A 2-m Wire With A Mass Of 60 G, Is Under Tension. A Transverse Wave, For Which The Frequency Is 550 Hz,

A 2-m Wire With A Mass Of 60 G, Is Under Tension. A Transverse Wave, For Which The Frequency Is 550 Hz, presents an intriguing scenario in wave physics, encompassing the principles of wave propagation, tension dynamics, and the properties of the medium involved. Understanding this setup requires a comprehensive exploration of the fundamental concepts of wave mechanics, the influence of tension on wave speed, and the calculation of various parameters such as wave velocity, tension force, and wavelength. This article provides a detailed analysis of these aspects, offering insights into the behavior of transverse waves on stretched wires, supported by relevant formulas and practical implications.

Understanding Transverse Waves on a Stretched Wire

What Is a Transverse Wave?

A transverse wave is a type of wave where particle displacement is perpendicular to the direction of wave propagation. In the context of a wire, when a transverse wave travels along it:
- Particles of the wire oscillate up and down (perpendicular to the length of the wire).
- The wave moves along the wire's length, carrying energy without transferring matter.

Key Characteristics of Transverse Waves

- Wavelength (λ): The distance over which the wave's shape repeats.
- Frequency (f): The number of oscillations or cycles per second, measured in Hertz (Hz).
- Wave Speed (v): The velocity at which the wave propagates along the medium.
- Amplitude: The maximum displacement from the equilibrium position.

Physical Properties of the Wire

Given Data

- Length of wire (L): 2 meters
- Mass of wire (m): 60 grams (0.06 kg)
- Frequency of wave (f): 550 Hz

Calculating Linear Mass Density (μ)

The linear mass density (μ) indicates how mass is distributed along the wire's length:

$$
\mu = \frac{m}{L}
$$

Plugging in the values:

$$
\mu = \frac{0.06\, \text{kg}}{2\, \text{m}} = 0.03\, \text{kg/m}
$$

This value is critical for understanding how the mass distribution affects wave speed.

Determining Wave Velocity

The wave speed (v) on a stretched string or wire is related to the tension (T) and the linear mass density (μ) by the wave equation:

$$
v = \sqrt{\frac{T}{\mu}}
$$

However, without the tension T explicitly given, we can still determine the wave velocity using the known frequency and wavelength once we find the wavelength.

Calculating the Wavelength (λ)

The relationship between wave speed, frequency, and wavelength is:

$$
v = f \times \lambda
$$

To find v, we need λ. But since λ is not given directly, we can express it in terms of the wave's properties once we determine v.

Given the nature of the problem, if the wave is traveling at a certain speed, and the frequency is 550 Hz, then:

$$
\lambda = \frac{v}{f}
$$

We will revisit this after calculating the wave velocity once more data or assumptions are made.

Calculating Tension in the Wire

Since the tension T influences wave speed, and the wave's frequency is known, the key is to find the tension based on the wave's behavior.

Suppose we know the wave's wavelength or can estimate it based on typical boundary conditions or harmonic modes. For simplicity, assuming the wave is the fundamental mode (first harmonic) with nodes at both ends, the wavelength

is:
$$
\lambda = 2L = 2 \times 2\, \text{m} = 4\, \text{m}
$$

This is a common assumption for standing waves on a string fixed at both ends.

Using the relation:
$$
v = f \times \lambda
$$
we get:
$$
v = 550\, \text{Hz} \times 4\, \text{m} = 2200\, \text{m/s}
$$

Now, using the wave velocity formula:
$$
v = \sqrt{\frac{T}{\mu}}
$$
we can solve for T:
$$
T = \mu v^2
$$
$$
T = 0.03\, \text{kg/m} \times (2200\, \text{m/s})^2
$$
$$
T = 0.03 \times 4,840,000
$$
$$
T = 145,200\, \text{N}
$$

This tension value seems unphysically high for a typical wire, indicating that either the assumption about the wavelength or the harmonic mode needs refinement. Alternatively, the actual wave may not be in the fundamental mode, or the tension could be adjusted accordingly.

Note: Realistically, tensions in wires are in the order of a few to a few hundred newtons, so this calculation suggests the wave might be a higher harmonic or the assumptions need revisiting.

Estimating the Wavelength and Tension More Realistically

Given the initial data, it's more practical to reverse engineer the tension based on typical tension ranges and realistic wave speeds.

Suppose the tension T is within a typical range (say 10 N to 1000 N). Let's select T = 50 N for demonstration:

$$
v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{50}{0.03}} \approx \sqrt{1666.67} \approx 40.8\, \text{m/s}
$$

\]

Calculate the wavelength:
\[
\lambda = \frac{v}{f} = \frac{40.8}{550} \approx 0.074\, \text{m}
\]

This small wavelength indicates higher harmonic modes, which are common in practical scenarios.

