3. Find Y Using Log Differentiona) (x^3) Tanx=y B) Y=(sin X)^5x4. Dindequation Of Tangent LineTo Curve:

3. Find Y Using Log Differentiation (x^3) Tan x = y B) Y = (sin x)^5 x 4. Find the equation of the tangent line to the curve:

Introduction

In calculus, differentiating complex functions can sometimes be challenging, especially when the functions involve products, powers, and trigonometric expressions. Logarithmic differentiation is a powerful technique that simplifies the differentiation process for such functions. This article aims to guide you through the process of finding the derivative of the given functions using log differentiation, as well as determining the tangent line at a specific point on the curve.

Understanding Logarithmic Differentiation

What is Logarithmic Differentiation?

Logarithmic differentiation involves taking the natural logarithm (ln) of both sides of an equation to simplify the differentiation of functions that are complicated products, quotients, or powers. This technique leverages properties of logarithms:

- ln(ab) = ln a + ln b
- ln(a/b) = ln a - ln b
- ln(a^b) = b ln a

By applying ln, differentiation becomes more manageable, especially when the original function involves exponents and products.

When to Use Logarithmic Differentiation

Log differentiation is particularly useful when:

- The function involves products of functions.
- The function has variables raised to variable powers.
- The function is a composite of multiple functions with powers and products.

Part A: Find Y Using Log Differentiation for (x^3) Tan x = y

Step 1: Write the equation

Given:

$$
x^3 \tan x = y
$$

Our goal is to find $\frac{dy}{dx}$.

Step 2: Apply natural logarithm to both sides

$$
\ln y = \ln (x^3 \tan x)
$$

Using the properties of logarithms:

$$
\ln y = \ln x^3 + \ln \tan x
$$

$$
\ln y = 3 \ln x + \ln \tan x
$$

Step 3: Differentiate both sides with respect to x

Applying differentiation:

$$
\frac{1}{y} \frac{dy}{dx} = 3 \cdot \frac{1}{x} + \frac{1}{\tan x} \cdot \sec^2 x
$$

Recall that:

$$
\frac{d}{dx} \ln x = \frac{1}{x}
$$

and

$$
\frac{d}{dx} \ln \tan x = \frac{1}{\tan x} \cdot \sec^2 x
$$

Step 4: Solve for $\frac{dy}{dx}$

Multiply both sides by y:

$$
\frac{dy}{dx} = y \left(\frac{3}{x} + \frac{\sec^2 x}{\tan x} \right)
$$

Recall that $y = x^3 \tan x$, so:

$$
\boxed{
\frac{dy}{dx} = x^3 \tan x \left(\frac{3}{x} + \frac{\sec^2 x}{\tan x} \right)
}
$$

Step 5: Simplify the derivative

Simplify the expression inside the parentheses:

$$
\frac{3}{x} + \frac{\sec^2 x}{\tan x}
$$

Note that:

$$
\frac{\sec^2 x}{\tan x} = \frac{\sec^2 x}{\frac{\sin x}{\cos x}} = \frac{\sec^2 x \cdot \cos x}{\sin x} = \frac{\sec^2 x \cos x}{\sin x}
$$

Since $\sec x = \frac{1}{\cos x}$, then:

$$
\sec^2 x = \frac{1}{\cos^2 x}
$$

So,

$$
\frac{\sec^2 x \cos x}{\sin x} = \frac{\frac{1}{\cos^2 x} \cdot \cos x}{\sin x} = \frac{\frac{1}{\cos x}}{\sin x} = \frac{1}{\cos x \sin x}
$$

Putting it all together:

$$
\frac{dy}{dx} = x^3 \tan x \left(\frac{3}{x} + \frac{1}{\sin x \cos x} \right)
$$

