5. The Sum Of Two Numbers Is 15. The Difference Between Five Times The First Number And Three Times The

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Introduction

Mathematics is a fascinating subject that often involves solving equations and understanding relationships between numbers. One common type of problem involves two unknown numbers and their relationships, such as sums, differences, and multiples. A classic example is when the sum of two numbers is given, and there's a need to find the individual numbers based on additional conditions. This article explores a specific problem: "The sum of two numbers is 15. The difference between five times the first number and three times the second number." We will analyze this problem thoroughly, providing step-by-step solutions, explanations, and practical applications to help you master similar algebraic challenges.

Understanding the Problem

Before diving into the solution, it is essential to understand what the problem asks:

- The sum of two numbers (say, x and y) is 15.
- The difference between five times the first number and three times the second number is a certain value (which is typically given or to be determined).

In many algebraic problems, the key is to translate words into mathematical equations. Here, the problem provides:

1. $x + y = 15$
2. An expression involving x and y, such as $5x - 3y$.

Depending on the context, the problem might specify the value of $5x - 3y$, or ask to analyze its properties. For clarity, we'll consider the case where the problem asks to find the value of $5x - 3y$ given the sum of the numbers.

Setting Up the Equations

Step 1: Define Variables

Let:

- x = the first number
- y = the second number

Step 2: Write Known Relationships

From the problem:

- Sum of the two numbers:

$$
x + y = 15
$$

- Expression involving multiples:

$$
5x - 3y
$$

Our goal is to find the value of $5x - 3y$, possibly in terms of the given sum.

Solving the Problem Step-by-Step

Step 1: Express one variable in terms of the other

From the sum:

$$
x + y = 15 \implies y = 15 - x
$$

Step 2: Substitute into the target expression

The expression $5x - 3y$ becomes:

$$
5x - 3(15 - x) = 5x - 45 + 3x = (5x + 3x) - 45 = 8x - 45
$$

Thus, the value of $5x - 3y$ depends on x, which is unknown.

Step 3: Find possible values of x

Since x and y are numbers satisfying the sum $x + y = 15$, and no additional constraints are given, x can be any real number between $-\infty$ and ∞, with y adjusted accordingly.

- For example, if $x = 0$, then $y = 15$, and

$$
5x - 3y = 8(0) - 45 = -45
$$

- If $x = 5$, then $y = 10$, and

$$
8(5) - 45 = 40 - 45 = -5
$$

- If $x = 15$, then $y = 0$, and

$$
8(15) - 45 = 120 - 45 = 75
$$

Step 4: General solution

The value of $5x - 3y$ varies depending on x. To find a specific value, additional constraints are necessary, such as:

- The two numbers are integers.
- The numbers satisfy certain inequalities.
- The problem states the value of $5x - 3y$.

If, for example, the problem asks to find the maximum or minimum value of $5x - 3y$, then we can proceed accordingly.

Optimizing the Expression $5x - 3y$

Assuming no further constraints, and wanting to find the maximum or minimum value of $5x - 3y$ given $x + y = 15$, we can analyze the expression as a function of x:

$$
\text{Expression} = 8x - 45
$$

- Since x can be any real number, the expression is unbounded unless constraints are applied.

Scenario 1: Both numbers are non-negative

Suppose $x, y \geq 0$. Then:

$$
x + y = 15 \implies y = 15 - x \geq 0 \implies x \leq 15
$$

and

$$
x \geq 0
$$

Thus, $x \in [0, 15]$.

- At $x = 0$:

$$
8(0) - 45 = -45
$$

- At $x = 15$:

$$
8(15) - 45 = 120 - 45 = 75
$$

Therefore, over the interval $[0, 15]$, $8x - 45$ ranges from -45 to 75. The maximum value is 75 at $x=15$, and the minimum is -45 at $x=0$.

Scenario 2: Both numbers are integers

If the numbers are integers, x can be any integer from 0 to 15, and the same maximum and minimum apply.

Graphical Representation

Understanding the relationship visually can help clarify the problem.

- The line $x + y = 15$ in the coordinate plane is a straight line passing through points $(0, 15)$ and $(15, 0)$.
- The expression $5x - 3y$ can be represented as a family of lines with different constant values, indicating the level curves.

Plotting these lines helps visualize how the value of $5x - 3y$ changes depending on the chosen point along the line $x + y = 15$.

Practical Applications of the Problem

Understanding problems involving sums and multiples of numbers has real-world significance in various fields:

1. Budget Allocation

Suppose you have a total of \$15 to distribute between two projects or departments, with constraints or goals involving multiples of the allocated funds. Calculating the difference between scaled amounts can help optimize resource allocation.

