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5. Find The Distance Between The Point Q = (2,5,6) And The Line I(T) = (-1,1,1) + T(1,0,1)

Calculating the shortest distance between a point and a line in three-dimensional space is a fundamental problem in vector geometry. This problem involves understanding the spatial relationships between a specific point and a line, and it leverages vector operations such as subtraction, dot product, and cross product. In this section, we will explore the method to find the distance from the point Q = (2,5,6) to the given line I(T) = (-1,1,1) + T(1,0,1).

Understanding the Given Data

- **Point Q:** (2, 5, 6)
- **Line I(T):** passes through point P_0 = (-1, 1, 1) and has a direction vector v = (1, 0, 1)

The line can be parametrized as:

$I(T) = P_0 + T \cdot v$

Conceptual Approach

The shortest distance from a point to a line in three-dimensional space can be found using the following method:

1. Identify a point on the line, P_0.
2. Determine the vector from P_0 to the given point Q, which is Q - P_0.
3. Calculate the cross product of this vector with the direction vector of the line, v.
4. Compute the magnitude of the cross product.

5. Calculate the magnitude of the direction vector v.
6. Use the formula for the shortest distance:

$$\text{Distance} = |(Q - P_0) \times v| / |v|$$

Step-by-Step Calculation

Step 1: Find the vector $Q - P_0$

$Q = (2, 5, 6)$

$P_0 = (-1, 1, 1)$

$Q - P_0 = (2 - (-1), 5 - 1, 6 - 1) = (3, 4, 5)$

Step 2: Cross product of $(Q - P_0)$ and v

$v = (1, 0, 1)$

$$(Q - P_0) \times v = \begin{vmatrix} i & j & k \\ 3 & 4 & 5 \\ 1 & 0 & 1 \end{vmatrix}$$

Calculating the determinant:

- i component: $(4)(1) - (5)(0) = 4 - 0 = 4$
- j component: $- [(3)(1) - (5)(1)] = - (3 - 5) = - (-2) = 2$
- k component: $(3)(0) - (4)(1) = 0 - 4 = -4$

Thus, $(Q - P_0) \times v = (4, 2, -4)$

Step 3: Magnitude of the cross product

$|(Q - P_0) \times v| = \sqrt{4^2 + 2^2 + (-4)^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$

Step 4: Magnitude of the direction vector v

$|v| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{1 + 0 + 1} = \sqrt{2}$

Step 5: Calculate the distance

Distance = $|(Q - P_0) \times v| / |v| = 6 / \sqrt{2} = 6\sqrt{2} / 2 = 3\sqrt{2}$

Final Answer

The shortest distance from the point Q = (2, 5, 6) to the line I(T) = (-1, 1, 1) + T(1, 0, 1) is **$3\sqrt{2}$ units**.

6. Determine

Since the prompt for question 6 is incomplete, we will interpret it as an extension of the previous problem, possibly asking to determine the foot of the perpendicular from Q to the line or to find the point on the line closest to Q.

Finding the Foot of the Perpendicular (Closest Point on the Line to Q)

Objective

Find the point P on the line I(T) such that the distance between Q and P is minimized. This point P is the projection of Q onto the line, and it can be found using vector projection techniques.

Method

1. Express any point P on the line as $P = P_0 + T \cdot v$.
2. Find T such that the vector Q - P is perpendicular to v, i.e., $(Q - P) \cdot v = 0$.

Calculations

Step 1: Write P in terms of T

- P = (-1, 1, 1) + T(1, 0, 1)

Step 2: Set up the perpendicularity condition

- $(Q - P) \cdot v = 0$
- $Q - P = (2 - (-1 + T), 5 - (1 + 0 \cdot T), 6 - (1 + T)) = (3 - T, 4, 5 - T)$

Step 3: Compute the dot product with v

- $(Q - P) \cdot v = (3 - T)(1) + (4)(0) + (5 - T)(1) = (3 - T) + 0 + (5 - T) = (3 + 5) - T - T = 8 - 2T$

Step 4: Solve for T

- Set equal to zero: $8 - 2T = 0$
- $2T = 8$
- $T = 4$

Step 5: Find the closest point P on the line

- $P = P_0 + T \cdot v = (-1, 1, 1) + 4(1, 0, 1) = (-1 + 4, 1 + 0, 1 + 4) = (3, 1, 5)$

Conclusion

The point on the line closest to Q = (2, 5, 6) is **P = (3, 1, 5)**. The shortest distance between Q and the line is therefore the distance between Q and P, which can be computed as:

Distance between Q and P

$Q - P = (2 - 3, 5 - 1, 6 - 5) = (-1, 4, 1)$

$|Q - P| = \sqrt{(-1)^2 + 4^2 + 1^2} = \sqrt{1 + 16 + 1} = \sqrt{18} = 3\sqrt{2}$

This confirms that the minimal distance from Q to the line is indeed **$3\sqrt{2}$ units**, matching our previous calculation.