Summary of approximate parameters:
- Wave velocity: ~40.8 m/s (for T=50 N)
- Wavelength: ~0.074 m
- Tension: 50 N (as an example)

Implications and Practical Applications

Wave Speed and Tension Relationship

The direct relationship between tension and wave speed implies:
- Increasing tension increases wave speed.
- Adjusting tension allows control over wave properties, which is crucial in musical instruments, engineering, and material testing.

Applications in Engineering and Physics

Understanding the behavior of transverse waves on wires and strings is fundamental in:
- Designing musical instruments (guitar strings, violins)
- Structural engineering (cables, suspension bridges)
- Signal transmission in communication systems
- Material stress testing

Summary of Key Points

- The linear mass density (μ) is calculated from the mass and length of the wire.
- The wave velocity depends on both tension and μ.
- Assuming harmonic modes, the wavelength can be estimated based on the wire length.
- The tension can be deduced from the wave velocity and linear mass density.
- Realistic tension values are essential for practical applications and accurate modeling.

Conclusion

Analyzing a 2-meter wire with a mass of 60 grams under tension, supporting a transverse wave with a frequency of 550 Hz, involves understanding the interplay between tension, wave speed, wavelength, and harmonic modes. Through the fundamental wave equations and realistic assumptions, we can estimate the tension and wave parameters, which are vital in various scientific and engineering contexts. Mastery of these concepts enhances the ability to manipulate and utilize wave phenomena in practical applications, ensuring precise control over wave behavior in technological and natural systems.

Meta Description:
Explore the physics of a 2-meter wire with 60 grams mass supporting a transverse wave at 550 Hz. Learn how to calculate tension, wave speed, and wavelength with detailed explanations and practical insights.

Frequently Asked Questions

What is the wave velocity on a 2-meter wire with a mass of 60 grams when a transverse wave of 550 Hz is produced?

The wave velocity can be calculated using the formula $v = f\lambda$. First, find the wavelength $\lambda = 2$ m (since the wave spans the entire wire length for a fundamental mode). Then, $v = 550$ Hz × 2 m = 1100 m/s.

How do you determine the tension in the wire given the wave frequency and mass?

The tension T can be found using the wave velocity formula $v = \sqrt{(T/\mu)}$, where μ is the linear mass density. First, calculate μ = mass/length = 0.06 kg / 2 m = 0.03 kg/m. Then, $T = \mu v^2 = 0.03$ kg/m × $(1100 \text{ m/s})^2 \approx 36{,}300$ N.

What is the linear mass density of the wire in this scenario?

The linear mass density μ is calculated as μ = mass / length = 0.06 kg / 2 m = 0.03 kg/m.

What is the significance of the frequency 550 Hz in the context of the wave on the wire?

The frequency 550 Hz indicates how many oscillations occur per second for the transverse wave traveling along the wire, affecting the wave's speed, wavelength, and the tension required to sustain such oscillations.

If the tension in the wire is increased, how does it affect the wave velocity and frequency?

Increasing the tension in the wire increases the wave velocity ($v = \sqrt{(T/\mu)}$). If the frequency remains constant at 550 Hz, an increase in tension results in a higher wave velocity and potentially a longer wavelength; otherwise, to maintain the same wavelength, the wave speed must increase accordingly.

Additional Resources

A 2-m wire with a mass of 60 g, under tension, producing a transverse wave at a frequency of 550 Hz is a fascinating subject that intertwines fundamental principles of physics, material science, and wave mechanics. This scenario offers a rich context for exploring how tension, mass distribution, and wave properties interplay to generate specific vibrational behaviors in stretched strings or wires. Understanding these dynamics not only deepens our grasp of classical physics but also has practical implications for musical instrument design, communication systems, and engineering applications.

Fundamentals of Wave Propagation in Strings and Wires

Nature of Transverse Waves

Transverse waves are characterized by particle displacement perpendicular to the direction of wave propagation. When a wire is plucked, struck, or otherwise disturbed, energy propagates along its length, creating oscillations that move perpendicular to the wire's axis. These waves are essential in musical instruments like guitars or violins, where string vibrations produce sound.

Wave Equation and Key Parameters

The behavior of transverse waves on a stretched string is governed by the classical wave equation:

$$ v = \sqrt{\frac{T}{\mu}} $$

where:
- v is the wave velocity,
- T is the tension in the wire,
- µ is the mass per unit length.

The wave velocity influences the frequency and wavelength of the propagating wave, with the fundamental frequency (first harmonic) being a primary focus when analyzing vibrating strings.

Given Data and Their Significance

To analyze the system, let's extract and interpret the provided data:

- Length of the wire (L): 2 meters
- Mass of the wire (m): 60 grams (or 0.060 kg)
- Frequency (f): 550 Hz

This data allows us to determine various properties, including the mass per unit length, wave velocity, and tension, which are critical for understanding the wire's vibrational characteristics.

Calculating the Mass Per Unit Length (µ)

The mass per unit length is a fundamental parameter influencing wave speed and frequency.

$$
\mu = \frac{m}{L} = \frac{0.060\, \text{kg}}{2\, \text{m}} = 0.030\, \text{kg/m}
$$

This value indicates that each meter of the wire carries 30 grams of mass, affecting how quickly waves can travel along it and, consequently, the frequencies it can support.

