3) AndWhat Is The Equation, In Point-slope Form, Of The Line Thatis Perpendicular To The Given Line And

3) And What Is The Equation, In Point-slope Form, Of The Line That Is Perpendicular To The Given Line And is a fundamental question in coordinate geometry that explores how to determine the equation of a line perpendicular to a given line, especially when expressed in point-slope form. Understanding this concept is essential for students and professionals working in mathematics, engineering, physics, and related fields, as it lays the foundation for analyzing geometric relationships, solving systems of equations, and graphing lines accurately.

In this comprehensive guide, we will delve into the concepts behind perpendicular lines, the significance of the point-slope form, and the step-by-step process to find the equation of a perpendicular line to a given line. We will also explore practical examples, common mistakes, and tips for mastering this topic.

Understanding the Basics: Lines in Coordinate Geometry

Before addressing the main question, it is crucial to understand some foundational concepts related to lines in coordinate geometry.

Line Equation Forms

Lines in the Cartesian plane can be expressed in various forms, each useful in different contexts:

- **Slope-intercept form:** $y = mx + b$, where m is the slope and b is the y-intercept.
- **Point-slope form:** $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope.
- **Standard form:** $Ax + By = C$, where A, B, and C are constants.

For our purposes, the point-slope form is particularly useful because it directly involves a known point and the slope of the line.

What Does It Mean for Two Lines to Be Perpendicular?

Perpendicular lines are lines that intersect at a right angle (90 degrees). Their slopes are related in a specific way:

Slope Relationship for Perpendicular Lines

- If the slope of the first line is m_1, then the slope of a line perpendicular to it, m_2, is given by:

$m_2 = -1 / m_1$, provided $m_1 \neq 0$.

- This means that the slopes are negative reciprocals of each other.

Special Cases

- If the given line is horizontal (slope $m_1 = 0$), then the perpendicular line is vertical, which cannot be expressed in slope-intercept form. Instead, it would be a vertical line with an equation $x = x_1$.
- If the given line is vertical (undefined slope), then the perpendicular line is horizontal, with an equation $y = y_1$.

Step-by-Step Process to Find the Equation of a Perpendicular Line in Point-slope Form

To find the equation of a line perpendicular to a given line and passing through a specific point, follow these steps:

Step 1: Identify the Given Line's Equation or Slope

- If you have the line's equation, convert it to slope-intercept form to find the slope m_1.
- If only two points on the line are given, calculate the slope using:

$m_1 = (y_2 - y_1) / (x_2 - x_1)$.

Step 2: Determine the Slope of the Perpendicular Line

- Use the slope relationship:

$m_2 = -1 / m_1$, assuming $m_1 \neq 0$.

- For special cases:

- Horizontal line ($m_1=0$): perpendicular line is vertical, $x = x_1$.
- Vertical line (m_1 undefined): perpendicular line is horizontal, $y = y_1$.

Step 3: Use the Point-slope Form to Write the Equation

- Once you have the slope m_2 and a point (x_1, y_1) through which the perpendicular line passes, write the equation as:

$y - y_1 = m_2(x - x_1)$.

Practical Examples

Let's explore some concrete examples to solidify the concepts.

Example 1: Given a Line with Known Slope

Suppose you are given the line:

$y = 2x + 3$

and a point (4, 5) through which the perpendicular line passes. Find the equation of the perpendicular line in point-slope form.

Solution:

- The slope of the given line:

$m_1 = 2$.

- Slope of the perpendicular line:

$m_2 = -1 / 2$.

- Using the point (4, 5):

$y - 5 = -1/2 (x - 4)$.

This is the equation in point-slope form.

Example 2: Given a Horizontal Line

Suppose the given line is $y = 7$, a horizontal line passing through all points where $y=7$.

- The slope of this line:

$m_1 = 0$.

- The perpendicular line must be vertical, with an equation:

$x = x_1$,

where x_1 is the x-coordinate of the point through which it passes.

