4.9.14Solve The Following System By Substitution.{y = X2 + 6x + 10y = -X+4Select The Correct Choice And

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Understanding how to solve systems of equations by substitution is a fundamental skill in algebra that enables students and mathematicians to find the exact values of variables that satisfy multiple equations simultaneously. In this comprehensive guide, we will explore the process of solving the system:

$$ y = x^2 + 6x + 10 $$
$$ y = -x + 4 $$

by substitution method, analyze key concepts, step-by-step procedures, and select the correct solution. This article aims to provide clarity for learners at various levels while optimizing for SEO keywords such as "solving systems by substitution," "algebraic system solutions," and "linear and quadratic systems."

Understanding the System of Equations

Before diving into the solution process, it’s essential to understand the types of equations involved in this system.

Types of Equations in the System

- The first equation, $y = x^2 + 6x + 10$, is a quadratic equation. It represents a parabola when graphed.
- The second equation, $y = -x + 4$, is a linear equation, which graphs as a straight line.

Goal of the Solution

The primary goal is to find the point(s) (x, y) that satisfy both equations simultaneously — meaning the points where the parabola and the line intersect.

Key Concepts in Solving Systems by Substitution

What is the Substitution Method?

The substitution method involves solving one of the equations for one variable and then substituting that expression into the other equation. This reduces the system to a single equation with one variable, which can be solved using algebraic techniques.

Why Use Substitution?

- Effective when one equation is already solved for a variable.
- Simplifies the process of solving systems involving both quadratic and linear equations.
- Provides exact solutions of the intersection points.

Important Tips for Using Substitution

- Always verify your solutions by substituting back into both original equations.
- Be cautious with the algebraic manipulations to avoid errors.
- Keep track of multiple solutions, especially in quadratic equations.

Step-by-Step Guide to Solve the System

Step 1: Recognize the Equations and Isolate a Variable

In this system, the first equation is already solved for y:

$$y = x^2 + 6x + 10$$

The second equation also expresses y:

$$y = -x + 4$$

Since both equations are equal to y, we can set them equal to each other:

$$x^2 + 6x + 10 = -x + 4$$

Step 2: Set Up the Equation for x

By equating the two expressions for y:

$$x^2 + 6x + 10 = -x + 4$$

Rearranged to standard quadratic form:

$$x^2 + 6x + 10 + x - 4 = 0$$

Simplify:

$$x^2 + (6x + x) + (10 - 4) = 0$$
$$x^2 + 7x + 6 = 0$$

Step 3: Solve the Quadratic Equation

The quadratic equation:

$$x^2 + 7x + 6 = 0$$

can be factored or solved using the quadratic formula. Let's factor:

Find two numbers that multiply to 6 and add to 7:

- 6 and 1, since $6 \times 1 = 6$ and $6 + 1 = 7$

Thus, factorization:

$$(x + 6)(x + 1) = 0$$

Solutions for x:

$$x + 6 = 0 \Rightarrow x = -6$$
$$x + 1 = 0 \Rightarrow x = -1$$

Step 4: Find Corresponding y Values

Substitute each x value into either original equation for y. Using the linear equation $y = -x + 4$:

- For $x = -6$:

$$ y = -(-6) + 4 = 6 + 4 = 10 $$

- For $x = -1$:

$$ y = -(-1) + 4 = 1 + 4 = 5 $$

Step 5: Write the Solution Points

The solutions where the parabola and line intersect are:

- $(-6, 10)$
- $(-1, 5)$

Verification of Solutions

To ensure accuracy, substitute the solutions into both original equations:

- For $(-6, 10)$:

- Equation 1: $y = x^2 + 6x + 10$

$$ 10 = (-6)^2 + 6(-6) + 10 = 36 - 36 + 10 = 10 \quad \checkmark $$

- Equation 2: $y = -x + 4$

$$ 10 = -(-6) + 4 = 6 + 4 = 10 \quad \checkmark $$

- For $(-1, 5)$:

- Equation 1:

$$ 5 = (-1)^2 + 6(-1) + 10 = 1 - 6 + 10 = 5 \quad \checkmark $$

- Equation 2:

$$ 5 = -(-1) + 4 = 1 + 4 = 5 \quad \checkmark $$

Both solutions satisfy the system, confirming their correctness.

Choosing the Correct Solution

When solving systems involving quadratic and linear equations via substitution, multiple solutions may exist, or sometimes no real solutions at all. In this example, we found two solutions. Understanding how to interpret and select the correct solution involves:

1. Graphical Interpretation: The solutions represent points of intersection between the parabola and the line.
2. Application Context: In real-life scenarios, consider the domain restrictions that may apply.
3. Algebraic Verification: Always verify solutions by substitution back into the original equations.

Additional Tips for Solving Systems by Substitution

- Use substitution when one variable is already isolated or easily solvable for.
- For more complex systems, consider substitution alongside other methods such as elimination or graphing.
- Always check solutions thoroughly to avoid algebraic mistakes.
- Be mindful of extraneous solutions, especially when dealing with square roots or denominators.

Conclusion

Solving systems of equations by substitution is a powerful technique, especially when dealing with systems involving both linear and quadratic equations. In the example provided, setting the two expressions for y equal led to a quadratic equation that was straightforward to factor. The solutions $(-6, 10)$ and $(-1, 5)$ reveal the points where the parabola $y = x^2 + 6x + 10$ intersects the line $y = -x + 4$. Mastering this method enhances problem-solving skills and deepens understanding of algebraic concepts.

Remember: Always verify your solutions and consider the context of the problem. With practice, solving systems by substitution becomes an intuitive and efficient process, essential for advanced mathematics, engineering, and science applications.

