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8. Find The Maximum Value Of F(x, Y)=x Y Subject To 3 X + y = 60. Also Find The Corresponding Point(s) (x, y)

Introduction

In the realm of optimization problems, constrained maximization and minimization are fundamental concepts with numerous applications in economics, engineering, and mathematics. One classic problem involves finding the maximum value of a function subject to a linear constraint. In this article, we will explore the problem of maximizing the function F(x, y) = x y under the constraint 3 x + y = 60. We will determine the maximum value of the function and identify the point(s) at which this maximum occurs.

Understanding the Problem

The Objective Function

The function we aim to maximize is:

$$ F(x, y) = x y $$

This is a bilinear function, representing the product of two variables, x and y.

The Constraint

The constraint provided is:

$$ 3 x + y = 60 $$

This is a linear equation representing a straight line in the xy-plane.

Goal

- Find the maximum value of $F(x, y)$ subject to the constraint.

- Find the point(s) (x, y) where this maximum occurs.

Mathematical Approach to the Problem

Step 1: Express one variable in terms of the other

Since the constraint relates x and y linearly, we can express y in terms of x:

$$y = 60 - 3 x$$

Step 2: Rewrite the objective function in terms of a single variable

Substitute y into $F(x, y)$:

$$F(x) = x (60 - 3 x)$$
$$F(x) = 60 x - 3 x^2$$

Now, the problem reduces to maximizing $F(x)$ over the feasible x-values.

Finding the Maximum of the Function $F(x)$

Step 1: Determine the domain

Since $y = 60 - 3x$, for y to be real and meaningful, we consider:

$$y \geq 0 \Rightarrow 60 - 3 x \geq 0 \Rightarrow x \leq 20$$

Similarly, for $x \geq 0$, $y \geq 0$ (assuming we are in the first quadrant), we have:

$$x \geq 0$$

Therefore, the domain of x is:

$$0 \leq x \leq 20$$

Step 2: Find critical points by differentiation

Differentiate $F(x)$:

$$ F'(x) = 60 - 6 x $$

Set derivative to zero to find critical points:

$$ 60 - 6 x = 0 $$
$$ 6 x = 60 $$
$$ x = 10 $$

Step 3: Confirm maximum using second derivative test

Second derivative:

$$ F''(x) = -6 $$

Since $F''(x) = -6 < 0$, the critical point at $x=10$ corresponds to a local maximum.

Calculating the Maximum Value

Step 1: Find y at $x=10$

$$ y = 60 - 3 (10) = 60 - 30 = 30 $$

Step 2: Compute $F(x, y)$ at this point

$$ F(10, 30) = 10 \times 30 = 300 $$

Thus, the maximum value of $F(x, y)$ under the given constraint is 300.

The Corresponding Point(s)

The point at which this maximum occurs is:

$$ (x, y) = (10, 30) $$

This point lies on the line $3x + y = 60$ and satisfies the constraints.

Additional Considerations

Boundary Points

In constrained optimization problems, it is crucial to evaluate the objective function at boundary points:

- When $x=0$:

$$y = 60 - 3(0) = 60$$
$$F(0, 60) = 0 \times 60 = 0$$

- When $x=20$:

$$y = 60 - 3(20) = 60 - 60 = 0$$
$$F(20, 0) = 20 \times 0 = 0$$

Since the maximum at the interior point $(10, 30)$ yields 300, which is greater than the boundary values, the maximum is confirmed at $(10, 30)$.

Summary of Results

Point (x, y)	Objective Function $F(x, y)$
(0, 60)	0
(10, 30)	300
(20, 0)	0

The maximum value of $F(x, y)$ is 300, occurring at the point (10, 30).

Practical Applications of This Optimization

This problem exemplifies the principles used in various real-life scenarios, such as:

- Profit maximization where resources are constrained.
- Production optimization in manufacturing with limited inputs.
- Economics for maximizing utility or output given resource limitations.
- Engineering design for optimal performance under constraints.

