A 100g Bead Slides Along A Frictionless Wire With The Parabolicshapely (2ma. Find An Expression For Ay,

A 100g Bead Slides Along A Frictionless Wire With The Parabolicshapely (2ma. Find An Expression For Ay,

Introduction to Bead Motion on a Parabolic Wire

Understanding the dynamics of a bead sliding along a wire is a fundamental problem in classical mechanics. When the wire has a specific shape, such as a parabola, the motion of the bead is governed by gravitational forces and the geometry of the wire. In this article, we explore the case of a bead with a mass of 100 grams sliding frictionlessly along a wire shaped like a parabola, specifically focusing on deriving an expression for the vertical component of acceleration, denoted as Ay.

This problem not only illustrates important principles in physics, such as conservation of energy and force analysis, but also demonstrates how geometry influences motion. By the end of this article, you'll have a clear understanding of how to derive Ay and analyze the bead's dynamics on the parabolic wire.

Physical Setup and Assumptions

Before diving into the mathematics, let's clarify the assumptions and the physical setup:

- The bead has a mass m = 0.1 kg (100 grams).
- The wire is shaped as a parabola described by a function y = f(x).
- The wire is frictionless, meaning no energy is lost to frictional forces.
- The motion occurs under gravity, with acceleration $g \approx 9.8 \text{ m/s}^2$ directed downward.
- The bead slides without slipping, constrained to the wire's shape.
- The coordinate system is chosen so that the positive y-axis points upward, and the x-axis is horizontal.

The main goal is to find an expression for the vertical acceleration component, Ay, which is the second derivative of the vertical position with respect to time.

Mathematical Foundations

To analyze the motion, we need to understand the geometry of the parabola and the forces acting on the bead.

Equation of the Parabolic Wire

Suppose the wire's shape is given by:

$$ y = f(x) = ax^2 + bx + c $$

where a, b, and c are constants defining the parabola's shape and position.

For simplicity, and without loss of generality, assume the parabola is centered at the origin and symmetric about the y-axis, such as:

$$ y = a x^2 $$

where a determines the "width" of the parabola.

Parameterization of the Bead's Position

The position of the bead can be described by a parameter $s(t)$, which measures the arc length along the wire from some reference point. Alternatively, we can express the position in terms of x:

$$
x(t), \quad y(t) = a x(t)^2
$$

The derivatives of position with respect to time are:

$$
\dot{x} = \frac{dx}{dt}, \quad \dot{y} = \frac{dy}{dt} = 2a x \dot{x}
$$

Similarly, the second derivatives are:

$$
\ddot{x} = \frac{d^2 x}{dt^2}
$$

$$
\ddot{y} = 2a \left(\dot{x}^2 + x \ddot{x}\right)
$$

The goal is to find $\ddot{y}$, which is the vertical acceleration component A_y.

Forces Acting on the Bead

The primary forces are:

- Gravity: $\vec{W} = m \vec{g}$, acting downward.
- Normal force $\vec{N}$, exerted by the wire on the bead, perpendicular to the wire's surface.

Since the wire is frictionless, the normal force does no work and acts perpendicular to the wire, ensuring the bead remains on the wire.

Components of Forces and Constraints

The normal force provides the necessary centripetal force component to follow the wire's curvature and balance the component of gravity perpendicular to the wire.

To analyze the motion, it's convenient to resolve forces along the tangent and normal directions to the wire at the bead's position.

Deriving the Expression for A_y

Let's proceed step by step to derive an explicit expression for the vertical acceleration A_y.

Step 1: Express the Tangential and Normal Directions

The tangent vector $\vec{T}$ at point (x, y) is proportional to:

$$
\vec{T} = \left(1, \frac{dy}{dx}\right) = \left(1, 2a x\right)
$$

The magnitude of $\vec{T}$ is:

$$
|\vec{T}| = \sqrt{1 + (2a x)^2}
$$

The unit tangent vector $\hat{t}$ is:

$$
\hat{t} = \frac{1}{|\vec{T}|} (1, 2a x)
$$

\]

Similarly, the unit normal vector $\hat{n}$, perpendicular to $\hat{t}$, can be written as:

$$
\hat{n} = \frac{1}{|\vec{T}|} (-2a x, 1)
$$

Note: The normal vector is chosen to point towards the center of curvature.

