27) Select The Collection Of Sets That Forms A Partition Of: {1,2,3,4,5,6,7,8} A. {1,2,5,7} {3,4} \{8\}

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Understanding the concept of partitions in set theory is fundamental in various fields of mathematics, including combinatorics, probability, and algebra. When given a set like {1,2,3,4,5,6,7,8} and a collection of subsets such as {1,2,5,7}, {3,4}, and {8}, the task often involves determining whether this collection forms a partition of the original set. This article provides a comprehensive guide to identifying such partitions, exploring the criteria involved, and applying these principles to the given set and subsets. Whether you're a student, educator, or enthusiast, understanding how to select the correct collection of sets that partition a set is vital for mastering set theory concepts.

Understanding Partitions in Set Theory

What Is a Partition?

In set theory, a partition of a set \(S\) is a collection of non-empty, disjoint subsets of \(S\) whose union equals \(S\). In simpler terms, the set is divided into groups (subsets) that do not overlap, and every element in the original set belongs to exactly one of these groups.

Key Characteristics of a Partition:
- Non-empty subsets: Each subset in the collection must contain at least one element.
- Disjointness: No two subsets share any common elements.
- Union equals original set: The union of all subsets must be the entire original set.

Why Are Partitions Important?

Partitions are crucial because they allow us to break down complex sets into manageable, non-overlapping parts. This is especially useful in statistical analysis, probability distributions, and algebraic structures where understanding the internal composition of a set leads to deeper insights.

Criteria for a Collection of Sets to Form a Partition

Before analyzing the provided collection, it's essential to understand the fundamental criteria that must be met:

1. All subsets are non-empty: No subset should be empty.
2. Mutually disjoint: No element appears in more than one subset.
3. Cover the entire set: The union of all subsets must be equal to the original set.

Mathematically:
Let $\{A_i\}_{i \in I}$ be a collection of subsets of S. Then, this collection is a partition of S if:
- $A_i \neq \emptyset$ for all i,
- $A_i \cap A_j = \emptyset$ for all $i \neq j$,
- $\bigcup_{i \in I} A_i = S$.

Analyzing the Given Collection of Sets

Given the set $S = \{1,2,3,4,5,6,7,8\}$ and the collection:
- $A = \{1,2,5,7\}$,
- $B = \{3,4\}$,
- $C = \{8\}$.

The question is: Does this collection form a partition of S?

Step 1: Check if all subsets are non-empty

- $A = \{1,2,5,7\}$ – non-empty.
- $B = \{3,4\}$ – non-empty.
- $C = \{8\}$ – non-empty.

All subsets are non-empty, satisfying the first criterion.

Step 2: Check for mutual disjointness

- $A \cap B = \{1,2,5,7\} \cap \{3,4\} = \emptyset$.
- $A \cap C = \{1,2,5,7\} \cap \{8\} = \emptyset$.
- $B \cap C = \{3,4\} \cap \{8\} = \emptyset$.

No overlaps exist between the subsets. The second criterion is satisfied.

Step 3: Check if the union equals the original set \(S\)

Calculate the union:
\[
A \cup B \cup C = \{1,2,5,7\} \cup \{3,4\} \cup \{8\} = \{1,2,3,4,5,7,8\}
\]

Compare with \(S = \{1,2,3,4,5,6,7,8\}\).

Notice that element 6 is missing from the union.

Conclusion: Does the collection form a partition?

Since the union of the subsets does not include 6, this collection does not form a partition of the entire set \(S\).

Identifying the Correct Collection of Sets That Form a Partition

Given the analysis, the current collection is incomplete because element 6 is not included. To form a valid partition, the collection must be expanded to include all elements of \(S\) without overlaps.

Possible Correct Partitions

- Partition 1:
- \(A = \{1,2,5,7\}\)
- \(B = \{3,4\}\)
- \(C = \{8\}\)
- Missing element: 6, which needs to be included in one of the subsets.

- Partition 2:
- Include element 6 in its own subset:
- \(A = \{1,2,5,7\}\)
- \(B = \{3,4\}\)
- \(C = \{8,6\}\)

Check if this collection satisfies the criteria:
- All subsets are non-empty.
- They are mutually disjoint.

- Their union is $\{1,2,3,4,5,6,7,8\} = S$.

Thus, the collection:

$$
\boxed{
\{A = \{1,2,5,7\}, \quad B = \{3,4\}, \quad C = \{8,6\}\}
}
$$

forms a valid partition of S.