- If the perpendicular line passes through $(3, y)$, then its equation:

$x = 3$.

Since the line is vertical, it cannot be expressed in point-slope form but is still perpendicular.

Example 3: Given a Vertical Line

Suppose the given line is $x = -2$, a vertical line.

- Its slope is undefined.

- The perpendicular line is horizontal, with slope $m_2 = 0$.

- If it passes through $(x, y) = (x, 4)$, then its equation:

$y = 4$.

Again, this is a horizontal line, not in point-slope form, but perpendicular.

Common Mistakes and How to Avoid Them

When working with perpendicular lines and point-slope equations, students often encounter errors. Here are some common mistakes and tips to avoid them:

- **Forgetting the Negative Reciprocal:** Remember that the slope of the perpendicular line is the negative reciprocal of the original slope. For example, if original slope is 3, perpendicular slope is -1/3.

- **Confusing Horizontal and Vertical Lines:** Horizontal lines have slope 0, and their perpendicular counterparts are vertical, which cannot be expressed in slope form. Conversely, vertical lines have undefined slope, and their perpendicular lines are horizontal.

- **Using the Wrong Point:** Ensure you are using the correct point through which the perpendicular line passes when applying the point-slope formula.

- **Ignoring Special Cases:** Always check if the given line is horizontal or vertical, as the perpendicular line's equation will not be in slope form.

Tips for Mastery and Practice

To become proficient in finding the equation of perpendicular lines in point-slope form, consider the following tips:

- Practice with different types of lines: Work with lines in slope-intercept form, standard form, and points to build versatility.
- Visualize the geometry: Drawing the lines and points helps in understanding the relationships and verifying solutions.
- Use graphing tools: Software like Desmos or GeoGebra can help visualize the lines and check your equations.
- Memorize the slope relationship: Remember that the slopes are negative reciprocals for perpendicular lines.
- Solve varied problems: Tackle problems involving special cases, such as vertical and horizontal lines, to deepen understanding.

Conclusion

Understanding how to find the equation of a line perpendicular to a given line in point-slope form is a vital skill in coordinate geometry. It involves identifying the original line's slope, determining the negative reciprocal for the perpendicular line, and then applying the point-slope formula with a known point. Whether working with lines in slope-intercept form, standard form, or special cases like horizontal or vertical lines, the core principles remain consistent.

Mastering this process enhances problem-solving skills and provides a solid foundation for more advanced topics in mathematics, such as analytic geometry, calculus, and vector analysis. Regular practice, visualization, and understanding the underlying concepts will lead to confidence and proficiency in working with perpendicular lines in various contexts.

By following the steps outlined in this guide and paying attention to common pitfalls, you can confidently determine the equation of any line perpendicular to a given line, expressed in point-slope form, and apply this knowledge to real-world and academic problems.

Frequently Asked Questions

What is the general form of the equation of a line in point-slope form?

The general form of the point-slope equation is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope of the line.

How do you find the slope of a line perpendicular to a given line?

To find the slope of a line perpendicular to a given line, take the negative reciprocal of the original line's slope. If the original slope is m, the perpendicular slope is -1/m.

What is the process to write the equation of a line perpendicular to a given line in point-slope form?

First, determine the slope of the given line, then find its negative reciprocal for the perpendicular line. Next, use a point on the perpendicular line and the new slope to write the equation in point-slope form: $y - y_1 = m\_perpendicular(x - x_1)$.

Can you give an example of writing the equation of a perpendicular line in point-slope form?

Yes. Suppose the given line has slope 2, and the point (3, 4) lies on the perpendicular line. The perpendicular slope is -1/2. The equation is y - 4 = -1/2(x - 3).

Why is the negative reciprocal important when finding the perpendicular line's equation?

Because two lines are perpendicular if and only if their slopes are negative reciprocals, meaning their slopes multiply to -1, ensuring the lines intersect at right angles.

What information do you need to write the point-slope

form of a perpendicular line?