Part B: Find the derivative of $y = (\sin x)^5 \times 4$

Step 1: Simplify the function

Given:

$$
y = 4 (\sin x)^5
$$

The constant 4 can be factored out:

$$
y = 4 \times (\sin x)^5
$$

Step 2: Use the chain rule

Differentiating:

$$
\frac{dy}{dx} = 4 \times 5 (\sin x)^4 \times \cos x
$$

because:

$$
\frac{d}{dx} (\sin x)^5 = 5 (\sin x)^4 \times \frac{d}{dx} \sin x = 5 (\sin x)^4 \times \cos x
$$

Step 3: Final derivative expression

$$
\boxed{
\frac{dy}{dx} = 20 (\sin x)^4 \cos x
}
$$

Part C: Finding the Equation of the Tangent Line to the Curve

Suppose we want to find the tangent line at a specific point $x = x_0$ on the curve $y = (\sin x)^5 \times 4$.

Step 1: Find the point on the curve

Calculate y at $x = x_0$:

$$
y_0 = 4 (\sin x_0)^5
$$

Step 2: Find the slope of the tangent line

Using the derivative obtained earlier:

\[

$$
m = \frac{dy}{dx} \bigg|_{x = x_0} = 20 (\sin x_0)^4 \cos x_0
$$

Step 3: Write the equation of the tangent line

Using the point-slope form:

$$
Y - y_0 = m (X - x_0)
$$

which becomes:

$$
Y - 4 (\sin x_0)^5 = 20 (\sin x_0)^4 \cos x_0 (X - x_0)
$$

This is the equation of the tangent line to the curve at $x = x_0$.

Summary and Key Takeaways

- Logarithmic differentiation is a powerful tool for differentiating functions involving products and powers.
- Applying natural logarithms simplifies complex derivatives, especially when dealing with functions like $x^3 \tan x$ or $(\sin x)^5$.
- Always differentiate carefully, leveraging properties of logarithms and the chain rule.
- Finding the tangent line involves calculating the derivative at a point and using the point-slope form of a line.

Conclusion

Mastering the technique of log differentiation enhances your ability to handle complex derivatives efficiently. By understanding the process outlined above, you can confidently differentiate functions involving products, powers, and trigonometric functions. Additionally, applying these derivatives to find tangent lines provides valuable insights into the behavior of curves, essential in both theoretical and applied calculus contexts. Whether dealing with academic problems or real-world applications, these tools are fundamental for advanced mathematical analysis.

Frequently Asked Questions

How do you find the derivative of y = (x^3) tan x using logarithmic differentiation?

Take natural logs on both sides: ln y = ln(x^3) + ln(tan x). Differentiate implicitly: (1/y) dy/dx = 3/x + sec^2 x / tan x. Then, multiply both sides by y to find dy/dx.

What is the step-by-step process of applying logarithmic differentiation to y = (x^3) tan x?

1. Take ln of both sides: ln y = 3 ln x + ln tan x. 2. Differentiate implicitly: (1/y) dy/dx = (3/x) + (sec^2 x / tan x). 3. Solve for dy/dx: dy/dx = y [(3/x) + (sec^2 x / tan x)]. 4. Substitute y back as (x^3) tan x.

How can logarithmic differentiation help differentiate y = (sin x)^5 4?

Rewrite as y = 4 (sin x)^5. Take ln both sides: ln y = ln 4 + 5 ln sin x. Differentiate: (1/y) dy/dx = 0 + 5 cot x. Multiply both sides by y: dy/dx = y 5 cot x = 4 (sin x)^5 5 cot x.

How do you find the derivative of y = (sin x)^5 4 using logarithmic differentiation?

Express as y = 4 (sin x)^5. Take natural logs: ln y = ln 4 + 5 ln sin x. Differentiate: (1/y) dy/dx = 0 + 5 cot x. Then, dy/dx = y 5 cot x = 4 (sin x)^5 5 cot x.

What is the derivative of y = (x^3) tan x after applying logarithmic differentiation?

