2) 5 2 (6) Consider The Matrix E = 3 4 (a) Compute The Eigenvalues Of E. (b) Compute An Eigenvector For

2) 5 2 (6) Consider The Matrix E = 3 4 (a) Compute The Eigenvalues Of E. (b) Compute An Eigenvector For

Understanding the eigenvalues and eigenvectors of matrices is fundamental in various fields such as linear algebra, physics, engineering, and data science. These concepts help us analyze linear transformations, stability of systems, and principal components in datasets. In this article, we delve into the process of computing eigenvalues and eigenvectors for a specific matrix, referred to as matrix E, which is given as:

$$
E = \begin{bmatrix} 3 & 4 \\ \text{?} & \text{?} \end{bmatrix}
$$

Note: The original prompt appears to have some missing or ambiguous entries for the matrix E. Assuming a typical 2x2 matrix with known entries, we will proceed with an example matrix for clarity. If the actual matrix differs, the process remains similar.

Understanding Eigenvalues and Eigenvectors

Before diving into calculations, it’s essential to understand what eigenvalues and eigenvectors are.

Eigenvalues

An eigenvalue of a matrix A is a scalar λ such that there exists a non-zero vector v satisfying:

$$
A v = \lambda v
$$

This equation signifies that applying the matrix A to the vector v results in a scaled version of v, with the scaling factor being λ.

Eigenvectors

The vector v associated with an eigenvalue λ is called an eigenvector. Eigenvectors point in directions that are invariant under the transformation represented by A.

Computing Eigenvalues of Matrix E

The first step is to determine the eigenvalues of the matrix E. The general process involves solving the characteristic equation:

$$
\det(E - \lambda I) = 0
$$

where I is the identity matrix of the same size as E, and $\det$ denotes the determinant.

Step-by-Step Calculation

Assuming the matrix E is:

$$
E = \begin{bmatrix} 3 & 4 \\ a & b \end{bmatrix}
$$

where a and b are known entries (for illustration, let’s assume $a=2$ and $b=6$):

$$
E = \begin{bmatrix} 3 & 4 \\ 2 & 6 \end{bmatrix}
$$

1. Form the matrix $E - \lambda I$:

$$
E - \lambda I = \begin{bmatrix} 3 - \lambda & 4 \\ 2 & 6 - \lambda \end{bmatrix}
$$

2. Compute the determinant:

$$
\det(E - \lambda I) = (3 - \lambda)(6 - \lambda) - (4)(2)
$$

$$
= (3 - \lambda)(6 - \lambda) - 8
$$

3. Expand the determinant:

$$
(3)(6) - 3\lambda - 6\lambda + \lambda^2 - 8 = 18 - 3\lambda - 6\lambda + \lambda^2 - 8
$$

$$
= (18 - 8) + (-3\lambda - 6\lambda) + \lambda^2 = 10 - 9\lambda + \lambda^2
$$

4. Set the characteristic polynomial to zero:

$$
\lambda^2 - 9\lambda + 10 = 0
$$

5. Solve for λ:

Using quadratic formula:

$$
\lambda = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 1 \times 10}}{2} = \frac{9 \pm \sqrt{81 - 40}}{2} = \frac{9 \pm \sqrt{41}}{2}
$$

Thus, the eigenvalues are:

$$
\boxed{
\lambda_{1,2} = \frac{9 \pm \sqrt{41}}{2}
}
$$

Calculating Eigenvectors

Once the eigenvalues are known, we can find the corresponding eigenvectors.

General Approach

For each eigenvalue λ, substitute back into $(E - \lambda I) v = 0$ and solve for the vector v.

Example: Eigenvector for $\lambda = \frac{9 + \sqrt{41}}{2}$

Using the matrix E:

$$
E - \lambda I = \begin{bmatrix} 3 - \lambda & 4 \\ 2 & 6 - \lambda \end{bmatrix}
$$

Substituting $\lambda = \frac{9 + \sqrt{41}}{2}$:

$$
3 - \lambda = 3 - \frac{9 + \sqrt{41}}{2} = \frac{6 - 9 - \sqrt{41}}{2} = \frac{-3 - \sqrt{41}}{2}
$$

$$
6 - \lambda = 6 - \frac{9 + \sqrt{41}}{2} = \frac{12 - 9 - \sqrt{41}}{2} = \frac{3 - \sqrt{41}}{2}
$$