Conclusion

In conclusion, the problem of maximizing $F(x, y) = xy$ subject to $3x + y = 60$ is effectively solved through substitution and calculus. The key steps involve expressing one variable in terms of the other, differentiating the resulting function, and analyzing critical points within the feasible domain. The maximum value obtained is 300, achieved at the point (10, 30). This example underscores the importance of mathematical techniques such as substitution, differentiation, and boundary analysis in solving constrained optimization problems.

Additional Tips for Solving Similar Problems

- Always identify the feasible domain based on the constraints.
- Consider boundary points as potential candidates for maxima or minima.
- Use derivatives to find critical points within the domain.
- Confirm the nature of critical points using second derivative tests.
- Interpret the results within the context of the problem to ensure practical relevance.

References

- Mathematics for Economics and Business by Ian Jacques
- Calculus: Early Transcendentals by James Stewart
- Online resources on constrained optimization and Lagrange multipliers

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By understanding and applying these methods, students and professionals can effectively tackle a wide array of optimization problems involving linear constraints and bilinear functions.

Frequently Asked Questions

What is the problem asking for in the function F(x, y) = xy subject to 3x + y = 60?

The problem asks to find the maximum value of the function $F(x, y) = xy$ given the constraint $3x + y = 60$, and to determine the point(s) (x, y) where this maximum occurs.

How do we express y in terms of x from the constraint 3x + y = 60?

Solving for y gives $y = 60 - 3x$.

What is the new function to optimize after substituting $y = 60 - 3x$ into $F(x, y)$?

The function becomes $F(x) = x (60 - 3x) = 60x - 3x^2$.

How do we find the maximum value of $F(x) = 60x - 3x^2$?

We take the derivative of $F(x)$, set it equal to zero, and solve for x: $F'(x) = 60 - 6x = 0$.

What is the value of x that maximizes $F(x)$?

Solving $60 - 6x = 0$ gives $x = 10$.

What is the corresponding y-value at the maximum point?

Substituting $x = 10$ into $y = 60 - 3x$ yields $y = 60 - 3(10) = 30$.

What is the maximum value of $F(x, y) = xy$ under the given constraint?

The maximum value is $F(10, 30) = 10 30 = 300$.

Are there any other points where the maximum occurs, or is it unique?

The maximum is unique at the point $(x, y) = (10, 30)$.

What is the final answer summary for the maximum value and the point where it occurs?

The maximum of $F(x, y) = xy$ subject to $3x + y = 60$ is 300, occurring at the point (10, 30).

Additional Resources

Find the Maximum Value of $F(x, y) = xy$ Subject To $3x + y = 60$ is a classic optimization problem in calculus and linear programming. It involves maximizing the product of two variables under a linear constraint, a common scenario in economics, engineering, and operational research. Such problems are fundamental in understanding how to maximize or minimize a certain quantity given specific restrictions. In this article, we will explore this problem in depth, breaking down the steps involved, methods to solve it, and the interpretation of results.

Understanding the Problem

The problem asks us to find the maximum value of the function $F(x, y) = xy$, which represents the product of two variables x and y. The constraint is given as $3x + y = 60$, which is a linear equation restricting the possible values of x and y. Our goal is to determine the point(s) (x, y) on the line $3x + y = 60$ where the product xy reaches its maximum.

This problem is an example of constrained optimization, where we seek the maximum (or minimum) of a function subject to one or more constraints. It is a common type of problem in calculus and can be approached systematically using methods such as substitution, Lagrange multipliers, or by analyzing the boundary conditions.

Mathematical Approach to the Problem

Step 1: Express y in terms of x

Since the constraint is $3x + y = 60$, we can solve for y:

$$ y = 60 - 3x $$

This allows us to convert the problem into a single-variable function:

$$ F(x) = x \times y = x \times (60 - 3x) $$

which simplifies to:

$$ F(x) = 60x - 3x^2 $$

The domain of x is determined by the restriction that $y \geq 0$ (assuming non-negative variables) or based on the problem context.

