are known.

Applying the Law of Cosines to Find the Hypotenuse

Scenario Description

Suppose you are given a triangle where two sides are both of length 3 units, and you are asked to find the length of the hypotenuse. The problem may involve:

- Two sides of length 3 units each, with a known included angle between them, or
- The need to assume or determine the angle between these sides.

To proceed, we need to clarify the specifics:

- Is the triangle a right triangle? (If so, Pythagoras applies, but the problem suggests using the Law of Cosines.)
- Is the included angle between the two sides known or assumed?

For the purpose of this article, we will consider the general case where the included angle C between the two sides of length 3 is known, and we want to find the length of side c (the hypotenuse).

Step-by-Step Solution

Let's assume the given data:

- Sides $a = 3$, $b = 3$
- Included angle C between sides a and b is known; for example, $C = 60^\circ$

Step 1: Write Out the Law of Cosines Formula

$$
c^2 = a^2 + b^2 - 2ab \cos C
$$

Step 2: Plug in the Known Values

$$
c^2 = 3^2 + 3^2 - 2 \times 3 \times 3 \times \cos 60^\circ
$$

$$
c^2 = 9 + 9 - 18 \times \cos 60^\circ
$$

Recall that $\cos 60^\circ = 0.5$, so:

$$
c^2 = 18 - 18 \times 0.5
$$

$$
c^2 = 18 - 9 = 9
$$

Step 3: Solve for c

$$
c = \sqrt{9} = 3
$$

In this case, the hypotenuse length c is also 3 units. This makes sense geometrically because a triangle with two sides of 3 and an included angle of 60° results in a side length of 3 units.

Note: If the angle C were 90°, the Law of Cosines reduces to the Pythagorean theorem:

$$
c^2 = a^2 + b^2 - 2ab \times 0 = a^2 + b^2
$$

which is the classic right triangle relation.

Generalizing the Problem

Case 1: Known Side Lengths and Included Angle

When two sides and the included angle are known, use the Law of Cosines directly, as demonstrated above.

Example:

- $a = 3$, $b = 3$, $C = 60^\circ$

Solution steps:

1. Write the formula
2. Substitute known values
3. Calculate $\cos C$
4. Find c

Case 2: Known Side Lengths Without the Included Angle

If only the side lengths are known and the angle is unknown, additional information, such as another side or an angle, is needed to find the hypotenuse.

Special Considerations

When the Triangle Is Right-Angled

In the special case where the triangle has a right angle, the Law of Cosines simplifies to the Pythagorean theorem:

$$
c^2 = a^2 + b^2
$$

This is often faster to use when the right angle is known, and the hypotenuse is sought.

When the Included Angle Is Not Known

If the angle between two sides of length 3 is unknown, but the third side or other angles are known, the Law of Cosines can be rearranged to solve for the unknown angle:

$$
\cos C = \frac{a^2 + b^2 - c^2}{2ab}
$$

This allows for solving the angle when side lengths are known.

Conclusion

The Law of Cosines is an essential tool in trigonometry for solving triangles that are not right-angled. It provides a direct relationship between side lengths and angles, allowing for the calculation of unknown sides or angles based on available measurements. In the specific problem of finding the hypotenuse of a triangle with two sides of length three units, the Law of Cosines can be applied straightforwardly when the included angle is known, as demonstrated with an example involving a 60° angle. When the triangle is right-angled, the Pythagorean theorem offers a quicker solution. Mastery of this law enhances problem-solving capabilities in various geometric contexts, from simple triangles to complex applications in engineering, navigation, and physics.

Frequently Asked Questions

What is the Law of Cosines formula?

The Law of Cosines states that for any triangle with sides a, b, and c, and the angle opposite side c being γ, the formula is $c^2 = a^2 + b^2 - 2ab \cos(\gamma)$.

How do you apply the Law of Cosines to find the hypotenuse when two sides are known?

If you know two sides and the included angle, you can use the Law of Cosines to find the third side: $c^2 = a^2 + b^2 - 2ab \cos(\gamma)$. For a right triangle, where the angle is 90°, $\cos(90°) = 0$, simplifying the formula.

What is the hypotenuse of a triangle with two sides each measuring 3 units and a right angle between

them?

Since it's a right triangle with sides 3 and 3, the hypotenuse is found using the Pythagorean theorem: $\sqrt{(3^2 + 3^2)} = \sqrt{(9 + 9)} = \sqrt{18} \approx 4.24$ units.

Can the Law of Cosines be used to find the hypotenuse in a right triangle?