The matrix becomes:

\[

\begin{bmatrix}

\frac{-3 - \sqrt{41}}{2} & 4 \\

2 & \frac{3 - \sqrt{41}}{2}

\end{bmatrix}

\]

To find \(v = (x, y)^T\) satisfying:

\[

(E - \lambda I) v = 0

\]

we derive the system:

\[

\left(\frac{-3 - \sqrt{41}}{2} \right) x + 4 y = 0

\]

or equivalently:

\[

\left(\frac{-3 - \sqrt{41}}{2} \right) x = -4 y

\]

Choosing \(y=1\):

\[

x = \frac{-4}{\left(\frac{-3 - \sqrt{41}}{2} \right)} = \frac{-4 \times 2}{-3 - \sqrt{41}} = \frac{-8}{-3 -

\sqrt{41}}

\]

Simplify numerator and denominator:

\[

x = \frac{8}{3 + \sqrt{41}}

\]

The eigenvector corresponding to this eigenvalue is:

\[

v_1 = \left(\frac{8}{3 + \sqrt{41}}, 1 \right)

\]

Similarly, you can find the eigenvector for \(\lambda = \frac{9 - \sqrt{41}}{2}\) following the same process.

Applications of Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors have a broad spectrum of applications across various disciplines:

- **Diagonalization of Matrices:** Simplifies matrix powers and functions, especially useful in solving differential equations.

- **Principal Component Analysis (PCA):** Reduces high-dimensional data to principal components, which are eigenvectors of the covariance matrix.

- **Quantum Mechanics:** Determines energy levels of quantum systems where operators have eigenvalues representing measurable quantities.

- **Stability Analysis:** Analyzes the stability of equilibrium points in dynamical systems based on eigenvalues of Jacobian matrices.

- **Vibration Analysis:** Identifies natural frequencies and modes in mechanical systems.

Conclusion

Computing eigenvalues and eigenvectors is a critical skill in linear algebra, providing insight into the intrinsic properties of matrices and the transformations they represent. The process involves forming the characteristic polynomial, solving for eigenvalues, and then determining the corresponding eigenvectors. Whether analyzing physical systems, reducing data dimensions, or solving differential equations, mastering these concepts empowers you to approach complex problems more effectively. Remember, the specific entries of the matrix influence the calculations, so always ensure to work with the correct matrix data for accurate results.

Further Reading and Resources

- Linear Algebra and Its Applications by David C. Lay
- Khan Academy’s Linear Algebra Course
- MIT OpenCourseWare: Linear Algebra Lecture Series

- Online tools like Wolfram Alpha for eigenvalue and eigenvector calculations

By understanding and applying these principles diligently, you can unlock a deeper comprehension of matrix behavior and their applications across science and engineering.

Frequently Asked Questions

How do you compute the eigenvalues of the matrix E = [[3, 4], [6, 5]]?

To compute the eigenvalues, set up the characteristic equation $\det(E - \lambda I) = 0$. For matrix E = [[3, 4], [6, 5]], the characteristic polynomial is $(3 - \lambda)(5 - \lambda) - (4)(6) = 0$, which simplifies to $(3 - \lambda)(5 - \lambda) - 24 = 0$. Expanding gives $(15 - 3\lambda - 5\lambda + \lambda^2) - 24 = 0$, or $\lambda^2 - 8\lambda - 9 = 0$. Solving this quadratic yields the eigenvalues $\lambda = [4 \pm \sqrt{(16 + 9)}] = [4 \pm \sqrt{25}] = [4 \pm 5]$, so the eigenvalues are $\lambda = 9$ and $\lambda = -1$.

What is the process to find an eigenvector corresponding to a given eigenvalue of matrix E?

To find an eigenvector for a specific eigenvalue λ, substitute λ into $(E - \lambda I)$ and solve the homogeneous system $(E - \lambda I) v = 0$ for the vector v. This involves setting up the equations from $(E - \lambda I)$ and solving for the components of v, usually by expressing one variable in terms of others or using row reduction.

Can you provide an eigenvector for the eigenvalue $\lambda = 9$ of matrix E?

Yes. For $\lambda = 9$, compute $(E - 9I)$: [[3 - 9, 4], [6, 5 - 9]] = [[-6, 4], [6, -4]]. Solving $(-6)v_1 + 4v_2 = 0$, we get $4v_2 = 6v_1$, so $v_2 = (3/2)v_1$. Choosing $v_1 = 2$, $v_2 = 3$, an eigenvector is [2, 3].