Keywords: solving systems by substitution, algebraic system solutions, quadratic and linear equations, intersection points, algebra practice, math problem solving

Frequently Asked Questions

What is the first step to solve the system using substitution when $y = x^2 + 6x + 10$ and $y = -x + 4$?

Substitute $y = x^2 + 6x + 10$ into the second equation to eliminate y and solve for x.

How do you find the value of x after substituting y in the system?

Set $x^2 + 6x + 10$ equal to $-x + 4$ and solve the resulting quadratic equation.

What is the quadratic equation obtained after substitution in this system?

$x^2 + 7x + 6 = 0$, derived from setting $x^2 + 6x + 10 = -x + 4$.

What are the solutions for x after solving the quadratic equation?

$x = -1$ or $x = -6$, obtained by factoring $x^2 + 7x + 6 = 0$.

How do you find the corresponding y-values once x is known?

Substitute each x-value back into $y = x^2 + 6x + 10$ to find the corresponding y-values.

Additional Resources

Solving Systems of Equations by Substitution is a fundamental skill in algebra that allows students and learners to find solutions where two or more equations intersect. In this comprehensive guide, we will walk through the process of solving a specific system of equations step-by-step, illustrating the method of substitution clearly and thoroughly. Our focus will be on the system:

$$ y = x^2 + 6x + 10 $$
$$ y = -x + 4 $$

This example demonstrates the practical application of substitution, a versatile method especially useful when one of the equations is solved for one variable in terms of the other. Let’s explore how to approach this problem systematically, ensuring clarity for learners at various levels.

Understanding the System of Equations

Before diving into solutions, it’s crucial to understand what a system of equations represents. In this context, each equation describes a relationship between the variables x and y. The solution to the system is the point(s) (x, y) where both equations are satisfied simultaneously—that is, the point(s) at which their graphs intersect.

Our given system:

1. $y = x^2 + 6x + 10$ – a quadratic (parabola)
2. $y = -x + 4$ – a straight line

Graphically, the solution involves finding the intersection point(s) of the parabola and the line.

Step 1: Recognizing the Substitution Method

Since both equations are solved for y, we can leverage substitution directly. The key insight is:

- The value of y in both equations is equal.
- Therefore, we can set the right-hand sides equal to each other:

$$ x^2 + 6x + 10 = -x + 4 $$

This step simplifies the system from two equations with two variables to a single equation with one variable, which we can solve more straightforwardly.

Step 2: Set Up the Equation for x

Equation for substitution:

$$ x^2 + 6x + 10 = -x + 4 $$

Bring all terms to one side:

$$ x^2 + 6x + 10 + x - 4 = 0 $$

Combine like terms:

$$ x^2 + (6x + x) + (10 - 4) = 0 $$

$$ x^2 + 7x + 6 = 0 $$

This is a quadratic equation in standard form.

Step 3: Solve the Quadratic Equation

Quadratic equations can be solved through various methods: factoring, quadratic formula, or completing the square.

Factoring Method

Check if the quadratic factors easily:

$$ x^2 + 7x + 6 = 0 $$

Find two numbers that multiply to 6 and add to 7:

- 6 and 1

So, the factors are:

$$ (x + 6)(x + 1) = 0 $$

Solutions for x:

$$ x + 6 = 0 \Rightarrow x = -6 $$
$$ x + 1 = 0 \Rightarrow x = -1 $$

Step 4: Find Corresponding y Values

Recall the original equations for y:

$$ y = -x + 4 $$

Plug in each x value to find y:

For $x = -6$:

$$ y = -(-6) + 4 = 6 + 4 = 10 $$

For $x = -1$:

$$ y = -(-1) + 4 = 1 + 4 = 5 $$

Final Solutions:

x	y
-6	10
-1	5

The solutions to the system are $(-6, 10)$ and $(-1, 5)$.

Graphical Interpretation

- The point $(-6, 10)$ lies on both the parabola $y = x^2 + 6x + 10$ and the line $y = -x + 4$.
- Similarly, $(-1, 5)$ is the intersection point of the same two graphs.

Plotting these points along with their respective graphs can visually verify the solutions. The parabola opens upward, and the line slopes downward, intersecting at these two points.

Additional Tips for Solving Systems by Substitution

- Choose the equation that is easiest to solve for a variable: When one equation is already solved for a variable, substitution becomes straightforward.
- Double-check your algebra: Mistakes in combining like terms or factoring are common pitfalls.
- Verify solutions: Plug each solution back into both original equations to confirm they satisfy both.

Summary

The method of solving systems by substitution involves:

1. Expressing one variable in terms of the other (or using the given expressions).
2. Substituting this expression into the other equation to create a single-variable equation.
3. Solving the resulting equation for the variable.
4. Substituting back to find the corresponding value of the other variable.
5. Verifying solutions by plugging them into the original equations.

This approach simplifies complex systems and provides clear insight into their solutions, especially when one of the equations is already solved for a variable or easily manipulated.

Practice Problems

To strengthen your understanding, try solving these systems using substitution:

1. \(y = 2x + 3 \) and \(y = x^2 - 1 \)
2. \(y = -x^2 + 4x \) and \(y = 3x - 2 \)
3. \(y = x^2 + 2x + 1 \) and \(y = -x^2 + 4 \)

Working through these problems will enhance your familiarity with the substitution method and prepare you for more complex systems.

Conclusion

Mastering the solve the following system by substitution approach is essential for tackling a wide variety of algebraic problems. By understanding the process—setting equations equal when both are solved for the same variable, transforming into a single-variable quadratic, and then back-substituting—you develop a powerful tool that extends into calculus, physics, engineering, and beyond. Keep practicing, and you'll find that solving systems becomes more intuitive and efficient over time.

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