Step 2: Force Components Along Tangent and Normal Directions

Applying Newton’s second law along the tangent and normal directions:

- Along the tangent:

$$
m a_t = \vec{F} \cdot \hat{t}
$$

- Along the normal:

$$
m a_n = \vec{F} \cdot \hat{n}
$$

where:

$$
a_t = \text{tangential acceleration}, \quad a_n = \text{normal (centripetal) acceleration}
$$

The gravitational force components are:

$$
\vec{W} = (0, -mg)
$$

The normal force $\vec{N}$ acts perpendicular to the wire, i.e., in the direction of $\hat{n}$.

Since the bead slides without friction, the net force in the normal direction provides the centripetal acceleration:

$$
m a_n = N - mg \cos \theta
$$

But more straightforward is to resolve gravity into components along $\hat{t}$ and $\hat{n}$:

$$
\vec{W} \cdot \hat{t} = -mg \sin \phi
$$
$$
\vec{W} \cdot \hat{n} = -mg \cos \phi
$$

where ϕ is the angle between the tangent and the horizontal:

$$
\sin \phi = \frac{2a x}{|\vec{T}|}
$$
$$
\cos \phi = \frac{1}{|\vec{T}|}
$$

This allows calculating the components of gravity along the tangent and normal directions.

Step 3: Expressing A_y in Terms of Known Quantities

Recall that:

$$
\ddot{y} = 2a (\dot{x}^2 + x \ddot{x})
$$

and

$$
\dot{y} = 2a x \dot{x}
$$

The tangential acceleration along the wire is related to the second derivatives:

$$
a_t = \frac{d |\vec{v}|}{dt}
$$

Alternatively, since the motion is constrained, it is often more straightforward to use energy methods or Lagrangian mechanics to relate the accelerations.

Using Energy Conservation to Find A_y

Given the frictionless setup, total mechanical energy is conserved:

$$
E = \text{Potential Energy} + \text{Kinetic Energy}
$$

\]

Assuming the bead starts from rest at height y_0, the energy at any point is:

$$
\frac{1}{2} m v^2 + m g y = \text{constant}
$$

Deriving accelerations directly from energy considerations involves differentiating the velocity, but for the purpose of finding A_y, a more precise approach uses the equations of motion along the wire's shape.

Final Expression for A_y

Combining the force analysis and the kinematic relations, the vertical acceleration A_y can be expressed as:

$$
A_y = \ddot{y} = 2a (\dot{x}^2 + x \ddot{x})
$$

From the equations of motion along the tangent:

$$
m a_t = -mg \sin \phi + N \cos \phi
$$

But since the normal force N acts perpendicular to the wire and does not influence tangential acceleration, and considering the constraint, the tangential acceleration relates primarily to the component of gravity along the wire:

$$
m a_t = -mg \sin \phi
$$

Given that:

$$
a_t = \frac{d |\vec{v}|}{dt}
$$

and

\[
|\vec{v}| = \

Frequently Asked Questions

What is the significance of the parabolic shape in the wire for the bead's motion?

The parabolic shape ensures that the bead's potential energy varies in a way that influences its acceleration and velocity along the wire, often resulting in predictable, analyzable motion similar to projectile trajectories.

How do you derive the expression for the component of acceleration Ay for a bead on a parabolic wire?

To find Ay, you apply Newton's second law along the vertical direction, considering the shape of the wire, the gravitational force, and the tension components, leading to an expression involving the derivative of the parabola and the bead's position.

What assumptions are typically made when analyzing a bead sliding on a frictionless parabolic wire?

Assumptions include neglecting friction and air resistance, treating the bead as a point mass, considering the wire's shape as fixed and smooth, and assuming the gravitational field is uniform.

How does the mass of the bead (100g) influence its acceleration along the wire?

Since the wire is frictionless, the mass does not affect the acceleration; the acceleration depends solely on the shape of the wire and gravity, illustrating the principle that inertial mass cancels out in such scenarios.

What is the general form of the parabola used in the wire's shape for this problem?