Similarly, how do you find an eigenvector for □ = -1 for matrix E?

For □ = -1, compute (E + I): [[3 + 1, 4], [6, 5 + 1]] = [[4, 4], [6, 6]]. Solving 4v□ + 4v□ = 0 yields v□ = -v□. Choosing v□ = 1, v□ = -1, an eigenvector is [1, -1].

What is the significance of eigenvalues and eigenvectors in linear algebra?

Eigenvalues and eigenvectors are fundamental in analyzing linear transformations—they reveal directions that are scaled but not rotated, help in matrix diagonalization, and are essential in applications like stability analysis, principal component analysis, and quantum mechanics.

Additional Resources

Consider The Matrix E = \(\begin{bmatrix} 3 & 4 \\ 2 & 6 \end{bmatrix}\): An In-Depth Eigenvalue and Eigenvector Analysis

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Introduction

In the realm of linear algebra, matrices serve as fundamental tools that model complex systems across physics, engineering, computer science, and beyond. Among the myriad properties of matrices, eigenvalues and eigenvectors stand out as essential concepts that unlock insights into matrix behavior, stability analysis, and transformations. This article delves into a specific matrix:

\[
E = \begin{bmatrix} 3 & 4 \\ 2 & 6 \end{bmatrix}
\]

focusing on a comprehensive investigation of its eigenvalues and eigenvectors. Through rigorous computation and analysis, we aim to illuminate the intrinsic characteristics of this matrix, providing clarity for students, researchers, and practitioners alike.

Fundamental Concepts and Significance

Before embarking on the calculations, it is vital to understand the significance of eigenvalues and eigenvectors:

- Eigenvalues (λ) are scalars indicating how a matrix stretches or compresses vectors in its transformation space.
- Eigenvectors ($\mathbf{v}$) are vectors that only scale when transformed by the matrix, i.e., $E\mathbf{v} = \lambda \mathbf{v}$.

These concepts are crucial in diagonalization, stability analysis, quantum mechanics, vibration analysis, and many other fields.

Part 1: Computing Eigenvalues of Matrix E

The Characteristic Polynomial

The eigenvalues of a matrix are roots of its characteristic polynomial:

$$
\det(E - \lambda I) = 0
$$

where I is the identity matrix.

For the matrix:

$$
E - \lambda I =
\begin{bmatrix} 3 - \lambda & 4 \\ 2 & 6 - \lambda \end{bmatrix}
$$

the determinant is:

$$
\det(E - \lambda I) = (3 - \lambda)(6 - \lambda) - (4)(2)
$$

Expanding:

$$
(3 - \lambda)(6 - \lambda) - 8
$$

which simplifies to:

$$
(3 \times 6) - 3\lambda - 6\lambda + \lambda^2 - 8 = 18 - 9\lambda + \lambda^2 - 8
$$

$$
= \lambda^2 - 9\lambda + 10
$$

Solving the Quadratic Equation

Set the characteristic polynomial equal to zero:

$$
\lambda^2 - 9\lambda + 10 = 0
$$

Apply the quadratic formula:

$$
\lambda = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(10)}}{2} = \frac{9 \pm \sqrt{81 - 40}}{2} = \frac{9 \pm \sqrt{41}}{2}
$$

Thus, the eigenvalues are:

$$
\boxed{
\lambda_1 = \frac{9 + \sqrt{41}}{2} \quad \text{and} \quad \lambda_2 = \frac{9 - \sqrt{41}}{2}
}
$$

which are real and distinct, indicating that matrix E is diagonalizable over the real numbers.

Part 2: Computing Eigenvectors of Matrix E

Having identified the eigenvalues, the next step involves finding the corresponding eigenvectors.

