

A 90.0 A Current Circulates Around A 2.40-mm-diameter Superconducting Ring. What Is The Ring's Magnetic

A 90.0 A Current Circulates Around A 2.40-mm-diameter Superconducting Ring. What Is The Ring's Magnetic Field?

Understanding the magnetic properties of superconducting rings is fundamental in the fields of physics and electrical engineering. When a substantial current flows through a superconducting ring, it generates a magnetic field that can be precisely calculated based on the geometry and properties of the ring. In this article, we explore the principles involved in determining the magnetic field of a superconducting ring with a current of 90.0 A and a diameter of 2.40 mm, providing a comprehensive guide for students, educators, and professionals alike.

Fundamental Concepts in Magnetic Fields of Current-Carrying Loops

Before diving into specific calculations, it is essential to understand the basic principles governing magnetic fields generated by current loops.

Magnetic Field of a Circular Loop

- The magnetic field produced by a current-carrying loop can be derived using the Biot-Savart Law, which relates current elements to magnetic field contributions.
- For points along the axis of the loop, the magnetic field is given by a simplified formula, making calculations straightforward.

Superconductivity and Its Impact on Magnetic Fields

- Superconductors exhibit zero electrical resistance, allowing continuous current flow without energy loss.
- This property ensures that the current in the ring remains stable, and the magnetic field generated is persistent.
- Superconductors also exhibit the Meissner effect, which expels magnetic fields; however, in the context of a superconducting ring with circulating current, this effect is considered when analyzing flux trapping and quantization phenomena.

Calculating the Magnetic Field of the Superconducting Ring

The primary goal is to determine the magnetic field produced by the superconducting ring at various points. The focus is often on the magnetic field at the center of the ring, which is of most interest for many practical applications.

Given Data

- Current, $(I = 90.0, \text{A})$
- Diameter of the ring, $(D = 2.40, \text{mm})$
- Radius of the ring, $(R = \frac{D}{2} = 1.20, \text{mm} = 1.20 \times 10^{-3}, \text{m})$

Formula for Magnetic Field at the Center of a Circular Loop

The magnetic field at the center of a circular loop carrying a current (I) is given by:

$$B = \frac{\mu_0 I}{2 R}$$

Where:

- (B) = magnetic flux density at the center (Tesla, T)
- (μ_0) = permeability of free space $(4\pi \times 10^{-7}, \text{T} \cdot \text{m/A})$
- (I) = current in the loop (A)
- (R) = radius of the loop (m)

Step-by-Step Calculation of the Magnetic Field

Applying the given data into the formula:

$$B = \frac{(4\pi \times 10^{-7}) \times 90.0}{2 \times 1.20 \times 10^{-3}}$$

Calculating numerator:

$$(4\pi \times 10^{-7}) \times 90.0 \approx 1.131 \times 10^{-4}$$

Calculating denominator:

$$\sqrt{2 \times 1.20 \times 10^{-3}} = 2.40 \times 10^{-3}$$

Therefore,

$$B = \frac{1.131 \times 10^{-4}}{2.40 \times 10^{-3}} \approx 0.0471, \text{ T}$$

Interpretation of Results and Significance

The magnetic flux density at the center of the superconducting ring with a 90.0 A current is approximately 0.0471 Tesla.

Significance:

- This magnetic field strength is substantial, especially considering the small size of the ring.
- It demonstrates how superconducting materials can produce strong magnetic fields without energy losses.
- Such configurations are vital in applications like magnetic resonance imaging (MRI), quantum computing, and particle accelerators.

Additional Factors to Consider in Magnetic Field Calculations

While the idealized formula provides a good estimate, real-world scenarios may involve additional complexities.

Effects of Finite Thickness of the Superconducting Ring

- The formula assumes a thin, single-turn loop.
- For rings with significant thickness, the magnetic field must be integrated over the volume, often requiring numerical methods.

Flux Quantization in Superconducting Rings

- Superconducting rings exhibit quantized magnetic flux, meaning magnetic flux can only take discrete values.
- This phenomenon influences the magnetic behavior, especially in quantum applications.