Summary

- The shortest distance from the point Q = (2, 5, 6) to the line $I(T) = (-1,1,1) + T(1,0,1)$ is **$3\sqrt{2}$** units.
- The closest point on the line to Q is **(3, 1, 5)**.
- The method involves vector subtraction, cross product calculations, and solving for the parameter T that minimizes the distance.

Additional Remarks

Understanding the geometric relationships in three-dimensional space enables solving various problems involving distances, projections, and shortest paths. The techniques demonstrated here are broadly applicable in fields such as computer graphics, physics, engineering, and more.

Frequently Asked Questions

How do I find the shortest distance between the point Q = (2, 5, 6) and the line given by I(T) = (-1, 1, 1) + T(1, 0, 1)?

To find the shortest distance, first find the vector from the point Q to a point on the line, for example, from Q to I(0) = (-1, 1, 1). Then, compute the cross product of this vector with the direction vector of the line, (1, 0, 1). The magnitude of this cross product divided by the magnitude of the direction vector gives the shortest distance.

What is the formula for calculating the distance from a point to a line in 3D space?

The shortest distance 'd' from a point Q to a line passing through point P0 with direction vector v is given by: d = |(Q - P0) × v| / |v|, where '×' denotes the cross product.

How do I compute the vector from the point Q to a point on the line for this problem?

Choose a point on the line, such as I(0) = (-1, 1, 1). Then, subtract P0 from Q: (2 - (-1), 5 - 1, 6 - 1) = (3, 4, 5).

What are the steps to find the cross product in this context?

Given vectors A = (3, 4, 5) and v = (1, 0, 1), compute A × v using the determinant method: |i j k|; |3 4 5|; |1 0 1|, which results in (41 - 50, 51 - 31, 30 - 41) = (4, 2, -4).

How do I calculate the magnitude of the cross product to find the distance?

Calculate the magnitude of the vector (4, 2, -4): |(4, 2, -4)| = sqrt(4^2 + 2^2 + $(-4)^2$) = sqrt(16 + 4 + 16) = sqrt(36) = 6.

What is the magnitude of the line's direction vector in this problem?

The direction vector is v = (1, 0, 1). Its magnitude is |v| = sqrt(1^2 + 0^2 + 1^2) = sqrt(1 + 0 + 1) = sqrt(2).

How do I compute the final shortest distance between Q and the line?

Divide the magnitude of the cross product by the magnitude of the direction vector: 6 / sqrt(2) = 6 / 1.4142 ≈ 4.24 units.

Can I verify the result by geometric intuition or alternative methods?

Yes, you can verify by projecting the vector from P0 to Q onto the direction vector and checking the perpendicular component, or use vector projection formulas to confirm the shortest distance.

Additional Resources

Finding the distance between a point and a line is a fundamental problem in coordinate geometry with applications spanning physics, engineering, computer graphics, and more. When given a point Q = (2, 5, 6) and a line defined parametrically as I(T) = (-1, 1, 1) + T(1, 0, 1), understanding how to accurately compute the shortest distance between them is essential. This guide provides a detailed, step-by-step approach to solving this problem, breaking down the concepts, formulas, and calculations involved.

Understanding the Problem

Given:
- A point Q in 3D space: Q = (2, 5, 6)
- A line defined parametrically: I(T) = (-1, 1, 1) + T(1, 0, 1)

Goal: Find the shortest distance from the point Q to the line I(T).

Conceptual Framework

Why is this important?

Determining the distance from a point to a line in three dimensions is a common task when analyzing spatial relationships. It can be used to:

- Calculate the shortest path or minimal separation between objects
- Determine the proximity of a point to a trajectory
- Solve geometric problems involving perpendicular distances

Key ideas involved:

- Vector operations: dot product, cross product
- Parametric equations: expressing lines in terms of a parameter T
- Perpendicular distance: the shortest distance is along the line perpendicular to the given line passing through the point

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Step-by-Step Breakdown

Step 1: Understand the line's parametric form

The line I(T) is given as:
- Point on the line: P_0 = (-1, 1, 1)
- Direction vector: D = (1, 0, 1)

Any point P(T) on the line can be written as:

$$ P(T) = P_0 + T \times D = (-1 + T, 1 + 0 \times T, 1 + T) $$

Step 2: Visualize the problem

The shortest distance from Q to line I(T) is the length of the perpendicular segment from Q to some point P(T) on the line. The goal is to find the value of T that minimizes this distance.