Yes, but in right triangles, it's often easier to use the Pythagorean theorem. The Law of Cosines reduces to the Pythagorean theorem when the included angle is 90°.

What is the significance of the cosine value when applying the Law of Cosines?

The cosine value determines how much the two sides contribute to the third side based on the included angle. For example, if the angle is 90°, $\cos(90°)=0$, simplifying the formula to the Pythagorean theorem.

How do you find the hypotenuse if the two sides are 3 units each and the angle between them is not specified?

If the angle is not specified, and the sides are adjacent, you cannot directly use the Law of Cosines without knowing the included angle. If it's a right triangle, the hypotenuse is $\sqrt{(3^2 + 3^2)} = \sqrt{18} \approx 4.24$ units.

What is the step-by-step process to find the hypotenuse of a triangle with sides of length 3, using the Law of Cosines?

1. Identify the included angle between the two sides. 2. Use the Law of Cosines: $c^2 = 3^2 + 3^2 - 2\ 3\ 3\ \cos(\gamma)$. 3. If the angle is 90°, $\cos(90°)=0$, so $c^2=9+9=18$. 4. Take the square root: $c=\sqrt{18} \approx 4.24$ units.

Is it necessary to use the Law of Cosines for finding the hypotenuse of a right triangle with sides 3 and 3?

No, for right triangles, the Pythagorean theorem is simpler and more direct: hypotenuse = $\sqrt{(3^2 + 3^2)} = \sqrt{18} \approx 4.24$ units. The Law of Cosines is more useful when the triangle is not right-angled or when the included angle is unknown.

Additional Resources

Law of Cosines and Calculating the Hypotenuse of an Isosceles Triangle with Two Sides of Length Three

Understanding the Law of Cosines and how it applies to various triangles is fundamental in trigonometry and geometry. In this detailed exploration, we will delve into the Law of Cosines, its formula, and how it can be used to determine the hypotenuse of a specific isosceles triangle with two equal sides of length three. This comprehensive analysis aims to clarify concepts, demonstrate calculations, and highlight practical applications.

Introduction to the Law of Cosines

The Law of Cosines is a powerful mathematical tool that relates the lengths of a triangle's sides to the cosine of one of its angles. It extends the Pythagorean theorem to non-right triangles, allowing us to solve for missing sides or angles in any triangle.

Historical Context

- Developed independently by mathematicians in ancient Greece and India.
- Serves as a bridge between the Pythagorean theorem (for right triangles) and the general case (oblique triangles).

Why Is the Law of Cosines Important?

- Enables calculation of unknown sides when two sides and an included angle are known.
- Allows determination of angles when all three sides are known.
- Useful in fields such as navigation, physics, engineering, and computer graphics.

Mathematical Formulation of the Law of Cosines

The Law of Cosines states that for any triangle $\triangle ABC$ with sides a, b, c opposite angles A, B, C respectively:

$$
c^2 = a^2 + b^2 - 2ab \cos C
$$

Similarly, for the other sides:

$$
a^2 = b^2 + c^2 - 2bc \cos A
$$
$$
b^2 = a^2 + c^2 - 2ac \cos B
$$

Key Components:

- a, b, c: lengths of the sides.
- A, B, C: measures of the angles opposite those sides.
- $\cos$: cosine of the respective angle.

Special Cases:

- When the triangle is right-angled, say $C = 90^\circ$, then $\cos C = 0$, and the Law of Cosines reduces to the Pythagorean theorem:

$$
c^2 = a^2 + b^2
$$

Application: Finding the Hypotenuse in an Isosceles Triangle with Sides of Length 3

Now, let's analyze a specific problem: an isosceles triangle where two sides are each of length 3, and we want to find its hypotenuse. The problem involves understanding whether the triangle is right-angled and how to apply the Law of Cosines effectively.

Understanding the Triangle's Nature

- Isosceles Triangle: Two sides are equal in length.
- Sides Given: $AB = AC = 3$ (assuming the equal sides are AB and AC).
- Third Side (Base): BC (unknown).
- Objective: Find the length of the hypotenuse, which typically refers to the longest side in a right triangle.

Key Considerations:

- Is the triangle right-angled?

- If not, can it be right-angled with the given side lengths?
- Or do we need to find the length of the third side to determine the hypotenuse?

Determining if the Triangle is Right-Angled

In an isosceles triangle with two sides of length 3, the third side's length determines whether the triangle can be right-angled.