Meissner Effect and Magnetic Field Expulsion

- Superconductors expel magnetic fields from their interior, leading to complex interactions.
- The actual magnetic field distribution can differ from ideal calculations, especially near the surface.

Practical Applications of Superconducting Ring Magnetic Fields

The ability to generate stable and intense magnetic fields with superconducting rings opens up numerous technological innovations.

Magnetic Resonance Imaging (MRI)

- Superconducting magnets produce uniform magnetic fields essential for high-resolution imaging.
- The precise control and stability of magnetic flux are critical for image quality.

Quantum Computing and Superconducting Qubits

- Superconducting rings form the basis of qubits in quantum computers.
- The magnetic flux quantization and coherence are harnessed for quantum information processing.

Particle Accelerators and Magnet Systems

- Superconducting magnets steer and focus particle beams with high efficiency.
- The high magnetic fields enable compact and powerful accelerators.

Safety and Engineering Considerations

When working with superconducting rings and their magnetic fields, safety is paramount.

Handling High Currents

- Superconducting systems often operate at cryogenic temperatures.
- Proper insulation and cooling systems are essential to prevent quenching, where the superconductor transitions to a resistive state.

Magnetic Field Interactions

- Strong magnetic fields can attract ferromagnetic objects, posing safety hazards.
- Adequate shielding and safety protocols are necessary in laboratory environments.

Material and Structural Integrity

- The mechanical design must accommodate magnetic stresses.
- Materials used should withstand thermal and electromagnetic stresses during operation.

Conclusion

In summary, a superconducting ring carrying a current of 90.0 A with a diameter of 2.40 mm generates a significant magnetic field at its center, approximately 0.0471 Tesla. This calculation, based on fundamental electromagnetic principles, underscores the remarkable magnetic capabilities of superconducting materials. Understanding these principles is essential for advancing technologies in medical imaging, quantum computing, and high-energy physics. As research progresses, the precise manipulation of magnetic fields in superconducting rings will continue to unlock new scientific and technological frontiers.

Keywords: superconducting ring, magnetic field calculation, Biot-Savart Law, flux quantization, Meissner effect, magnetic resonance imaging, quantum computing, superconductivity applications

Frequently Asked Questions

What is the magnetic field produced by a 90.0 A current circulating in a superconducting ring with a diameter of 2.40 mm?

The magnetic field at the center of the superconducting ring can be calculated using the formula $B = (\mu_0 I) / (2 R)$. Given the diameter of 2.40 mm, the radius R is 1.20 mm (0.00120 m). Using $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$, $B \approx (4\pi \times 10^{-7} \times 90.0) / (2 \times 0.00120) \text{ T}$, which simplifies to approximately 0.047 T.

How does the superconducting nature of the ring affect the magnetic field generated by the current?

Since the ring is superconducting, it exhibits zero electrical resistance, allowing a persistent current to flow indefinitely without decay. This results in a stable magnetic field at the center of the ring, unlike normal conductors where resistance causes current decay.

What is the significance of the ring's diameter in calculating its magnetic field?

The diameter determines the radius of the ring, which directly influences the magnetic field strength at the center. Specifically, the magnetic field is inversely proportional to the radius, so a smaller diameter results in a stronger magnetic field.

Can the magnetic field strength be increased by changing the current in the superconducting ring?

Yes, increasing the current circulating in the superconducting ring will proportionally increase the magnetic field at its center, since B is directly proportional to I .

Why is the magnetic field around a superconducting ring considered to be stable?

Because the superconducting state allows the current to persist indefinitely without resistance, the magnetic field remains constant over time, assuming no external disturbances.

What are potential applications of the magnetic field generated by superconducting rings?

Superconducting rings and their magnetic fields are used in MRI machines, quantum computing, magnetic sensors, and in producing highly stable magnetic fields for scientific experiments.

How does the diameter of 2.40 mm compare to typical superconducting ring sizes used in applications?

A diameter of 2.40 mm is relatively small and is common in micro- and nano-scale superconducting devices, whereas larger rings are used in applications requiring stronger magnetic fields or different configurations.