Additional Considerations and Variations

Are There Other Possible Partitions?

Yes. Partitions are not unique; multiple valid partitions of a set can exist depending on how the set elements are grouped.

Some alternative partitions include:

- Partition with singleton sets:

$$
\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}\}
$$

which is the finest possible partition.

- Partition aligning with the original subsets:

$$
\{\{1,2,5,7\}, \{3,4\}, \{6\}, \{8\}\}
$$

which includes element 6 as a singleton.

Implications in Real-World Applications

Partitions are useful in dividing data, categorizing items, and simplifying complex problems. For example:

- In probability theory: Partitions help define mutually exclusive events.
- In database normalization: Partitions can represent data segmentation.
- In clustering analysis: Dividing data points into non-overlapping groups.

Summary and Final Recommendations

- The collection \(\{ \{1,2,5,7\}, \{3,4\}, \{8\} \}\) does not form a partition of \(S\) because element 6 is missing.
- To form a proper partition, include element 6 in one of the subsets or create a singleton set \(\{6\}\).
- The correct partition collection could be:
\[
\boxed{
\{\{1,2,5,7\}, \{3,4\}, \{8,6\}\}
}
\]
which satisfies all criteria.

Practical tip: Always verify the union covers the entire original set and that subsets are mutually disjoint and non-empty.

Conclusion

Understanding how to select a collection of sets that form a partition of a set is a foundational skill in set theory. When analyzing the given set and subsets, careful examination of the union, disjointness, and non-emptiness reveals whether the collection qualifies as a partition. In this case, adjustments are necessary to include all elements, especially element 6, to ensure the collection forms a valid partition of \(\{1,2,3,4,5,6,7,8\} \). Mastering this process enhances comprehension of fundamental mathematical concepts and their applications across diverse disciplines.

Frequently Asked Questions

Does the collection { {1,2,5,7}, {3,4}, {8} } form a partition of the set {1,2,3,4,5,6,7,8}?

Yes, because the sets are mutually disjoint and their union equals the entire set {1,2,3,4,5,6,7,8}.

Are all elements of the set {1,2,3,4,5,6,7,8} covered in the given collection?

Yes, every element from 1 to 8 appears in exactly one of the subsets in the collection.

Do the subsets in the collection overlap?

No, the subsets are pairwise disjoint, which is necessary for a partition.

Is the set {6} included in the collection?

No, the collection does not include {6}, which means it does not cover the entire original set.

Does the collection { {1,2,5,7}, {3,4}, {8} } qualify as a partition of the set {1,2,3,4,5,6,7,8}?

No, because it does not include all elements; specifically, {6} is missing.

What condition must a collection of sets satisfy to be a partition of a set?

The sets must be mutually disjoint and their union must be the entire original set.

Is the subset {1,2,5,7} a valid part of the partition?

Yes, it is one of the subsets, and it is disjoint from the others.

Can the collection { {1,2,5,7}, {3,4}, {8} } be considered a partition of the original set?

No, because it does not include all elements; specifically, element 6 is missing.

What is missing in the collection for it to be a valid partition of {1,2,3,4,5,6,7,8}?

The subset {6} should be included to cover all elements of the set.

Additional Resources

Select the collection of sets that forms a partition of: {1, 2, 3, 4, 5, 6, 7, 8} is a fundamental problem in set theory, often encountered in mathematical logic, combinatorics, and related fields. Understanding how to determine whether a collection of subsets constitutes a partition of a larger set is crucial for various applications—from organizing data to analyzing relationships within sets. In this article, we will explore the concept of set partitions, analyze the given collection, and provide a detailed step-by-step guide to identify which collections of subsets form valid partitions of the set {1, 2, 3, 4, 5, 6, 7, 8}.

What Is a Partition of a Set?

Before diving into the specific problem, it’s essential to understand what constitutes a partition of a set.

Definition of a Partition

A partition of a set S is a collection of non-empty, disjoint subsets of S whose union is S itself. Formally, a collection $\{A_1, A_2, ..., A_n\}$ is a partition of S if:

- Non-emptiness: For each A_i, $A_i \neq \varnothing$.
- Pairwise Disjointness: For any $i \neq j$, $A_i \cap A_j = \varnothing$.
- Union Covers S: The union of all A_i equals S; that is, $A_1 \cup A_2 \cup ... \cup A_n = S$.

