

A 95onfidence Interval For The Mean Was Computed With A Sample Of Size 90 To Be (16,22). Then The Error

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Introduction

In statistical analysis, confidence intervals are essential tools used to estimate the range within which a population parameter, such as the mean, is likely to fall. When dealing with large sample sizes, understanding how to interpret confidence intervals and the associated error is crucial for making informed decisions. In this article, we will explore the process of computing a 95% confidence interval for a population mean based on a sample size of 90, which results in an interval of (16, 22). We will also analyze the meaning of the error term in this context, how it is calculated, and its significance in statistical inference.

Understanding Confidence Intervals

What Is a Confidence Interval?

A confidence interval (CI) is a range of values, derived from sample data, that is believed to contain the true population parameter with a specified level of confidence. The most common confidence level is 95%, indicating that if the same population is sampled repeatedly and intervals are computed each time, approximately 95% of these intervals will contain the true mean.

Why Use Confidence Intervals?

- To estimate population parameters when complete data is unavailable.
- To quantify the uncertainty associated with a sample estimate.
- To inform decision-making in research, business, medicine, and other fields.

Computing a 95% Confidence Interval for the Mean

The Formula

When the population standard deviation is unknown and the sample size is large ($n \geq 30$), the confidence interval for the population mean (μ) is calculated using the t-distribution:

$$\bar{x} \pm t_{\alpha/2, n-1} \left(\frac{s}{\sqrt{n}} \right)$$

$$\text{CI} = \bar{x} \pm t_{\{\alpha/2, n-1\}} \times \frac{s}{\sqrt{n}}$$

where:

- $\bar{x}$ = sample mean
- $t_{\{\alpha/2, n-1\}}$ = t-value for the desired confidence level and degrees of freedom
- s = sample standard deviation
- n = sample size

Key Components

- Sample Mean ($\bar{x}$): the average of the sample data.
- Sample Size (n): number of observations in the sample.
- Standard Error (SE): $\frac{s}{\sqrt{n}}$, indicating the variability of the sample mean.
- t-Value: from the t-distribution table corresponding to the confidence level and degrees of freedom.

Applying the Data: From (16, 22) to Error Calculation

Given Data

- Confidence level: 95%
- Sample size: $n=90$
- Confidence interval: (16, 22)

Step 1: Find the Sample Mean ($\bar{x}$)

The interval endpoints are 16 and 22, and the mean is at the midpoint:

$$\bar{x} = \frac{16 + 22}{2} = \frac{38}{2} = 19$$

Step 2: Determine the Margin of Error (E)

The margin of error is the amount added to and subtracted from the sample mean to create the confidence interval:

$$E = \text{Upper limit} - \bar{x} = 22 - 19 = 3$$

or

$$E = \bar{x} - \text{Lower limit} = 19 - 16 = 3$$

Thus, the margin of error ($E = 3$).

Understanding the Error in Confidence Interval

What Is the Error?

In the context of confidence intervals, the error or margin of error (E) quantifies the extent of uncertainty in the estimate. It reflects the maximum expected difference between the sample mean and the true population mean with the specified level of confidence.

Significance of the Error

- It provides a measure of precision; smaller errors indicate more precise estimates.
- It depends on the variability in the data (standard deviation), the sample size, and the desired confidence level.
- It helps in understanding the reliability of the estimate.

Calculating the Standard Error and the t-Value

Step 1: Find the Standard Error (SE)

Since the margin of error $(E = 3)$, and the t-value for 95% confidence with $(n-1=89)$ degrees of freedom is approximately 1.99 (from t-distribution tables):

$$E = t_{\alpha/2, n-1} \times \text{SE}$$

Rearranged to solve for SE:

$$\text{SE} = \frac{E}{t_{\alpha/2, n-1}} = \frac{3}{1.99} \approx 1.51$$

Step 2: Find the Sample Standard Deviation (s)

Using the standard error formula:

$$\text{SE} = \frac{s}{\sqrt{n}}$$

Solve for (s) :

$$s = \text{SE} \times \sqrt{n} = 1.51 \times \sqrt{90} \approx 1.51 \times 9.49 \approx 14.33$$

Thus, the estimated sample standard deviation is approximately 14.33.

