A Body Whose Mass Is 0.4 Kg Is Suspended From A Spring And Oscillates With A Period Of 2 S. By How Much

A Body Whose Mass Is 0.4 Kg Is Suspended From A Spring And Oscillates With A Period Of 2 S. By How Much

Understanding the physics behind oscillations is essential for grasping how simple harmonic motion works, especially when dealing with mass-spring systems. This article delves into the problem involving a mass attached to a spring, exploring how to determine key parameters such as the spring constant and maximum displacement. Whether you're a student preparing for exams or a physics enthusiast, this comprehensive guide will clarify the concepts and calculations involved.

Basics of Simple Harmonic Motion and Mass-Spring Systems

What Is Simple Harmonic Motion?

Simple Harmonic Motion (SHM) describes the repetitive, oscillatory movement of objects where the restoring force is directly proportional to the displacement but acts in the opposite direction. Classic examples include pendulums (for small angles) and mass-spring systems.

Mass-Spring System Overview

A mass-spring system consists of a mass attached to a spring fixed at one end. When displaced from its equilibrium position, the spring exerts a restoring force proportional to the displacement, leading to oscillations.

Key Parameters:
- Mass of the object (m)
- Spring constant (k)
- Displacement from equilibrium (x)
- Period of oscillation (T)
- Maximum displacement or amplitude (A)

Given Data and Objective

Let's analyze the problem with the following data:
- Mass of the body, $m = 0.4\, \text{kg}$
- Period of oscillation, $T = 2\, \text{s}$

Objective:
- To determine the maximum displacement A (amplitude) of the oscillation.

Note: The problem is often presented as "by how much does the mass oscillate," which refers to its maximum displacement from equilibrium.

Understanding the Period of Oscillation

Formula for the Period of a Mass-Spring System

The period T of oscillation for a mass-spring system undergoing SHM is given by:

$$
T = 2\pi \sqrt{\frac{m}{k}}
$$

Where:
- T is the period
- m is the mass
- k is the spring constant

Rearranging to find k:

$$
k = \frac{4\pi^2 m}{T^2}
$$

Calculating the Spring Constant k

Using the given data:

$$
k = \frac{4 \pi^2 \times 0.4\, \text{kg}}{(2\, \text{s})^2}
$$

Calculations:
- $4 \pi^2 \approx 39.478$
- $T^2 = 4$

Therefore:

$$
k = \frac{39.478 \times 0.4}{4} = \frac{15.7912}{4} \approx 3.9478\, \text{N/m}
$$

Result:
- Spring constant, $k \approx 3.95\, \text{N/m}$

Determining the Maximum Displacement (Amplitude)

Relationship Between Force, Displacement, and Energy in SHM

In simple harmonic motion, the maximum restoring force exerted by the spring is:

$$
F_{max} = k \times A
$$

Where:
- A is the amplitude or maximum displacement

The maximum potential energy stored in the spring is:

$$
U_{max} = \frac{1}{2} k A^2
$$

This energy transforms into kinetic energy at the equilibrium position, but for the purpose of finding maximum displacement, the amplitude can be inferred from initial conditions or through other parameters.

In the absence of initial displacement, how do we find A?

If the problem does not specify initial displacement or maximum velocity, typically, the amplitude A is given or can be calculated based on initial conditions. However, if the question asks "by how much" the mass oscillates, it usually refers to the maximum displacement, which is often determined based on energy considerations or initial conditions.

In this case, without additional data such as initial velocity or energy input, the maximum amplitude isn't directly specified. But, if the question implies the maximum displacement during oscillation, it is often related to the maximum velocity or initial displacement.

However, for demonstration purposes, suppose the system starts from rest at a maximum displacement A, and the period is known. We can explore the relationship between maximum velocity and amplitude for a complete picture.

Calculating Maximum Velocity and Displacement

Maximum Velocity in SHM

The maximum velocity v_{max} in simple harmonic motion is related to amplitude A:

$$
v_{max} = \omega A

$$
\]
$$

Where:
- ω is the angular frequency:

$$
\omega = \frac{2\pi}{T}
$$

Using the given period:

$$
\omega = \frac{2\pi}{2} = \pi \approx 3.1416\, \text{rad/s}
$$

Expressing Amplitude in Terms of Maximum Velocity

If, for instance, the maximum velocity is known or can be measured, the amplitude A can be found as:

$$
A = \frac{v_{max}}{\omega}
$$

Without specific velocity data, the best we can do is present the general relationships.

Summary of Key Calculations

Parameter	Calculation	Result
Spring constant k	$\frac{4 \pi^2 m}{T^2}$	3.95 N/m
Angular frequency ω	$\frac{2\pi}{T}$	3.14 rad/s

Note: If the problem provides initial velocity or energy, you could directly compute the amplitude A. For example, if the maximum velocity v_{max} is known, then:

$$
A = \frac{v_{max}}{\omega}
$$

Practical Application: Estimating Displacement

In real-world scenarios, understanding the maximum displacement is crucial for designing systems that withstand specific oscillation amplitudes.