Why Are Partitions Important?

Partitions allow us to break down complex sets into manageable, non-overlapping parts. This is useful in:

- Data segmentation
- Equivalence classes
- Problem simplification in combinatorics
- Mathematical proofs involving set decompositions

Analyzing the Given Collection of Sets

The problem provides the set S = {1, 2, 3, 4, 5, 6, 7, 8} and a collection of subsets:

- A. {1, 2, 5, 7}
- B. {3, 4}
- C. {8}

Our task: Select the collection of sets among these that form a partition of S.

Step-by-Step Approach to Determine a Valid Partition

To evaluate whether a collection forms a partition, follow these logical steps:

Step 1: Check for Non-Empty Subsets

- Are all the subsets non-empty?
- In the given collection:
- A. {1, 2, 5, 7} → Non-empty
- B. {3, 4} → Non-empty
- C. {8} → Non-empty

Result: All subsets are non-empty, satisfying the first criterion.

Step 2: Verify Disjointness of Sets

- Are the subsets pairwise disjoint?
- Check for overlaps:

- A and B:
- Elements in A: 1, 2, 5, 7
- Elements in B: 3, 4
- Intersection: none → disjoint

- A and C:
- Elements in A: 1, 2, 5, 7
- Elements in C: 8
- Intersection: none → disjoint

- B and C:
- Elements in B: 3, 4
- Elements in C: 8
- Intersection: none → disjoint

Result: All subsets are pairwise disjoint.

Step 3: Check if the Union of Sets Equals the Original Set S

- Union of all subsets:
- A ∪ B ∪ C: {1, 2, 5, 7} ∪ {3, 4} ∪ {8} = {1, 2, 3, 4, 5, 7, 8}
- Original set S: {1, 2, 3, 4, 5, 6, 7, 8}

- Compare:
- Union result: {1, 2, 3, 4, 5, 7, 8}
- S: {1, 2, 3, 4, 5, 6, 7, 8}

- Observation:
- Element 6 is missing from the union.

Result: The union does not cover the entire set S, because element 6 is not included.

Conclusion: Does the Collection Form a Partition?

Based on the above analysis:

- All subsets are non-empty.
- The subsets are pairwise disjoint.
- The union of the subsets does not cover the entire set S (missing element 6).

Therefore, the given collection { {1, 2, 5, 7}, {3, 4}, {8} } does not form a partition of S, because it doesn’t include all elements, particularly element 6.

How to Find a Valid Partition

To form a valid partition of S, the collection must:

1. Cover every element of S.
2. Have disjoint subsets.
3. Consist of non-empty subsets.

Potential Approach:

- Include element 6 in one of the subsets.
- For instance, modify the collection to:

- A. {1, 2, 5, 7}
- B. {3, 4}
- C. {8}
- D. {6}

- Now, check:

- Non-empty? Yes.
- Disjoint? Yes; no overlaps.
- Union? {1, 2, 5, 7} ∪ {3, 4} ∪ {8} ∪ {6} = {1, 2, 3, 4, 5, 6, 7, 8} = S.

- Conclusion: The collection { {1, 2, 5, 7}, {3, 4}, {8}, {6} } forms a valid partition.

Summary and Final Recommendations

- The collection { {1, 2, 5, 7}, {3, 4}, {8} } does not form a partition of the set {1, 2, 3, 4, 5, 6, 7, 8} because it omits element 6.
- To form a partition, you must include all elements of the set exactly once

across the subsets.
- When evaluating any collection for a partition:

1. Ensure all subsets are non-empty.
2. Check that subsets are pairwise disjoint.
3. Confirm that the union of all subsets equals the entire set.

Additional Practice

To deepen understanding, consider the following exercises:

- Given: { {1, 2}, {3, 4}, {5, 6, 7, 8} }

- Does this collection form a partition of {1, 2, 3, 4, 5, 6, 7, 8}?

- Given: { {1, 2, 3}, {3, 4, 5} }

- Is this collection a partition of some set?

Hint: Remember to check for overlaps and whether the union covers the entire set.

Final Thoughts

Determining whether a collection of sets forms a partition is a straightforward process once the criteria are understood. It requires careful analysis of set disjointness, non-emptiness, and complete coverage of the original set. Mastery of this concept empowers you to organize data, analyze equivalence relations, and solve complex problems in mathematics and computer science effectively.

Happy set partitioning!

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