You need a point on the perpendicular line and the slope of that perpendicular line, which is the negative reciprocal of the original line's slope.

How do you determine the slope of the original line when it is given in different forms?

If the line is given in slope-intercept form y = mx + b, the slope is m. If given in standard form Ax + By = C, solve for y to find the slope as -A/B.

Is the point-slope form of a line always suitable for perpendicular lines calculations?

Yes, the point-slope form is convenient for writing the equation of a perpendicular line once you have a point and the perpendicular slope.

What are common mistakes to avoid when writing the perpendicular line's equation in point-slope form?

Common mistakes include using the wrong slope (not the negative reciprocal), mixing up the point coordinates, or forgetting to simplify the equation properly.

How can I verify that my perpendicular line is correctly calculated?

Check that the slope of your perpendicular line is the negative reciprocal of the original line's slope and that the point (x_1, y_1) lies on your new line by substituting into the equation.

Additional Resources

Understanding the Equation of a Perpendicular Line in Point-Slope Form

When working with linear equations in algebra, a common and essential task is determining lines that are perpendicular to a given line. This concept is fundamental in coordinate geometry, influencing the study of slopes, angles, and geometric relationships. In this article, we will explore in detail how to find the equation, in point-slope form, of a line that is perpendicular to a given line. We will delve into the properties of slopes, the significance of point-slope form, and step-by-step methods to derive the desired line's equation.

Fundamentals of Line Equations and Slope

Before tackling the problem of perpendicular lines, it’s crucial to review some basic concepts:

1. The Equation of a Line in Slope-Intercept and Point-Slope Forms

- Slope-Intercept Form:
$$ y = mx + b $$
where m is the slope and b is the y-intercept.

- Point-Slope Form:
$$ y - y_1 = m(x - x_1) $$
where (x_1, y_1) is a known point on the line, and m is the slope.

The point-slope form is particularly useful when you know a point on the line and its slope, which is often the case in geometric problems.

2. The Concept of Slope and Its Role in Perpendicular Lines

- Definition of Slope:
The slope m of a line measures its steepness. It is calculated as:
$$ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} $$

- Perpendicular Lines and Slopes:
Two lines are perpendicular if they intersect at a right angle (90 degrees). The key property is:
$$ m_1 \times m_2 = -1 $$
where m_1 and m_2 are the slopes of the two lines.

This negative reciprocal relationship is the cornerstone of finding the slope of a line perpendicular to a given line.

Step-by-Step Approach to Find the Perpendicular Line's Equation

The process involves understanding the original line, determining its slope, calculating the perpendicular slope, and then writing the equation in point-slope form using a given point.

1. Identify the Slope of the Given Line

Suppose the given line is expressed in a standard form or slope-intercept form. For example:

- Given line: $y = 2x + 3$
The slope m_1 is 2.

- Given line: $3x - 4y = 7$
Rewrite in slope-intercept form:
$$-4y = -3x + 7 \Rightarrow y = \frac{3}{4}x - \frac{7}{4}$$
The slope $m_1 = \frac{3}{4}$.

- Given line in point-slope form:
$$y - 5 = 4(x + 2)$$
Expanding, $y = 4x + 13$, so $m_1 = 4$.

The first step is always to find or deduce the slope of the original line.

2. Calculate the Slope of the Perpendicular Line

Once the slope of the original line m_1 is known, find the perpendicular slope m_2:

- If $m_1 \neq 0$:
$$m_2 = -\frac{1}{m_1}$$

- Special cases:
- If the original line is vertical with undefined slope ($x = c$), then the perpendicular line is horizontal ($y = k$).
- If the original line is horizontal ($y = k$), then the perpendicular line is vertical ($x = c$).

For example:

- Original slope: $m_1 = 2$
Perpendicular slope: $m_2 = -\frac{1}{2}$

- Original slope: $m_1 = \frac{3}{4}$
Perpendicular slope: $m_2 = -\frac{4}{3}$

- Original line is vertical (e.g., $x = 5$), the perpendicular line is horizontal: $y = y_1$.