Step 2: Determine the domain of x

Given y = 60 - 3x, for y to be non-negative:

$$60 - 3x \geq 0$$
$$3x \leq 60$$
$$x \leq 20$$

Similarly, x should be ≥ 0 if we consider only non-negative solutions. Thus, the domain of x is:

$$0 \leq x \leq 20$$

Step 3: Maximize F(x) = 60x - 3x^2

Now, to find the maximum of F(x) over [0, 20], we differentiate F(x) with respect to x:

$$F'(x) = 60 - 6x$$

Set the derivative equal to zero to find critical points:

$$60 - 6x = 0$$
$$6x = 60$$
$$x = 10$$

This is a critical point in the domain.

Step 4: Verify the maximum at x = 10

To confirm whether this critical point corresponds to a maximum, we examine the second derivative:

$$F''(x) = -6$$

Since F''(x) is negative everywhere, the function is concave down, and the critical point at x = 10 is a maximum.

Next, find y at x = 10:

$$y = 60 - 3(10) = 60 - 30 = 30$$

The corresponding point is $(10, 30)$.

Maximum Value of $F(x, y)$ and the Corresponding Point

The maximum value of $F(x, y)$ occurs at $(x, y) = (10, 30)$. The maximum value of the product xy is:

$$F(10, 30) = 10 \times 30 = 300$$

This is the highest possible product under the given constraint.

Graphical Interpretation

To visualize this problem, consider the line $3x + y = 60$ in the xy-plane. The feasible region is the line segment between points where x and y are non-negative:

- When $x = 0$, $y = 60$
- When $y = 0$, $x = 20$

The graph of $F(x, y) = xy$ is a family of hyperbolas, but since we're restricted to the line segment, the maximum occurs at the critical point found.

Plotting the line and the hyperbolic curves can help in understanding how the product xy varies along the line, with the maximum at the point $(10, 30)$.

Alternative Methods to Solve the Problem

While substitution and differentiation are straightforward here, other methods include:

Lagrange Multipliers

This method introduces a multiplier λ to handle the constraint:

$$ \text{Maximize } xy $$
$$ \text{Subject to } 3x + y = 60 $$

Set up the Lagrangian:

$$ \mathcal{L}(x, y, \lambda) = xy - \lambda (3x + y - 60) $$

Partial derivatives:

$$
\frac{\partial \mathcal{L}}{\partial x} = y - 3\lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial y} = x - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} = 3x + y - 60 = 0
$$

Solve these equations simultaneously to find x, y, and λ. The solution will match the previous result, confirming the maximum point.

Pros and Cons of Different Methods

Method	Pros	Cons
Substitution & Derivatives	Simple, straightforward for single constraint	Less generalizable for multiple constraints
Lagrange Multipliers	Systematic, handles multiple constraints	Slightly more complex algebraically

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Features and Insights

- Optimality Point: The maximum product xy occurs at (10, 30), which balances the variables under the linear constraint.
- Maximum Value: 300, indicating the highest possible product given the constraint.
- Sensitivity: Slight variations in the constraint line (e.g., changing the total sum from 60) will shift the maximum point accordingly.

- Applicability: This approach can be generalized to similar optimization problems involving other functions and constraints.

Practical Applications

This problem exemplifies real-world scenarios such as:

- Profit maximization: where x and y represent quantities of products to be produced under resource constraints.
- Resource allocation: optimizing the product of two factors limited by a linear combination of resources.
- Engineering design: maximizing certain parameters constrained by physical or operational limits.

Conclusion

Finding the maximum of the function $F(x, y) = xy$ subject to a linear constraint like $3x + y = 60$ involves a systematic approach combining substitution, differentiation, and critical point analysis. The key steps include expressing one variable in terms of the other, deriving the one-variable function, and applying calculus to locate the maximum. The result indicates that the maximum product is 300 at the point (10, 30). This problem not only demonstrates fundamental optimization techniques but also highlights the importance of mathematical tools in solving practical problems involving constraints and maximization.

Summary of Results:

- Maximum value of $F(x, y)$: 300
- Corresponding point: (10, 30)
- Method used: Substitution and calculus (derivative test)

Understanding and mastering such optimization problems are essential skills in various fields, enabling effective decision-making under constraints.

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