The derivative is dy/dx = (x^3) tan x [(3/x) + sec^2 x / tan x]. Simplify to get dy/dx = (x^3) [tan x (3/x) + sec^2 x].

How do you find the slope of the tangent line to y = (x^3) tan x at a point using logarithmic differentiation?

Differentiate y using log differentiation to find dy/dx. Evaluate dy/dx at the given x-value. Then, use the point-slope form: $y - y_1 = m (x - x_1)$, where m = dy/dx at that point.

Can logarithmic differentiation be used for y = (sin x)^5 4? How?

Yes. Rewrite as y = 4 (sin x)^5. Take natural logs: ln y = ln 4 + 5 ln sin x. Differentiate both sides to find dy/dx, then substitute back y to get the derivative.

What are the benefits of using logarithmic differentiation in these types of functions?

It simplifies the differentiation of products, quotients, and powers, especially when functions involve variables raised to powers or multiplied together, making the process more manageable.

What is the derivative of the tangent line to the curve y = (x^3) tan x at a

specific point?

First, find dy/dx using logarithmic differentiation. Then, substitute the x-value into dy/dx to get the slope. Use point-slope form with the point coordinates to write the tangent line equation.

Additional Resources

Find Y Using Log Differentiation: A Deep Dive into Derivatives of Complex Functions

In the realm of calculus, differentiation techniques are foundational tools enabling us to analyze the behavior of functions, determine slopes of tangent lines, and understand rates of change. Among these methods, logarithmic differentiation stands out as a powerful technique, particularly when dealing with functions involving exponents, products, or quotients that are otherwise cumbersome to differentiate directly. This article explores the application of logarithmic differentiation to two intriguing functions, delves into their derivatives, and discusses how to find tangent lines to their curves.

Introduction to Logarithmic Differentiation

Logarithmic differentiation leverages the properties of logarithms to simplify the differentiation process of complex functions. It is especially useful when:

- The function is a product or quotient of multiple functions.
- The function involves variables raised to variable powers.
- Direct differentiation leads to unwieldy expressions.

Basic Procedure of Log Differentiation:

1. Take the natural logarithm of both sides of the function $y = f(x)$: $\ln y = \ln f(x)$.
2. Simplify the right side using logarithmic properties:
- $\ln(ab) = \ln a + \ln b$
- $\ln(a/b) = \ln a - \ln b$
- $\ln a^b = b \ln a$
3. Differentiate implicitly with respect to x:
- $\frac{1}{y} \frac{dy}{dx} =$ derivative of the right side.
4. Solve for $\frac{dy}{dx}$:
- $\frac{dy}{dx} = y \times$ (derivative of the right side).

This technique transforms complicated derivatives into manageable expressions, making it invaluable for functions with complex exponents or compositions.

Part 1: Differentiating $y = (x^3) \tan x$ Using Logarithmic Differentiation

Understanding the Function

The function:

$$ y = (x^3) \tan x $$

is a product of two functions: x^3 and $\tan x$. Direct differentiation using the product rule yields:

$$
\frac{dy}{dx} = 3x^2 \tan x + x^3 \sec^2 x
$$

However, when the functions involved become more complex, or when generalizing for different exponents, logarithmic differentiation offers a cleaner approach.

Applying Logarithmic Differentiation

1. Take the natural logarithm:

$$
\ln y = \ln (x^3 \tan x)
$$

2. Use logarithmic properties:

$$
\ln y = \ln x^3 + \ln \tan x = 3 \ln x + \ln \tan x
$$

3. Differentiate both sides implicitly:

$$
\frac{1}{y} \frac{dy}{dx} = 3 \times \frac{1}{x} + \frac{1}{\tan x} \times \sec^2 x
$$

Note that:

$$
\frac{d}{dx} \ln x = \frac{1}{x}
$$

and

$$
\frac{d}{dx} \ln \tan x = \frac{1}{\tan x} \times \sec^2 x
$$

4. Solve for $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = y \left(\frac{3}{x} + \frac{\sec^2 x}{\tan x} \right)
$$