Step 3: Find the vector from P_0 to Q

Construct the vector from the point on the line at T=0, P_0, to Q:

$$
\vec{v} = Q - P_0 = (2 - (-1), 5 - 1, 6 - 1) = (3, 4, 5)
$$

Step 4: Use the projection formula

The shortest distance from a point to a line is given by the formula:

$$
d = \frac{|\vec{v} \times \vec{D}|}{|\vec{D}|}
$$

where:
- $\vec{v} = Q - P_0$
- $\vec{D}$ is the direction vector of the line

This formula calculates the magnitude of the cross product of $\vec{v}$ and $\vec{D}$, divided by the magnitude of $\vec{D}$.

Step 5: Calculate the cross product $\vec{v} \times \vec{D}$

$$
\vec{v} \times \vec{D} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & 4 & 5 \\
1 & 0 & 1 \\
\end{vmatrix}
$$

Compute the determinant:

- $\mathbf{i}$ component:
$$
(4)(1) - (5)(0) = 4 - 0 = 4
$$

- $\mathbf{j}$ component:
$$
- [(3)(1) - (5)(1)] = - (3 - 5) = - (-2) = 2
$$

- $\mathbf{k}$ component:
$$
(3)(0) - (4)(1) = 0 - 4 = -4
$$

So:
$$
\vec{v} \times \vec{D} = (4, 2, -4)
$$

Step 6: Calculate the magnitude of the cross product

$$
|\vec{v} \times \vec{D}| = \sqrt{4^2 + 2^2 + (-4)^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6
$$

Step 7: Calculate the magnitude of the direction vector $\vec{D}$

$$
|\vec{D}| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{1 + 0 + 1} = \sqrt{2}
$$

Step 8: Compute the shortest distance

Using the formula:

$$
d = \frac{|\vec{v} \times \vec{D}|}{|\vec{D}|} = \frac{6}{\sqrt{2}} = 6 \times \frac{\sqrt{2}}{2} = 3 \sqrt{2}
$$

Final Result

The shortest distance from point Q = (2, 5, 6) to the line I(T) = (-1, 1, 1) + T(1, 0, 1) is:

$$ d = 3 \sqrt{2} $$ units

Additional Insights and Variations

Finding the exact point on the line closest to Q

If needed, we can also find the specific point $P(T^)$ on the line that is closest to Q by solving for T using the projection:

$$
T^ = \frac{(Q - P_0) \cdot D}{|D|^2}
$$

Calculations:

$$
(Q - P_0) \cdot D = (3, 4, 5) \cdot (1, 0, 1) = 3 \times 1 + 4 \times 0 + 5 \times 1 = 3 + 0 + 5 = 8
$$

$$
|D|^2 = 1^2 + 0^2 + 1^2 = 2
$$

$$
T^ = \frac{8}{2} = 4
$$

Thus, the closest point on the line is:

$$
P(4) = (-1 + 4, 1 + 0, 1 + 4) = (3, 1, 5)
$$

Verifying the shortest distance

Calculate the vector from Q to P(4):

$$
Q - P(4) = (2 - 3, 5 - 1, 6 - 5) = (-1, 4, 1)
$$

Magnitude:

$$
\sqrt{(-1)^2 + 4^2 + 1^2} = \sqrt{1 + 16 + 1} = \sqrt{18} = 3 \sqrt{2}
$$

which matches our previous calculation, confirming the accuracy.

Practical Applications

Understanding how to find the distance between a point and a line in 3D space is vital for several real-world applications:

- Collision detection in computer graphics and robotics
- Determining the shortest path for navigation
- Calculating proximity in engineering design
- Analyzing trajectories in physics simulations

Summary

To find the distance between Q = (2, 5, 6) and the line I(T) = (-1, 1, 1) + T(1, 0, 1):

1. Identify the point on the line (P_0) and the direction vector (D).
2. Construct the vector from P_0 to Q.
3. Calculate the cross product of this vector with D.
4. Determine the magnitudes of the cross product and D.
5. Compute the ratio to find the shortest distance.

The key formula simplifies the process, leveraging vector operations to provide an elegant solution. Mastery of these techniques enhances spatial reasoning and problem-solving skills in advanced mathematics and applied fields.

Remember: The shortest distance from a point to a line in 3D space is always found along the perpendicular segment, and vector algebra provides a powerful toolkit to compute it efficiently.

5 Find The Distance Between The Point Q 2 5 6 And The Line I T 1 1 1 T 1 0 1 6 Determine

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