Interpreting the Confidence Interval and Error

The Meaning of (16, 22)

The confidence interval suggests that, based on the sample, we are 95% confident that the true population mean lies between 16 and 22. The interval width is:

$$\begin{array}{l} \backslash \\ 22 - 16 = 6 \\ \backslash \end{array}$$

which indicates the range of plausible values for the population mean based on the data.

The Role of the Error (Margin of Error = 3)

- The margin of error (3) shows the amount of uncertainty around the sample mean of 19.
- The true mean is expected to be within 3 units above or below 19 with 95% confidence.
- The larger the margin of error, the less precise the estimate; conversely, a smaller margin indicates a more accurate estimate.

Factors Affecting the Margin of Error

Sample Size (n)

- Increasing the sample size reduces the standard error, thus decreasing the margin of error.
- Larger samples provide more reliable estimates of the population mean.

Variability in Data (s)

- Higher variability increases the standard deviation, leading to a larger margin of error.
- Ensuring data quality and reducing measurement error can improve precision.

Confidence Level

- Higher confidence levels (e.g., 99%) require a larger t-value, increasing the margin of error.
- Lower confidence levels (e.g., 90%) result in narrower intervals but less certainty.

Practical Example: Applying the Concepts

Suppose a researcher is studying the average weight of a certain species of fish. They take a sample of 90 fish and find the confidence interval for the mean weight to be (16, 22)

kilograms.

- The sample mean weight is 19 kg.
- The margin of error is 3 kg.
- The estimated standard deviation is approximately 14.33 kg.
- The researcher can state with 95% confidence that the true average weight of the species falls between 16 kg and 22 kg.

Understanding the error and the calculation process helps the researcher assess the reliability of their estimate and plan further data collection or analysis.

Additional Considerations

Assumptions in Confidence Interval Calculation

- The data follows a normal distribution, especially important with smaller samples.
- The sample is random and representative of the population.
- The sample size is sufficiently large ($n \geq 30$) for the Central Limit Theorem to apply, making the t-distribution approximation valid.

Limitations

- The confidence interval does not guarantee the true mean is within the interval in any single sample; it only indicates the long-term proportion of such intervals that would contain the true mean.
- Outliers or skewed data can affect the accuracy of the interval.

Conclusion

Calculating and interpreting confidence intervals is fundamental in statistics, providing a quantitative measure of the uncertainty associated with sample estimates. In our case, with a sample size of 90, a confidence interval of (16, 22) for the population mean, and a margin of error of 3, we gain valuable insights into the range within which the true mean likely resides.

By understanding the components—sample mean, standard deviation, standard error, t-value, and margin of error—researchers and analysts can better assess the reliability of their data and make informed decisions. The error term, in particular, highlights the importance of sample size, data variability, and confidence level in shaping the precision of statistical estimates.

References

- Moore, D. S., McCabe, G. P., & Craig, B. A. (2017). Introduction to the Practice of Statistics. W. H. Freeman.

- Ott, L. & Longnecker, M. (2015). An Introduction to Statistical Methods and Data Analysis. Cengage Learning.
- t-Distribution Table: [Source: Statistical tables]

Note: All calculations are approximate and based on standard statistical tables and assumptions.

Frequently Asked Questions

What is the confidence level associated with the 95% confidence interval (16, 22) for the mean?

The confidence level is 95%, meaning we are 95% confident that the true population mean lies within the interval (16, 22).

How do you calculate the margin of error from the given confidence interval (16, 22)?

The margin of error (E) is half the width of the confidence interval: $(22 - 16) / 2 = 3$.