Part 2.1: Eigenvector Corresponding to λ_1

$$
\lambda_1 = \frac{9 + \sqrt{41}}{2}
$$

Substitute into $E - \lambda_1 I$:

$$
\begin{bmatrix}
3 - \lambda_1 & 4 \\
2 & 6 - \lambda_1
\end{bmatrix}
$$

Calculating each element:

$$
3 - \lambda_1 = 3 - \frac{9 + \sqrt{41}}{2} = \frac{6 - 9 - \sqrt{41}}{2} = \frac{-3 - \sqrt{41}}{2}
$$

$$
6 - \lambda_1 = 6 - \frac{9 + \sqrt{41}}{2} = \frac{12 - 9 - \sqrt{41}}{2} = \frac{3 - \sqrt{41}}{2}
$$

The matrix becomes:

$$
\begin{bmatrix}
\frac{-3 - \sqrt{41}}{2} & 4 \\
2 & \frac{3 - \sqrt{41}}{2}
\end{bmatrix}
$$

To find an eigenvector $\mathbf{v} = \begin{bmatrix} x \\ y \end{bmatrix}$, solve:

$$
\left(\frac{-3 - \sqrt{41}}{2}\right) x + 4 y = 0
$$

or equivalently,

$$
\left(\frac{-3 - \sqrt{41}}{2}\right) x = -4 y
$$

Choose $y = 1$ for simplicity, then:

$$
x = \frac{-4}{\frac{-3 - \sqrt{41}}{2}} = \frac{-4 \times 2}{-3 - \sqrt{41}} = \frac{-8}{-3 - \sqrt{41}} = \frac{8}{3 + \sqrt{41}}
$$

Rationalize the denominator:

$$
x = \frac{8}{3 + \sqrt{41}} \times \frac{3 - \sqrt{41}}{3 - \sqrt{41}} = \frac{8(3 - \sqrt{41})}{(3)^2 - (\sqrt{41})^2} = \frac{8(3 - \sqrt{41})}{9 - 41} = \frac{8(3 - \sqrt{41})}{-32}
$$

Simplify:

$$
x = -\frac{8(3 - \sqrt{41})}{32} = -\frac{3 - \sqrt{41}}{4}
$$

Thus, an eigenvector corresponding to \(\lambda_1\) is:

\[
\boxed{
\mathbf{v}_1 = \begin{bmatrix} -\frac{3 - \sqrt{41}}{4} \\ 1 \end{bmatrix}
}
\]

Part 2.2: Eigenvector Corresponding to \(\lambda_2\)

\[
\lambda_2 = \frac{9 - \sqrt{41}}{2}
\]

Similarly, compute the entries:

\[
3 - \lambda_2 = 3 - \frac{9 - \sqrt{41}}{2} = \frac{6 - 9 + \sqrt{41}}{2} = \frac{-3 + \sqrt{41}}{2}
\]

\[
6 - \lambda_2 = 6 - \frac{9 - \sqrt{41}}{2} = \frac{12 - 9 + \sqrt{41}}{2} = \frac{3 + \sqrt{41}}{2}
\]

The matrix becomes:

\[
\begin{bmatrix}
\frac{-3 + \sqrt{41}}{2} & 4 \\

2 & \frac{3 + \sqrt{41}}{2}

\end{bmatrix}

\]

Set up the eigenvector equations:

\[

\left(\frac{-3 + \sqrt{41}}{2}\right) x + 4 y = 0

\]

Choose \(y = 1\):

\[

x = - \frac{4}{\frac{-3 + \sqrt{41}}{2}} = - \frac{4 \times 2}{-3 + \sqrt{41}} = - \frac{8}{-3 + \sqrt{41}} = \frac{8}{3 - \sqrt{41}}

\]

Rationalize the denominator:

\[

x = \frac{8}{3 - \sqrt{41}} \times \frac{3 + \sqrt{41}}{3 + \sqrt{41}} = \frac{8(3 + \sqrt{41})}{9 - 41} = \frac{8(3 + \sqrt{41})}{-32}

\]

Simplify:

\[

x = -\frac{8(3 + \sqrt{41})}{32} = -\frac{3 + \sqrt{41}}{4}

\]

Hence, an eigenvector corresponding to \(\lambda_2\) is:

\[

\boxed{

\mathbf{v}_2 = \begin{bmatrix} -\frac{3 + \sqrt{41}}{4} \\ 1 \end{bmatrix}

}

\]

Summary of Results

| Eigenvalue | Eigenvector (up to scalar multiples) |

| --- | --- |

| \(\lambda_1 = \frac{9 + \sqrt{41}}{2}\) | \(\begin{bmatrix} -\frac{3 - \sqrt{41}}{4} \\ 1 \end{bmatrix}\) |

| \(\lambda_2 = \frac{9 - \sqrt{41

2 5 2 6 Consider The Matrix E 3 4 A Compute The Eigenvalues Of E B Compute An Eigenvector For

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