Typically, the parabola can be expressed as $y = ax^2 + bx + c$, where the coefficients define the specific shape; in many physics problems, it is simplified to $y = kx^2$ for symmetry about the vertical axis.

How does energy conservation help in deriving the expression for Ay in this context?

Energy conservation relates the potential and kinetic energies at different points, allowing the determination of velocity components and, consequently, the acceleration Ay by differentiating the velocity with respect to time or position.

Can the expression for Ay be generalized for different shapes of wire, and if so, how?

Yes, by expressing the wire's shape as a function y(x), the acceleration components can be derived by differentiating y(x) to find slopes and curvature, then applying Newton's laws along the tangent and normal directions to obtain Ay.

Additional Resources

In-Depth Analysis of a 100g Bead Sliding Along a Frictionless Wire with a Parabolic Shape: Deriving the Expression for A_y

---

Introduction

Understanding the dynamics of a bead sliding along a wire is a classic problem in classical mechanics, often used to illustrate the principles of conservation of energy, forces, and motion along curved paths. When the wire takes a specific shape—here, a parabola—the problem becomes richer in analysis, requiring a careful derivation of the forces involved, especially the normal and tangential components, and how they influence the vertical acceleration A_y.

This review provides a comprehensive exploration into the motion of a 100g bead on a frictionless parabolically shaped wire, culminating in deriving a precise expression for A_y, the vertical acceleration component of the bead.

Physical Setup and Assumptions

Before delving into the mathematics, it’s critical to understand the physical conditions and assumptions:

- Mass of the Bead (m): 100 grams, which is $0.1\, \text{kg}$.
- Wire Shape: Parabolic, generally modeled as $y = k x^2$, where k is a constant defining the parabola's curvature.
- Frictional Forces: Assumed to be zero, simplifying the analysis to pure normal and tangential forces.
- Gravity (g): Acts downward with magnitude approximately $9.81\, \text{m/s}^2$.
- Coordinate System: A standard Cartesian coordinate system where y is vertical and x is horizontal.
- Motion: The bead slides without slipping, with its position described parametrically along the wire as a function of time.

Geometrical Characterization of the Wire

The shape of the wire is fundamental to the analysis. Consider the parabola:

$$
y = k x^2
$$

- Parameter k: Determines the steepness of the parabola.
- Derivative dy/dx: Indicates the slope of the wire at any point:

$$
\frac{dy}{dx} = 2k x
$$

- Tangent and Normal Directions: These are crucial for analyzing the forces.

Tangent and Normal Vectors

At any point on the wire:

- Tangent vector $\mathbf{t}$:

$$
\mathbf{t} = \left(1, \frac{dy}{dx}\right) = (1, 2k x)
$$

- Normal vector $\mathbf{n}$:

$$
\mathbf{n} = \left(-2k x, 1\right)
$$

- Unit tangent vector $\hat{\mathbf{t}}$:

$$
\hat{\mathbf{t}} = \frac{\mathbf{t}}{\sqrt{1 + (dy/dx)^2}} = \frac{(1, 2k x)}{\sqrt{1 + 4k^2 x^2}}
$$

- Unit normal vector $\hat{\mathbf{n}}$:

$$
\hat{\mathbf{n}} = \frac{(-2k x, 1)}{\sqrt{1 + 4k^2 x^2}}
$$

Dynamics of the Bead

The bead's motion can be described by Newton's second law in the presence of gravity and the normal force exerted by the wire:

$$
m \mathbf{a} = \mathbf{F}_g + \mathbf{N}

\]

Where:

- $\mathbf{a}$: acceleration vector
- $\mathbf{F}_g = m \mathbf{g}$: gravitational force
- $\mathbf{N}$: normal force exerted by the wire, always perpendicular to the wire surface.

Decomposition of Forces and Motion

Since the wire is frictionless, the normal force $\mathbf{N}$ acts perpendicular to the wire's surface, providing the necessary centripetal force for the bead's curved path. The component of gravity along and perpendicular to the wire influences the acceleration components.

Key points:

- The acceleration $\mathbf{a}$ can be decomposed into tangential a_t and normal a_n components.
- The normal component a_n is related to the curvature of the wire and the velocity v at a point:

$$
a_n = \frac{v^2}{R}
$$

where R is the radius of curvature at that point.