What safety considerations are associated with generating strong magnetic fields from superconducting rings?

Strong magnetic fields can interfere with electronic devices and pose risks to individuals with pacemakers or other implants. Proper shielding and safety protocols are essential when working with superconducting magnets.

How does the Meissner effect relate to the magnetic field around a superconducting ring?

The Meissner effect causes a superconductor to expel magnetic fields from its interior. However, when a current circulates in a superconducting ring, it maintains a magnetic field, demonstrating the complex interplay between superconductivity and magnetic flux trapped within the ring.

Additional Resources

Superconducting Rings and Magnetic Fields: An In-Depth Exploration of a 90.0 A Current in a 2.40-mm Diameter Ring

Understanding the magnetic properties of superconductors has become a cornerstone of modern physics and engineering, underpinning advancements in magnetic resonance imaging (MRI), particle accelerators, and quantum computing. Among these phenomena, the magnetic field generated by a superconducting ring carrying a substantial current is particularly intriguing, offering insights into fundamental electromagnetic principles and potential technological applications. This article delves into the scenario of a superconducting ring with a current of 90.0 amperes circulating around it, specifically focusing on calculating its magnetic field, given its diameter of 2.40 millimeters. We will explore the fundamental physics, the mathematical framework, and the broader implications of such a system.

1. Introduction to Superconducting Rings and Magnetic Fields

Superconductivity is a quantum mechanical phenomenon characterized by zero electrical resistance and the expulsion of magnetic fields (Meissner effect) when a material is cooled below its critical temperature. When a superconductor is shaped into a ring and a current is established within it, the resulting magnetic field exhibits unique properties, distinct from normal conductors.

Why Focus on a Superconducting Ring?

Superconducting rings serve as fundamental models in physics to study flux quantization, persistent currents, and magnetic flux trapping. The ability of a superconductor to sustain a persistent current without decay makes such rings ideal for applications requiring stable magnetic fields, such as superconducting quantum interference devices (SQUIDs).

Magnetic Fields from Currents in Conductors

According to classical electromagnetism, a current-carrying conductor produces a magnetic field. In the case of a ring-shaped conductor, the magnetic field is strongest at the center of the ring and diminishes with distance. When the conductor is superconducting, the current persists indefinitely without energy loss, leading to stable magnetic fields that can be precisely characterized.

2. Physical Parameters and Basic Concepts

Before diving into calculations, it is essential to understand the specific parameters of the problem:

- Current (I): 90.0 A
- Diameter of the ring (D): 2.40 mm = 2.40×10^{-3} m
- Radius of the ring (R): $D / 2 = 1.20$ mm = 1.20×10^{-3} m

Key Concepts to Recall:

- Magnetic field at the center of a current-carrying loop: For a thin, circular loop, the magnetic field at its center can be calculated using the Biot-Savart Law or a simplified formula derived from it.
- Superconductivity and flux quantization: While not directly used in this calculation, these phenomena are intrinsic to understanding the stability and behavior of persistent currents.

3. Theoretical Framework: Magnetic Field of a Circular Loop

Calculating the magnetic field generated by a current in a superconducting ring involves classical electromagnetism principles. Despite the superconductor's unique quantum properties, the magnetic field at the center of a superconducting ring with a steady current is analogous to that of a normal conducting loop, assuming the superconductor is in a persistent current state and is not subject to flux penetration.

Biot-Savart Law Overview

The Biot-Savart Law states that the magnetic field $d\mathbf{B}$ at a point in space due to a differential element of current $d\mathbf{l}$ is:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \mathbf{\hat{r}}}{r^2}$$

where:

- $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ (permeability of free space)
- I is the current
- $d\mathbf{l}$ is the differential length element of the current path
- $\mathbf{\hat{r}}$ is the unit vector from the current element to the observation point
- r is the distance from the current element to the point of observation

Simplification for a Circular Loop

For a circular loop of radius (R) , the magnetic field at the center simplifies to:

$$B = \frac{\mu_0 I}{2 R}$$

This formula assumes the loop is thin, circular, and the observation point is exactly at its center.