3. Use a Point on the Perpendicular Line

To write the equation in point-slope form, you need a specific point (x_1, y_1) that the perpendicular line passes through. This point can be:

- Given explicitly in the problem, or
- Derived from the context (e.g., the point of intersection between the original line and another geometric figure).

If the problem doesn't specify a point, it's common to choose a point known to be on the perpendicular line, such as the intersection point or a point provided in the problem.

4. Write the Equation in Point-Slope Form

Having the point (x_1, y_1) and the perpendicular slope m_2, the line's equation in point-slope form is:

$$ y - y_1 = m_2 (x - x_1) $$

This form is flexible and convenient because it directly incorporates known points and slopes.

Practical Examples and Applications

Let’s explore concrete examples to solidify the concepts.

Example 1: Find the Equation of a Line Perpendicular to $y = 3x + 4$ and Passing Through $(2, -1)$

Step 1: Identify the slope of the given line:
$$ m_1 = 3 $$

Step 2: Find the perpendicular slope:
$$ m_2 = -\frac{1}{3} $$

Step 3: Use the point $(2, -1)$:
$$ y - (-1) = -\frac{1}{3}(x - 2) $$

Step 4: Simplify:
$$ y + 1 = -\frac{1}{3}x + \frac{2}{3} $$

Final Equation (Point-Slope Form):
$$ y + 1 = -\frac{1}{3}(x - 2) $$

This is the required line in point-slope form.

Example 2: Find the Equation of a Perpendicular Line to $x = 7$ Passing Through $(4, 5)$

Step 1: Recognize the original line: $x = 7$ is vertical, slope undefined.

Step 2: The perpendicular line must be horizontal: $y = k$.

Step 3: Use the point $(4, 5)$:
Since the line passes through this point and is horizontal, the equation is simply:
$$y = 5$$

In point-slope form, this can be written as:
$$y - 5 = 0 \times (x - 4)$$

which simplifies to
$$y = 5$$

Special Cases and Considerations

While the above methods are generally straightforward, some special situations require attention:

1. Horizontal and Vertical Lines

- Original line is horizontal: $y = k$ (slope 0).
The perpendicular line is vertical: $x = c$.
Since point-slope form cannot represent vertical lines (which have undefined slope), the equation is written in standard form $x = c$.

- Original line is vertical: $x = c$.
The perpendicular line is horizontal: $y = y_1$.

2. Lines with Zero or Infinite Slopes

- When dealing with lines with zero slope, the perpendicular line must have an undefined slope, and vice versa.

3. Choosing the Point

- If no specific point is provided, you may choose a point on the original line or any point of interest on the perpendicular line to write the equation.

4. Multiple Perpendicular Lines

- The process described applies to any point and line, but there are infinitely many perpendicular lines passing through different points. Clarify the point of passage to specify the exact line.

Summary and Key Takeaways

- To find the equation in point-slope form of a line perpendicular to a given line, first determine the slope of the original line.
- The perpendicular slope is the negative reciprocal of the original slope.
- Use the known point the new line passes through and the perpendicular slope to write the equation in point-slope form.
- Be mindful of special cases involving vertical and horizontal lines, which require alternative forms.
- Practice with diverse examples to master the concepts, including lines in various forms and passing through different points.

Conclusion

Understanding how

3 Andwhat Is The Equation In Point Slope Form Of The Line Thatis Perpendicular To The Given Line And

3 Andwhat Is The Equation In Point Slope Form Of The Line Thatis Perpendicular To The Given Line And

Related Articles

- & RecommendationsSkill PlantsEureka Math - 7th Grade>5.8 Solve Two-step Equations: Word Problems
- When Susan B. Anthony Registered To Vote, Some Said She Broke The Law.Which Statements Name Results Of
- $1,000 Par Value Zero-coupon Bonds (ignore Liquidity Premiums) Bond Years To Maturity

Yield To Maturity

Back to Home