2. Mixture and Composition Problems

In chemistry or manufacturing, mixing two components to reach a specific total quantity involves similar algebraic equations. Adjusting proportions to achieve desired properties relies on understanding such relationships.

3. Profit and Cost Analysis

In business, determining the difference between scaled profits or costs of two products based on total sales or expenses can be modeled using similar equations.

4. Educational Purposes

Problems like these are foundational in algebra education, helping students develop skills in translating word problems into mathematical equations and solving for unknowns.

Variations and Extensions

The basic problem can be extended or modified to explore different mathematical scenarios:

Variation 1: Find the numbers given the difference

Suppose the difference between five times the first number and three times the second number is a specific value, say D. Then:

$$
5x - 3y = D
$$

Along with:

$$
x + y = 15
$$

We can analyze:

$$
8x - 45 = D \implies x = \frac{D + 45}{8}
$$

and

$$
y = 15 - x = 15 - \frac{D + 45}{8}
$$

Variation 2: Find the sum of the two numbers given the difference

If, instead, the difference $5x - 3y$ is known, and the sum is 15, then:

$$
x + y = 15
$$

\[
5x - 3y = D
\]

This forms a system of equations that can be solved for \(x \) and \(y \):

\[
\begin{cases}
x + y = 15 \\
5x - 3y = D
\end{cases}
\]

Solve for \(x \):

\[
x = 15 - y
\]

Substitute into the second equation:

\[
5(15 - y) - 3y = D
\]
\[
75 - 5y - 3y = D
\]
\[
75 - 8y = D
\]
\[
8y = 75 - D
\]
\[
y = \frac{75 - D}{8}
\]

And:

\[
x = 15 - y = 15 - \frac{75 - D}{8} = \frac{120 - (75 - D)}{8} = \frac{120 - 75 + D

Frequently Asked Questions

What is the main goal when solving the problem 'The sum of two numbers is 15' and relating it to their difference?

The main goal is to find the two numbers that satisfy both the sum and the difference conditions, often by setting up and solving algebraic equations.

How do you set up equations for the problem involving the sum of two numbers and their multiple differences?

Let the two numbers be x and y. The sum gives $x + y = 15$, and the difference condition involves an expression like $5x - 3y$, which you set equal to a known value or analyze further based on the problem context.

What algebraic methods are useful for solving problems involving the sum and difference of two numbers?

Methods such as substitution, elimination, and forming systems of equations are useful for solving such problems efficiently.

Can you provide an example of how to solve for two numbers given their sum and a difference involving multiples?

Yes. For example, if $x + y = 15$ and $5x - 3y = D$ (a known difference), you can solve the first for $y = 15 - x$, then substitute into the second to find x and y.

What common mistakes should be avoided when solving these types of problems?

Common mistakes include mixing up the signs, mislabeling variables, or substituting incorrectly. Always double-check algebraic steps and ensure equations are set up correctly.

How does understanding the relationship between the sum and difference help in solving real-world problems?

It helps in modeling situations where two quantities are related by addition and a multiple difference, enabling accurate calculation of individual values in fields like finance, physics, and engineering.

Are there specific strategies to simplify solving for the two numbers in this problem?

Yes, expressing one variable in terms of the other using the sum, then substituting into the difference expression simplifies the problem significantly.

What additional information is needed to fully solve the problem involving the sum and the difference of two numbers?

You need the value of the difference expression (e.g., the specific value of $5x - 3y$) or additional conditions to determine the exact numbers.

How can understanding this problem help develop general problem-solving skills in algebra?

It enhances skills in setting up equations from word problems, manipulating algebraic expressions, and applying logical reasoning to find solutions efficiently.

Additional Resources

The Sum Of Two Numbers Is 15. The Difference Between Five Times The First Number And Three Times The is a classic algebraic problem that serves as an excellent example for understanding systems of equations, problem-solving strategies, and the application of algebra in real-world scenarios. This type of problem is foundational in mathematics education, providing students with the skills necessary to approach more complex problems involving multiple variables. In this comprehensive review, we will explore the problem's formulation, methods to solve it, its educational significance, and practical applications.

Understanding the Problem Statement

At its core, the problem involves two unknown numbers, say x and y, with the following conditions:
- The sum of the two numbers equals 15:
$x + y = 15$
- The difference between five times the first number and three times the second number is considered, though the problem statement appears incomplete. Typically, such problems specify an equality, for example:
$5x - 3y$ = some value

For the purpose of this review, we will assume the full problem states:

"The sum of two numbers is 15. The difference between five times the first number and three times the second number is a specific value, say 5."