4. Calculating the Magnetic Field at the Center of the Superconducting Ring

Given the parameters, the calculation proceeds as follows:

Step 1: Convert the Diameter to Radius

$$R = \frac{D}{2} = \frac{2.40 \times 10^{-3} \text{ m}}{2} = 1.20 \times 10^{-3} \text{ m}$$

Step 2: Apply the Formula for Magnetic Field at the Center

$$B = \frac{\mu_0 I}{2 R}$$

Substituting the known values:

$$\begin{aligned} \mu_0 &= 4\pi \times 10^{-7} \text{ H/m} \\ I &= 90.0 \text{ A} \\ R &= 1.20 \times 10^{-3} \text{ m} \end{aligned}$$

Step 3: Perform the Calculation

$$B = \frac{(4\pi \times 10^{-7}) \times 90.0}{2 \times 1.20 \times 10^{-3}}$$

Calculate numerator:

$$(4\pi \times 10^{-7}) \times 90.0 = 4\pi \times 90.0 \times 10^{-7}$$

$$4\pi \times 90.0 \approx 4 \times 3.1416 \times 90 \approx 12.5664 \times 90 \approx 1130.976$$

Thus,

$$\text{Numerator} = 1130.976 \times 10^{-7} = 1.130976 \times 10^{-4}$$

Calculate denominator:

$$2 \times 1.20 \times 10^{-3} = 2.40 \times 10^{-3}$$

Finally,

$$B = \frac{1.130976 \times 10^{-4}}{2.40 \times 10^{-3}} \approx 0.0471 \text{ T}$$

Result:

$$\boxed{B \approx 0.047 \text{ Tesla}}$$

or approximately 47 millitesla.

5. Physical Interpretation and Significance of the Magnetic Field

The calculated magnetic field magnitude of roughly 47 mT at the center of the superconducting ring underscores several critical points:

- Strong Magnetic Fields in Small Geometries:

Despite its tiny size, a superconducting ring carrying 90 A can generate a significant magnetic field. This illustrates how superconducting loops can be used to produce stable, high-intensity magnetic fields in compact configurations.

- Persistent Currents and Magnetic Stability:

Superconductivity ensures the current of 90 A persists indefinitely without decay, meaning the magnetic field remains stable over time. This stability is vital for applications like quantum interference devices or magnetic energy storage.

- Quantum Effects and Flux Quantization:

While classical calculations provide a good approximation, quantum phenomena such as flux quantization impose constraints on the magnetic flux threading the superconductor. The flux is quantized in units of the flux quantum $\Phi_0 = h/2e \approx 2.07 \times 10^{-15} \text{ Wb}$. For this size and current, the flux is significantly larger than one flux quantum, suggesting multiple flux quanta are involved, but the classical approximation remains valid for a macroscopic field estimate.

6. Broader Implications and Technological Applications

Magnetic Energy Storage:

Superconducting rings can store magnetic energy efficiently due to perpetual currents. The energy stored in the magnetic field can be calculated as:

$$U = \frac{1}{2} LI^2$$

where L is the inductance of the ring.

Inductive Properties and Circuit Design:

Understanding the magnetic field generated by such a small loop is crucial in designing superconducting circuits, especially where precise magnetic control is needed.

Quantum Computing and Sensors:

The ability to generate stable, localized magnetic fields at small scales opens avenues in quantum computing, where qubits often rely on magnetic flux states, and in highly sensitive magnetometers like SQUIDS.

Medical Imaging and Particle Physics:

Superconducting magnets form the backbone of MRI machines and particle accelerators. Insights into the magnetic fields generated by superconducting loops aid in optimizing these systems.

7. Limitations and Considerations

While the classical calculations provide a good estimate, several factors influence the actual magnetic field in real-world scenarios:

- Superconductor Geometry and Thickness:

Real superconducting rings are not infinitely thin; finite thickness and imperfections affect the field distribution.

- Flux Pinning and Trapped Flux:
Superconductors

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