Determining the Wave Velocity (v)

The relationship between wave velocity, frequency, and wavelength is:

$$
v = f \times \lambda
$$

For the fundamental mode (assuming the wire vibrates as a string fixed at both ends), the wavelength is twice the length of the wire:

$$
\lambda = 2L = 4\, \text{m}
$$

Thus,

$$
v = 550\, \text{Hz} \times 4\, \text{m} = 2200\, \text{m/s}
$$

The wave propagates along the wire at 2200 meters per second, a typical speed for steel or similar materials under tension.

Calculating the Tension (T) in the Wire

Using the wave velocity formula:

$$
v = \sqrt{\frac{T}{\mu}}
$$

Rearranged to solve for T:

$$
T = \mu v^2
$$

Substituting known values:

$$
T = 0.030\, \text{kg/m} \times (2200\, \text{m/s})^2
$$

$$
T = 0.030 \times 4,840,000 = 145,200\, \text{N}
$$

This indicates a tension of approximately 145.2 kN is required to sustain a wave at 550 Hz with the given parameters.

Note: Such a high tension suggests that either the wire is made of a very strong material like steel or the frequency is associated with a higher harmonic mode. For typical laboratory or musical wire, tensions are much lower, implying the wave may not be in the fundamental mode or the initial assumptions might need refinement.

Implications of High Tension and Material Considerations

The calculated tension is notably high, prompting questions about practicality and material choice:

- Material Strength: Common wires like steel can withstand high tension, but 145 kN exceeds typical values for small laboratory wires. This suggests the wire could be a high-tensile steel or an alloy designed for such stress levels.
- Frequency and Mode of Vibration: If the wire is vibrating at such a high frequency, it might be in a higher harmonic mode, which would have a shorter wavelength and thus a lower tension for the same frequency.
- Safety and Structural Integrity: Applying such tension requires robust fixtures and careful handling, emphasizing the importance of safety considerations in experimental setups.

Analyzing the Mode of Vibration

The fundamental frequency often dominates in basic analysis, but higher modes can also produce the same frequency under different conditions.

Harmonic Frequencies:

$$
f_n = n \times \frac{v}{2L}
$$

where:
- n is the mode number (integer),
- v is wave velocity,

Given the frequency of 550 Hz, if this is not the fundamental, it could be a higher harmonic. Let's check for possible mode numbers:

$$
n = \frac{2Lf}{v}
$$

Assuming the earlier calculated wave velocity (2200 m/s):

$$
n = \frac{2 \times 2\, \text{m} \times 550\, \text{Hz}}{2200\, \text{m/s}} = \frac{2200}{2200} = 1
$$

This indicates the wave is likely in the fundamental mode, supporting our earlier assumption.

Practical Applications and Real-World Relevance

Understanding the interplay of tension, mass distribution, and wave frequency has tangible applications:

- Musical Instruments: String tension and mass per unit length determine pitch. Precise control allows musicians and instrument builders to craft desired tonal qualities.
- Communication Technologies: Vibrating wires serve as transmission lines in certain radio and telecommunication devices, where wave speed and frequency are critical.
- Structural Engineering: Knowledge of wave propagation informs the design of bridges, buildings, and other structures to withstand vibrational stresses, especially in seismic zones.
- Scientific Research: Laboratory experiments often involve controlling tension and mass distribution to study wave mechanics, resonance, and material properties.

Limitations and Assumptions in the Analysis

While the calculations provide valuable insights, several assumptions underpin them:

- Uniform Density: The wire is assumed to have a uniform mass distribution.
- Ideal Conditions: No damping or energy loss due to air resistance or internal friction.
- Fixed Boundary Conditions: The wire is perfectly fixed at both ends, supporting standing waves.
- Material Homogeneity: The wire's material properties are consistent throughout.
- Neglecting External Factors: Temperature, humidity, and other environmental factors are not considered.

Real-world deviations from these assumptions can influence actual wave behavior, necessitating experimental validation.

Conclusion: Bridging Theory and Practice

The exploration of a 2-meter wire with a mass of 60 grams, under tension and vibrating at 550 Hz, exemplifies the intricate relationship between physical parameters and vibrational phenomena. Through systematic calculation, we've deduced that achieving such a wave state requires significant tension and is influenced heavily by the material properties and boundary conditions.

This analysis underscores the importance of fundamental physics principles in diverse applications—from musical acoustics to structural engineering—and highlights how meticulous calculations inform practical design choices. As technology advances, understanding wave mechanics at such granular levels remains crucial for innovation, safety, and the continued pursuit of scientific knowledge.

In essence, the study of wave dynamics in stretched wires not only illuminates the elegance of classical physics but also empowers engineers and scientists to manipulate and harness these phenomena for a broad spectrum of technological advancements.

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