Recall that $y = x^3 \tan x$, so plugging back:

$$
\boxed{
\frac{dy}{dx} = x^3 \tan x \left(\frac{3}{x} + \frac{\sec^2 x}{\tan x} \right)
}
$$

This simplifies to:

$$
\frac{dy}{dx} = x^3 \tan x \times \frac{3}{x} + x^3 \tan x \times \frac{\sec^2 x}{\tan x}
$$

which further simplifies to:

$$
\frac{dy}{dx} = 3x^2 \tan x + x^3 \sec^2 x
$$

Remarkably, the derivatives obtained via logarithmic differentiation match those derived directly using the product rule, confirming the method's validity.

Part 2: Differentiating $y = (\sin x)^5 x^4$ Using Logarithmic Differentiation

Understanding the Function

The second function:

$$
y = (\sin x)^5 x^4
$$

is a product of two functions, with one being a power of $\sin x$. The presence of $(\sin x)^5$ makes direct differentiation more complex, especially when considering the chain rule.

Applying Logarithmic Differentiation

1. Take the natural logarithm:

$$
\ln y = \ln (\sin x)^5 + \ln x^4
$$

2. Simplify:

$$
\ln y = 5 \ln \sin x + 4 \ln x
$$

3. Differentiate both sides:

$$
\frac{1}{y} \frac{dy}{dx} = 5 \times \frac{1}{\sin x} \times \cos x + 4 \times \frac{1}{x}
$$

Here, we've used:

$$
\frac{d}{dx} \ln \sin x = \frac{\cos x}{\sin x} = \cot x
$$

4. Solving for $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = y \left(5 \cot x + \frac{4}{x} \right)
$$

Recall that $y = (\sin x)^5 x^4$, so:

$$
\boxed{
\frac{dy}{dx} = (\sin x)^5 x^4 \left(5 \cot x + \frac{4}{x} \right)
}
$$

This expression provides the derivative of the function, simplifying the differentiation process especially for functions involving powers of trigonometric functions.

Part 3: Finding the Equation of the Tangent Line to the Curve

Having found the derivatives, a natural progression is to determine the tangent line at a specific point $x = x_0$. The tangent line at x_0 is given by:

$$
y = y(x_0) + \left. \frac{dy}{dx} \right|_{x = x_0} (x - x_0)
$$

where:

- $y(x_0)$ is the function value at x_0.
- $\left. \frac{dy}{dx} \right|_{x = x_0}$ is the derivative evaluated at x_0.

Procedure to find the tangent line:

1. Choose a point x_0 based on the context or specific problem.
2. Compute $y(x_0)$ using the original function.
3. Compute $\frac{dy}{dx}$ at x_0.
4. Plug into the tangent line formula.

Conclusion: The Power and Utility of Logarithmic Differentiation

Logarithmic differentiation serves as an essential technique in the calculus toolkit for handling complex functions. Its ability to convert multiplication into addition, powers into products, and quotients into differences simplifies the differentiation process significantly.

In the context of the functions analyzed:

- For \(y = (x^3) \tan x \), the method confirms the derivative obtained via the product rule and offers a streamlined approach for more complicated variants.
- For \(y = (\sin x)^5 x^4 \), it simplifies differentiating a product involving a power of a trigonometric function, which would otherwise require more intricate chain and product rule applications.

Implications for practitioners and students: Mastery of logarithmic differentiation enhances problem-solving efficiency, especially in advanced calculus, differential equations, and real-world applications involving exponential growth, decay, or oscillatory phenomena.

Final Remarks

Understanding how to find \(y \) using log differentiation not only deepens comprehension of derivative rules but also broadens the scope of functions that can be tackled systematically. Whether analyzing the slope of a curve or deriving tangent lines, this technique proves indispensable in both academic and applied mathematics.

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