A Bag Contains 3 Red Marbles, 6 Blue Marbles And 4 Green Marbles. If Three Marbles Are Drawn Out Of The

A Bag Contains 3 Red Marbles, 6 Blue Marbles And 4 Green Marbles. If Three Marbles Are Drawn Out Of The bag without replacement, various probability questions and combinatorial considerations arise. This scenario is a classic example used to illustrate concepts in probability theory, combinatorics, and statistics. Understanding the different outcomes and calculating the likelihood of specific events can be both insightful and practical, especially in game theory, decision making, and educational contexts. In this article, we will explore the problem in detail, analyze the possible outcomes, perform probability calculations, and discuss related concepts such as expected values and combinatorial reasoning.

Understanding the Composition of the Bag

Before diving into probability calculations, it is essential to understand the initial composition of the marbles in the bag.

Number of Marbles by Color

- Red Marbles: 3
- Blue Marbles: 6
- Green Marbles: 4

Total number of marbles:
\[
3 + 6 + 4 = 13
\]

This total forms the sample space from which the three marbles will be drawn.

Assumption of Randomness

For the purpose of the probability calculations, we assume that each marble has an equal chance of being drawn, and the draws are made randomly without replacement (i.e., once a marble is drawn, it is not put back into the bag).

Calculating Basic Probabilities of Drawing Specific Marbles

Understanding the probability of drawing certain types of marbles involves combinatorial calculations.

Sample Space and Total Number of Ways to Draw 3 Marbles

The total number of ways to select any 3 marbles from 13 is given by the combination:

$$
\binom{13}{3} = \frac{13!}{3! \times 10!} = 286
$$

This is the denominator for many probability calculations.

Probability of Drawing Specific Color Combinations

Suppose we are interested in the probability of drawing a particular combination, such as:

- All three marbles are of different colors
- All three marbles are of the same color
- Exactly two marbles of one color and one of another

Let's analyze these scenarios:

1. Drawing 3 Marbles of Different Colors

To satisfy this, the selected marbles must include at least one red, one blue, and one green.

Number of ways:

$$
\text{Red choices} = \binom{3}{1} = 3
$$

$$
\text{Blue choices} = \binom{6}{1} = 6
$$

$$
\text{Green choices} = \binom{4}{1} = 4
$$

Total ways:

$$
3 \times 6 \times 4 = 72
$$

Probability:

$$
P(\text{all different colors}) = \frac{72}{286} \approx 0.2517
$$

2. Drawing 3 Marbles of the Same Color

Possible only for the colors with at least 3 marbles:
- Red: 3 marbles (all of them)
- Blue: 6 marbles
- Green: 4 marbles

Calculations:

- Red:

$$
\binom{3}{3} = 1
$$

- Blue:

$$
\binom{6}{3} = 20
$$

- Green:

$$
\binom{4}{3} = 4
$$

Total ways:

$$
1 + 20 + 4 = 25
$$

Probability:

$$
P(\text{all same color}) = \frac{25}{286} \approx 0.0874
$$

3. Drawing Exactly Two Marbles of One Color and One of Another

This involves multiple sub-cases, such as:

- Two reds and one blue or green
- Two blues and one red or green
- Two greens and one red or blue

Calculations:

- Two reds + one blue:

\[

$$
\binom{3}{2} \times \binom{6}{1} = 3 \times 6 = 18
$$
- Two reds + one green:
$$
3 \times 4 = 12
$$
- Two blues + one red:
$$
\binom{6}{2} \times \binom{3}{1} = 15 \times 3 = 45
$$
- Two blues + one green:
$$
15 \times 4 = 60
$$
- Two greens + one red:
$$
\binom{4}{2} \times \binom{3}{1} = 6 \times 3 = 18
$$
- Two greens + one blue:
$$
6 \times 6 = 36
$$

Sum of all these arrangements:
$$
18 + 12 + 45 + 60 + 18 + 36 = 189
$$

Probability:
$$
P(\text{exactly two of one color, one of another}) = \frac{189}{286} \approx 0.6608
$$

Expected Values and Probabilities in Multiple Draws

Beyond individual combinations, understanding the expected number of marbles of each color drawn in the three draws can be insightful.

Expected Number of Marbles of a Specific Color

The expected value (mean) for each color can be calculated considering the probability of drawing each marble:

- Red:
$$
E(\text{Red}) = 3 \times \frac{3}{13} = \frac{9}{13} \approx 0.692
$$

\]
- Blue:

$$
E(\text{Blue}) = 6 \times \frac{3}{13} = \frac{18}{13} \approx 1.385
$$

- Green:

$$
E(\text{Green}) = 4 \times \frac{3}{13} = \frac{12}{13} \approx 0.923
$$

Note: These are approximate expectations considering the probability of each marble being drawn, assuming independence, which is valid for expectation calculations in sampling without replacement.

Combinatorial Analysis and Probabilistic Reasoning

Understanding the problem involves mastering combinatorics, which is the mathematical study of counting arrangements and selections.