- Case 1: The third side BC is less than or equal to 6 (since the sum of two sides must be greater than the third).
- Case 2: The third side is exactly 3, making the triangle equilateral.
- Case 3: The third side is longer than 3 but less than 6.

To find out whether the triangle is right-angled, we need to analyze the possible angles using the Law of Cosines.

Applying the Law of Cosines to Find the Third Side

Suppose the two equal sides are $AB = AC = 3$, and the included angle $\angle BAC$ is θ. Our goal is to determine the length of side BC, which we will denote as a.

Applying the Law of Cosines:

$$
a^2 = b^2 + c^2 - 2bc \cos A
$$

In this case:

- $b = c = 3$
- $A = \angle BAC$
- $a = BC$

So,

$$
a^2 = 3^2 + 3^2 - 2 \times 3 \times 3 \times \cos \theta
$$

$$
a^2 = 9 + 9 - 18 \cos \theta
$$

$$
a^2 = 18 - 18 \cos \theta
$$

From this, the length of BC depends on the measure of angle θ.

Special Cases:

- When $\theta = 90^\circ$, $\cos 90^\circ = 0$:

$$
a^2 = 18 - 0 = 18
$$

$$
a = \sqrt{18} = 3 \sqrt{2} \approx 4.2426
$$

This indicates that if the included angle is 90°, the third side BC is approximately 4.2426.

- When $\theta = 0^\circ$, $\cos 0^\circ = 1$:

$$
a^2 = 18 - 18 \times 1 = 0
$$

which is degenerate (the points coincide), so not a valid triangle.

- When $\theta = 180^\circ$, $\cos 180^\circ = -1$:

$$
a^2 = 18 - 18 \times (-1) = 18 + 18 = 36
$$

$$
a = 6
$$

So, the longest possible side BC in this configuration is 6, which occurs when the included angle is 180°, but in that case, the triangle is degenerate (points are colinear).

Calculating the Hypotenuse in a Specific Configuration

Suppose we are investigating a scenario where the triangle might be right-angled, and one of the sides is the hypotenuse.

Scenario 1: The triangle is right-angled at A.

- In this case, the side opposite A (say, BC) would be the hypotenuse.
- The other two sides AB and AC are both 3.

Applying the Pythagorean theorem:

$$
a^2 = 3^2 + 3^2 = 9 + 9 = 18
$$

$$
a = \sqrt{18} = 3 \sqrt{2} \approx 4.2426
$$

Thus, the hypotenuse BC would be approximately 4.2426 in this right-angled configuration.

Scenario 2: The triangle is not necessarily right-angled.

Using the Law of Cosines, if the third side a is known (say, 4.2426), we can verify if the triangle is right-angled by calculating the angle:

$$
\cos A = \frac{b^2 + c^2 - a^2}{2bc}
$$

For $b = c = 3$, $a \approx 4.2426$:

$$
\cos A = \frac{9 + 9 - 18}{2 \times 3 \times 3} = \frac{0}{18} = 0
$$

which confirms $A = 90^\circ$, consistent with a right triangle.

Summary of Key Calculations and Results

| Description | Calculation | Result |

|------------|-------------|--------|
| Length of side \(BC \) if the included angle \(A \) is \(90^\circ \) | \(a^2 = 18 \) | \(a = 3 \sqrt{2} \approx 4.2426 \) |
| Hypotenuse when the triangle is right-angled at \(A \) with equal sides 3 | \(c = \sqrt{3^2 + 3^2} \) | \(c \approx 4.2426 \) |

Practical Applications of the Law of Cosines in This Context

Understanding how to apply the Law of Cosines to specific problems such as this has numerous practical implications:

1. Navigation and Surveying: Determining distances between points when direct measurement isn't feasible.
2. Engineering Design: Calculating lengths of structural components at angles.
3. Robotics and Computer Graphics: Computing distances and angles between points for movement and rendering.
4. Astronomy: Calculating the distance between

[1 Write Out The Law Of Cosines And Find The Hypotenuse Of Triangle With 2 Sides Lengths Equal To Three]

1 Write Out The Law Of Cosines And Find The Hypotenuse Of Triangle With 2 Sides Lengths Equal To Three

Related Articles

- Use The Terms & Names List To Identify Each Sentence Online Or On Your Own Paper.A. OsceolaB. John
- What Does This Quote Mean "our Deepest Fear Is Not That We Are Inadequate. Our Deepest Fear Is That We
- When A Certain Constant Force Acts Upon An Object With Mass 4kg, The Acceleration Of The Object Is 13.

Back to Home