What is the formula to compute the standard error (SE) based on the sample size and margin of error?

$SE = E / z$ where z is the z-score corresponding to the confidence level (for 95%, $z \approx 1.96$). Given $E = 3$, $SE \approx 3 / 1.96 \approx 1.53$.

How can we estimate the sample standard deviation from the standard error and sample size?

Using the formula $SE = s / \sqrt{n}$, where s is the sample standard deviation. Rearranged as $s = SE \times \sqrt{n}$. With $SE \approx 1.53$ and $n=90$, $s \approx 1.53 \times \sqrt{90} \approx 1.53 \times 9.49 \approx 14.52$.

What does the sample size of 90 imply about the reliability of the confidence interval?

A larger sample size ($n=90$) generally leads to a more precise estimate of the population mean, resulting in a narrower confidence interval and increased reliability of the estimate.

If the confidence interval is (16, 22), what is the sample mean?

The sample mean is the midpoint of the interval: $(16 + 22) / 2 = 19$.

What are the potential implications if the calculated margin of error is large?

A large margin of error indicates greater uncertainty in the estimate of the population mean, possibly due to high variability in the data or a smaller sample size.

Additional Resources

A 95% Confidence Interval For The Mean Was Computed With A Sample Of Size 90 To Be (16, 22). Then The Error — this statement encapsulates a fundamental concept in inferential statistics that allows researchers and analysts to estimate population parameters with a specified level of confidence. Understanding how this confidence interval was derived, what it signifies, and how to interpret its components is crucial for making informed decisions based on data. This guide offers a comprehensive breakdown of the concepts involved, providing clarity on what the confidence interval represents, how to compute the error margin, and the broader implications for statistical analysis.

Introduction to Confidence Intervals

A confidence interval (CI) is a range of values, derived from sample data, that is believed to contain the true population parameter (like the mean) with a specified probability—commonly 95%. When we say "A 95% confidence interval for the mean was computed with a sample of size 90 to be (16, 22)", it indicates that, based on the sample data, we are 95% confident that the true population mean lies somewhere between 16 and 22.

Why Use Confidence Intervals?

- Estimate Population Parameters: They provide a range rather than a single point estimate, acknowledging uncertainty.
- Quantify Uncertainty: The width of the interval reflects the variability in the data and sample size.
- Inform Decision Making: They help determine whether a parameter falls within a critical range, based on statistical evidence.

Breaking Down the Given Data

Let's analyze the key components of the statement:

- Sample Size (n): 90
- Confidence Level: 95%
- Confidence Interval: (16, 22)

From this, we can infer:

- The sample mean ($\bar{x}$) and margin of error (E) are embedded within the interval.
- The interval is centered around the sample mean.
- The width of the interval reflects the variability and sample size.

Step 1: Determining the Sample Mean

Since the confidence interval is symmetric around the sample mean, the midpoint of the interval provides the sample mean:

$$\bar{x} = \frac{16 + 22}{2} = \frac{38}{2} = 19$$

Interpretation:

The best estimate of the population mean based on this sample is 19.

Step 2: Calculating the Margin of Error (E)

The margin of error is half the width of the confidence interval:

$$E = \frac{22 - 16}{2} = \frac{6}{2} = 3$$

Interpretation:

There is a ± 3 uncertainty around the sample mean, meaning the true mean is estimated to be within 3 units of 19 with 95% confidence.

Step 3: Understanding the Components of the Confidence Interval Formula

The general formula for a confidence interval for the mean when the population standard deviation (σ) is unknown is:

$$\text{CI} = \bar{x} \pm t^* \times \frac{s}{\sqrt{n}}$$

Where:

- $\bar{x}$ = sample mean
- s = sample standard deviation
- n = sample size
- t^* = critical t-value based on the confidence level and degrees of freedom ($df = n - 1$)

Note:

If the population standard deviation were known, the formula would use the z-distribution

instead of t.