Key Concepts in Combinatorics

- **Combination:** Selecting items without regard to order (e.g., $\binom{n}{k}$).
- **Permutation:** Arranging items where order matters.
- **Sample Space:** The set of all possible outcomes.

Application to the Marble Problem

Using combinations, we can enumerate all possible outcomes and then determine the probability of specific events. This approach is fundamental in calculating precise probabilities in sampling problems.

Real-Life Applications and Educational Importance

The problem of drawing marbles from a bag is more than an academic exercise; it has practical implications in various fields.

Applications in Probability and Statistics

- Quality Control: Sampling items to determine quality without inspecting every item.
- Gambling and Games: Understanding odds and probabilities to strategize.
- Medical Testing: Estimating disease prevalence through random sampling.

Educational Benefits

- Reinforces understanding of basic probability concepts.
- Develops skills in combinatorial reasoning.
- Enhances problem-solving abilities with real-world contexts.

Conclusion

Analyzing the scenario of drawing three marbles from a bag containing 3 red, 6 blue, and 4 green marbles provides a comprehensive view of probability theory and combinatorics. By calculating the total number of outcomes, various event probabilities, and expected values, we gain insight into how randomness and chance operate in simple systems. Whether for academic purposes, game strategies, or practical decision-making, mastering such problems equips individuals with foundational skills in understanding uncertainty and statistical reasoning.

In summary:
- The total number of possible outcomes when drawing three marbles from the bag is 286.
- The probability of drawing specific combinations can be precisely computed using combinations.
- Expectation calculations help predict average outcomes over multiple trials.
- These concepts serve as building blocks for more complex statistical analyses and real-world applications.

Understanding these principles not only enhances mathematical literacy but also provides tools to approach everyday problems involving chance and uncertainty with confidence and clarity.

Frequently Asked Questions

What is the total number of marbles in the bag?

The total number of marbles is 3 Red + 6 Blue + 4 Green = 13 marbles.

What is the probability of drawing a red marble in a single draw?

The probability of drawing a red marble is 3/13.

What is the probability of drawing exactly two blue marbles when three are drawn?

The probability is calculated as (Number of ways to choose 2 blue and 1 non-blue) divided by total ways to choose any 3 marbles. It is:

Number of ways: C(6,2) C(7,1) = 15 7 = 105
Total combinations: C(13,3) = 286

Probability = 105 / 286 ≈ 0.367.

What is the probability that all three drawn marbles are green?

Since there are only 4 green marbles, the probability of drawing all three green is C(4,3)/C(13,3) = 4/286 ≈ 0.014.

If three marbles are drawn without replacement, what is the probability that none are green?

Number of non-green marbles: 3 Red + 6 Blue = 9.
Number of ways to choose 3 from these 9: C(9,3) = 84.
Total ways to choose any 3 marbles: C(13,3) = 286.

Probability = 84 / 286 ≈ 0.294.

What is the probability of drawing at least one green marble in three draws?

It's easier to find the complement: probability of drawing no green marbles (all from red and blue).
Number of non-green marbles: 9.
Number of ways: C(9,3) = 84.

So, probability of no green = 84/286.
Therefore, probability of at least one green = 1 - (84/286) ≈ 1 - 0.294 = 0.706.

If three marbles are drawn, what is the probability that exactly one is green?

Number of ways: Choose 1 green from 4 and 2 non-green from 9.
Ways: C(4,1) C(9,2) = 4 36 = 144.
Total combinations: C(13,3) = 286.

Probability = 144 / 286 ≈ 0.503.

Additional Resources

Comprehensive Analysis of Marble Draw Probabilities from a Mixed Bag

Understanding the probability and combinatorics involved in drawing marbles from a collection is a fundamental aspect of probability theory and statistics. This detailed review aims to explore the problem: A bag contains 3 Red marbles, 6 Blue marbles, and 4 Green marbles. If three marbles are drawn out of the bag without replacement, what are the various probabilities and outcomes associated with this scenario?

Introduction to the Problem Context

Drawing marbles from a bag is a classic example used in probability lessons to illustrate concepts such as sample space, events, combinations, and probability calculations. The problem involves a bag with a total of:

- Red marbles: 3
- Blue marbles: 6
- Green marbles: 4

Total marbles in the bag: 3 + 6 + 4 = 13

The key question is: When three marbles are drawn without replacement, what are the probabilities of different outcomes?

These outcomes can include various scenarios, such as drawing all marbles of the same color, two of one color and one of another, or one of each color, among others.

Understanding the Fundamental Concepts

Before delving into the specific calculations, it's essential to review some fundamental principles:

Sample Space and Events

- Sample Space (S): All possible combinations of 3 marbles drawn from 13.
- Event (E): A specific subset of outcomes within the sample space, such as drawing two blue marbles and one green marble.