Radius of Curvature R

The radius of curvature for a curve $y = y(x)$ is given by:

$$
R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2 y}{dx^2}\right|}
$$

For the parabola $y = k x^2$:

$$
\frac{dy}{dx} = 2k x
$$
$$
\frac{d^2 y}{dx^2} = 2k
$$

Thus,

\[
R = \frac{\left[1 + 4k^2 x^2\right]^{3/2}}{2k}

\]

This expression describes how the curvature varies along the parabola, influencing the normal acceleration.

Conservation of Energy

Since the bead slides frictionlessly, total mechanical energy is conserved:

$$
E = \text{Potential Energy} + \text{Kinetic Energy} = \text{constant}
$$

Assuming initial conditions where the bead starts at position x_0, y_0:

$$
m g y_0 + \frac{1}{2} m v_0^2 = m g y + \frac{1}{2} m v^2
$$

At any point,

$$
v^2 = 2 g (y_0 - y) + v_0^2
$$

which relates the velocity at any position to initial conditions.

Deriving A_y: Vertical Acceleration

The primary goal is to find the vertical component of acceleration, A_y. This requires analyzing the acceleration vector $\mathbf{a}$, its components, and how the forces project onto the vertical axis.

Step 1: Express the acceleration in terms of tangential and normal components:

$$
\mathbf{a} = a_t \hat{\mathbf{t}} + a_n \hat{\mathbf{n}}
$$

- The tangential acceleration a_t is along the direction of motion.
- The normal acceleration $a_n = v^2 / R$.

Step 2: Find the components in the y-direction:

$$
A_y = a_t \hat{t}_y + a_n \hat{n}_y
$$

where:

$$
\hat{\mathbf{t}} = \frac{(1, 2k x)}{\sqrt{1 + 4k^2 x^2}}
$$

$$
\hat{\mathbf{n}} = \frac{(-2k x, 1)}{\sqrt{1 + 4k^2 x^2}}
$$

Thus,

$$
A_y = a_t \frac{2k x}{\sqrt{1 + 4k^2 x^2}} + a_n \frac{1}{\sqrt{1 + 4k^2 x^2}}
$$

Calculating the Tangential Acceleration a_t

From energy considerations, the tangential acceleration relates to the change in velocity:

$$
a_t = \frac{d v}{d t}
$$

Using the chain rule:

$$
a_t = \frac{d v}{d s} \cdot v
$$

where s is the arc length along the wire.

Alternatively, in a more general way, considering the forces:

$$
m a_t = \text{component of gravity along the wire}
$$

which can be obtained by projecting gravity onto the tangent direction:

$$
\text{Component of gravity along tangent} = m g \sin \theta
$$

where θ is the angle of the wire at the point.

Given:

\[

\frac{dy}{dx} = 2k x
\]

the slope \(\tan \theta = 2k x\), so:

\[
\sin \theta = \frac{2k x}{\sqrt{1 + 4k^2 x^2}}
\]

Hence,

\[
a_t = g \sin \theta = g \frac{2k x}{\sqrt{1 + 4k^2 x^2}}
\]

Final Expression for \(A_y\)

Putting all parts together:

\[
A_y = a_t \frac{2k x}{\sqrt{1 + 4k^2 x^2}} + a_n \frac{1}{\sqrt{1 + 4k^2 x^2}}
\]

Substituting \(a_t\) and \(a_n\):

\[
A_y = \left(g \frac{2k x}{\sqrt{1 + 4k^2 x^2}} \right) \frac{2k x}{\sqrt{1 + 4k^

A 100g Bead Slides Along A Frictionless Wire With The Parabolicshapely 2ma Find An Expression For Ay

A 100g Bead Slides Along A Frictionless Wire With The Parabolicshapely 2ma Find An Expression For Ay

Related Articles

- Which Devices Does The Author Use To Establish The Mood? Check The Four Best Choices.details About The
- Using The Correlation For The Second Virial Coefficient (Pitzer Correlation), Find The Molar Volume Of
- Use The Following Information To Answer The Next Two Questions. Consider The Phillips Curvet = T - (ut

Back to Home