Step 4: Estimating the Standard Error

Given the margin of error and the t-value, we can estimate the standard error:

$$E = t^* \times \frac{s}{\sqrt{n}}$$

Rearranged:

$$\frac{s}{\sqrt{n}} = \frac{E}{t^*}$$

We know:

- $(E = 3)$
- $(n = 90)$

Next, determine (t^*) for a 95% confidence level with 89 degrees of freedom.

Step 5: Finding the Critical t-Value

For a 95% confidence interval:

- Degrees of freedom: $(df = n - 1 = 89)$
- The critical t-value (t^*) corresponds to the value where the area in each tail is 2.5%.

Using a t-distribution table or calculator:

$$t^* \approx 1.987$$

(Values may vary slightly depending on the source, but approximately 1.987 is accurate for $df=89$.)

Step 6: Computing the Standard Error

Now, calculate the standard error:

$$\frac{s}{\sqrt{n}} = \frac{E}{t^*} = \frac{3}{1.987} \approx 1.509$$

Step 7: Estimating the Sample Standard Deviation

Given:

$$\sqrt{\frac{s}{\sqrt{n}}} \approx 1.509$$

And knowing:

$$s = \text{standard deviation}$$

Since:

$$s = 1.509 \times \sqrt{n} = 1.509 \times \sqrt{90}$$

Calculate $(\sqrt{90} \approx 9.487)$:

$$s \approx 1.509 \times 9.487 \approx 14.33$$

Interpretation:

The estimated sample standard deviation is approximately 14.33.

Broader Implications and Insights

Confidence Interval Width and Variability

The width of the confidence interval reflects the data's variability and the sample size. A wider interval (like 16 to 22) indicates substantial variability or uncertainty. The large estimated standard deviation (~ 14.33) supports this.

Effect of Sample Size

A sample size of 90 is generally considered sufficient for many statistical analyses, but high variability in data inflates the confidence interval width. Increasing the sample size reduces the standard error, leading to narrower intervals, assuming variability remains constant.

Practical Interpretation

- Population Mean Estimate: Based on this data, we are 95% confident that the true

population mean lies between 16 and 22.

- Margin of Error: The ± 3 margin indicates the precision of this estimate.
- Decision-Making: If, for example, the population mean relates to a quality measurement, the interval suggests the process may need refinement if specific thresholds are not within this range.

Additional Considerations

Assumptions Underlying the Confidence Interval

- Normality: The data is approximately normally distributed, especially important for smaller samples. With a sample size of 90, the Central Limit Theorem generally justifies normal approximation.
- Random Sampling: The sample must be representative of the population.
- Independence: Observations should be independent.

Limitations and Potential Errors

- Sampling Bias: If the sample isn't representative, the interval may be inaccurate.
- Misinterpretation: The confidence level (95%) does not mean there's a 95% probability the true mean is within this specific interval; rather, that 95% of such intervals computed from many samples will contain the true mean.

Summary of Key Points

- The confidence interval (16, 22) was derived from a sample of size 90, with a 95% confidence level.
- The sample mean is 19, with a margin of error of 3.
- The estimated standard deviation of the sample data is approximately 14.33.
- Critical t-value for 95% confidence and 89 degrees of freedom is roughly 1.987.
- The interval provides an estimated range for the true population mean, reflecting the data's variability and the sample size.

Final Thoughts

Understanding A 95% Confidence Interval For The Mean Was Computed With A Sample Of Size 90 To Be (16, 22). Then The Error enables statisticians and analysts to interpret the precision and reliability of their estimates. It emphasizes the importance of sample size, variability, and confidence levels in statistical inference. Properly interpreting confidence intervals supports more informed decisions across research, quality control, policy-making, and beyond.

By mastering these concepts, you can critically evaluate statistical reports, design better studies, and communicate findings effectively, ensuring that data-driven insights are both accurate and meaningful.

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