Combinatorics and Calculations

- Number of ways to choose k objects from n: Given by the combination formula:

$$
\binom{n}{k} = \frac{n!}{k! (n - k)!}
$$

- Total number of ways to draw 3 marbles from 13:

$$
\binom{13}{3}
$$

- Probability of an event:

$$
P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}
$$

Calculating Total Possible Outcomes

First, determine the total number of ways to draw 3 marbles from 13:

$$
\binom{13}{3} = \frac{13!}{3! \times 10!} = \frac{13 \times 12 \times 11}{3 \times 2 \times 1} = 286
$$

This total forms the denominator for all probability calculations.

Analyzing Possible Color Combinations in the Draw

When drawing 3 marbles, various combinations based on colors are possible:

- All three marbles are of the same color.
- Two marbles are of one color, and the third is of another.
- All three marbles are of different colors.

Let’s analyze each category step by step.

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1. All Three Marbles of the Same Color

Red:
- Number of ways to select 3 red marbles from 3 available:

$$
\binom{3}{3} = 1
$$

Blue:
- Number of ways to select 3 blue marbles from 6:

$$
\binom{6}{3} = 20
$$

Green:
- Number of ways to select 3 green marbles from 4:

$$
\binom{4}{3} = 4
$$

Total same-color outcomes:

$$
1 + 20 + 4 = 25
$$

Probability:

$$
P(\text{all same color}) = \frac{25}{286} \approx 0.0874 \text{ or } 8.74\%
$$

---

2. Two Marbles of One Color and One of Another

This scenario involves three sub-cases:

- Two Red + One Other
- Two Blue + One Other
- Two Green + One Other

Let's analyze each.

Two Red + One Other

- Ways to choose 2 red marbles:

$$
\binom{3}{2} = 3
$$

- Remaining one marble from the other colors (6 blue + 4 green = 10 marbles):

$$
\binom{10}{1} = 10
$$

- Total outcomes:

$$
3 \times 10 = 30
$$

Two Blue + One Other

- Ways to choose 2 blue:

$$
\binom{6}{2} = 15
$$

- One marble from red and green (3 + 4 = 7):

$$
\binom{7}{1} = 7
$$

- Total outcomes:

$$
15 \times 7 = 105
$$

Two Green + One Other

- Ways to choose 2 green:

$$
\binom{4}{2} = 6
$$

- One marble from red and blue (3 + 6 = 9):

$$
\binom{9}{1} = 9
$$

- Total outcomes:

$$
6 \times 9 = 54

\]

Total outcomes for two of one color and one of another:

$$
30 + 105 + 54 = 189
$$

Probability:

$$
P(\text{two of one color, one of another}) = \frac{189}{286} \approx 0.6608 \text{ or } 66.08\%
$$

---

3. All Three Marbles of Different Colors

- To select one marble of each color:

$$
\binom{3}{1} \times \binom{6}{1} \times \binom{4}{1} = 3 \times 6 \times 4 = 72
$$

Probability:

$$
P(\text{all different colors}) = \frac{72}{286} \approx 0.2517 \text{ or } 25.17\%
$$

Summary of Outcomes and Probabilities

Scenario	Number of Favorable Outcomes	Probability (%)
All three marbles of the same color	25	8.74%
Two of one color and one of another	189	66.08%
One marble of each color	72	25.17%

Note: The sum of probabilities exceeds 100% because these are mutually exclusive events, and their sum should be approximately 100%. Let's verify:

$$
8.74\% + 66.08\% + 25.17\% \approx 100\%
$$

Deeper Dive: Conditional Probabilities and Specific Cases

Beyond the general probabilities, we can compute specific conditional probabilities, such as:

- The probability that all three marbles are blue given that at least one is green.
- The probability of drawing exactly two green marbles in the three draws, regardless of the third.

Probability of Drawing Exactly Two Green Marbles

Number of ways:

- Select 2 green marbles:

$$
\binom{4}{2} = 6
$$

- Select 1 marble from remaining 9 marbles (3 red + 6 blue):

$$
\binom{9}{1} = 9
$$

- Total:

$$
6 \times 9 = 54
$$

Probability:

$$
P(\text{exactly two green}) = \frac{54}{286} \approx 0.1888 \text{ or } 18.88\%
$$

Similarly, the probability of exactly one green marble:

- Select 1 green:

$$
\binom{4}{1} = 4
$$

- Select 2 from remaining 9 marbles:

\[

$$
\binom{9}{2} = 36
$$

- Total:

$$
4 \times 36 = 144
$$

Probability:

$$
P(\text{exactly one green}) = \frac{144}{286} \approx 0.5035 \text{ or } 50.35\%
$$

And with no green marbles:

- Select all 3 from red and blue:

$$
\binom{3 + 6}{3} = \binom{9}{3} = 84
$$

Probability:

$$
P(\text{no green}) = \frac{84}{286} \approx 0.2937 \text{ or } 29.37\%
$$

Expected Values and Averages

Understanding the expected number of marbles of a certain color drawn in multiple trials can be insightful. For a single draw:

- Probability of drawing a red marble:

$$
P(\text{red}) = \frac{3}{13} \approx 0.2308
$$

- Blue:

$$
\frac{6}{13} \approx 0.4615
$$

- Green:

\[
\frac{4}{

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