Thus, the full set of equations becomes:

1. $x + y = 15$
2. $5x - 3y = 5$

Mathematical Formulation and Approach

Setting Up Equations

The first step involves translating the word problem into algebraic equations, a crucial skill in problem-solving:

- Equation 1 (Sum): $x + y = 15$
- Equation 2 (Difference): $5x - 3y = 5$

From here, the goal is to find the values of x and y that satisfy both equations simultaneously.

Methods of Solution

Several methods are commonly used to solve such systems:

- Substitution Method
- Elimination Method
- Graphical Method
- Using Matrices (for advanced students)

In this review, we focus primarily on substitution and elimination, as they are most accessible and illustrative.

Solving the System of Equations

Using Substitution

1. From Equation 1, express y in terms of x:
$y = 15 - x$

2. Substitute into Equation 2:
$5x - 3(15 - x) = 5$

3. Simplify:
$5x - 45 + 3x = 5$
$8x - 45 = 5$

4. Solve for x:
$8x = 50$
$x = 50 / 8 = 6.25$

5. Find y:
$y = 15 - 6.25 = 8.75$

Solution:
$x = 6.25$, $y = 8.75$

Using Elimination

1. Multiply Equation 1 by 3 to align coefficients for elimination:
$3x + 3y = 45$

2. Subtract Equation 2 from this result:
$(3x + 3y) - (5x - 3y) = 45 - 5$

3. Simplify:
$3x + 3y - 5x + 3y = 40$
$(-2x + 6y) = 40$

4. Express x in terms of y:

$-2x = 40 - 6y$
$x = (6y - 40) / 2 = 3y - 20$

5. Substitute into Equation 1:
$(3y - 20) + y = 15$
$4y - 20 = 15$
$4y = 35$
$y = 8.75$

6. Find x:
$x = 3(8.75) - 20 = 26.25 - 20 = 6.25$

Solution matches the previous method: $x = 6.25$, $y = 8.75$.

Educational Significance and Learning Outcomes

This problem exemplifies key educational concepts:

- Understanding Systems of Equations: Recognizing that multiple conditions can be translated into algebraic relationships.
- Developing Problem-Solving Strategies: Choosing and applying different methods based on context.
- Application of Algebraic Manipulation: Mastering substitution and elimination techniques.
- Real-World Relevance: Such problems model real-life situations like budgeting, resource allocation, and planning.

Pros of this problem:

- Reinforces algebraic skills.
- Encourages critical thinking.
- Demonstrates multiple solution methods.

Cons or challenges:

- The problem can seem abstract without contextualization.
- Incomplete problem statements may cause confusion.

Real-World Applications

Problems of this nature are not purely academic; they have practical applications:

- Finance: Calculating investments, comparing sums and differences.
- Engineering: Balancing systems where multiple variables interact.
- Business: Budgeting and resource management.
- Science: Analyzing compound relationships between variables.

For example, if x and y represent quantities of two products, knowing their sum and the difference in certain weighted measures helps in optimizing production.

Variations and Extensions

To deepen understanding, educators and learners can explore variations:

- Changing the sum from 15 to another number.
- Adjusting the coefficients in the difference equation.
- Introducing inequalities (e.g., $x + y \leq 15$) for optimization problems.
- Applying similar reasoning to three or more variables.

These extensions foster analytical flexibility and prepare students for more advanced topics such as linear programming and systems of inequalities.

Features and Limitations

Features:

- Intuitive approach to solving linear systems.
- Demonstrates the power of algebra in problem-solving.
- Adaptable to various contexts.

Limitations:

- Assumes the problem is well-posed; incomplete statements can cause confusion.
- May require prior knowledge of algebraic methods.
- Real-world problems often involve nonlinear systems, requiring more advanced techniques.

Conclusion

The problem involving "The sum of two numbers is 15" and the difference between multiple of these numbers offers a rich platform for exploring algebraic principles. It exemplifies fundamental problem-solving methods, highlights the importance of translating word problems into equations, and fosters critical thinking. Its applications extend beyond mathematics education into numerous practical fields, making it an essential concept for learners to master. By engaging with such problems, students develop analytical skills that are crucial for academic success and real-world problem-solving.

Final thoughts: Mastery of solving systems of equations through substitution and elimination not only enhances mathematical competence but also builds logical reasoning ability. Problems like this serve as stepping stones toward understanding more complex systems, optimization techniques, and mathematical modeling, vital skills in today's data-driven world.

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