Example:

Suppose the initial velocity of the mass is zero, but it is displaced and released from rest. The maximum displacement (amplitude) can be determined if energy or initial conditions are known. Conversely, if the initial velocity is known, the amplitude is directly calculable.

Conclusion: How Much Does the Mass Oscillate?

Based on the fundamental properties of simple harmonic motion and the data provided, the key goal is to find the maximum displacement or amplitude A. Given the period $T = 2\, \text{s}$ and mass $m = 0.4\, \text{kg}$, the spring constant k is approximately 3.95 N/m.

Without explicit initial displacement or velocity, the maximum amplitude cannot be precisely calculated from the period alone. However, if additional information such as initial conditions or maximum velocity is provided, the amplitude can be determined using the relationships:

$$
A = \frac{v_{max}}{\omega}
$$

or

$$
A = \sqrt{\frac{2U_{max}}{k}}
$$

This comprehensive understanding allows for better analysis of oscillating systems, vital in engineering, physics experiments, and various technological applications.

Final Note: Always ensure all necessary parameters are available to perform complete calculations. The period alone, combined with mass, allows us to find the spring constant, which is essential for analyzing the oscillatory behavior of the system.

Frequently Asked Questions

What is the mass of the body suspended from the spring in the given problem?

The mass of the body is 0.4 kg.

What is the period of oscillation for the body attached to the spring?

The period of oscillation is 2 seconds.

How can we determine the spring constant (k) from the given data?

Using the formula $T = 2\pi\sqrt{(m/k)}$, rearranged as $k = (4\pi^2 m)/T^2$, we can calculate k.

What is the value of the spring constant (k) for this oscillation?

Calculating k: $k = (4\ \pi^2\ 0.4\ kg) / (2\ s)^2 \approx (39.4784\ 0.4) / 4 \approx 15.79136 / 4 \approx 3.95$ N/m.

How does the mass of the body influence the period of oscillation in a spring-mass system?

The period increases with increasing mass, following the relationship $T = 2\pi\sqrt{(m/k)}$, so heavier masses result in longer oscillation periods.

Additional Resources

A Body Whose Mass Is 0.4 Kg Is Suspended From A Spring And Oscillates With A Period Of 2 S. By How Much

---

Introduction: Unraveling the Physics Behind Oscillations

Oscillations are a fundamental phenomenon observed in various systems—from the gentle sway of a pendulum to the complex vibrations of musical instruments and the motion of mechanical components. At the core of many oscillatory systems lies the simple yet profound concept of harmonic motion, often modeled through the classic example of a mass attached to a spring.

Imagine a scenario where a body with a mass of 0.4 kg is suspended from a spring, and it oscillates back and forth with a period of 2 seconds. Such a setup is not just a classroom demonstration; it embodies the principles of Hooke's Law, simple harmonic motion (SHM), and the interplay of elastic force and inertia.

This article delves into the physics governing this system—exploring how the mass, the spring's properties, and the oscillation period interrelate. By the end, readers will gain a comprehensive understanding of how to analyze such oscillatory systems, derive key parameters like the spring constant, and appreciate the elegant mathematics underpinning harmonic motion.

Understanding Simple Harmonic Motion and Its Parameters

What Is Simple Harmonic Motion?

Simple Harmonic Motion (SHM) refers to a repetitive, oscillatory movement about an equilibrium position, where the restoring force is directly proportional to the displacement and acts in the opposite direction. Mathematically, it's characterized by sinusoidal functions—sine and cosine—describing displacement, velocity, and acceleration over time.

Key Parameters in SHM

- Mass (m): The object attached to the spring.
- Spring Constant (k): A measure of the spring's stiffness.
- Period (T): The time taken for one complete oscillation.
- Frequency (f): The number of oscillations per unit time ($f = 1/T$).
- Angular Frequency (ω): The rate of change of phase in radians per second, given by $\omega = 2\pi f = 2\pi/T$.

Understanding how these parameters relate allows us to analyze the system's behavior and determine unknown quantities like the spring constant.

The Relationship Between Period, Mass, and Spring Constant

Fundamental Equation of SHM for a Mass-Spring System

The period T of oscillation for a mass m attached to a spring with spring constant k is given by the formula:

$$
T = 2\pi \sqrt{\frac{m}{k}}
$$

This relationship emerges from solving the differential equations governing the motion, considering Hooke's Law ($F = -kx$) and Newton's second law ($F = m a$).

Deriving the Spring Constant

Given the period $T = 2\, \text{s}$ and the mass $m = 0.4\, \text{kg}$, we can rearrange the formula to solve for the spring constant k:

$$
k = \frac{4\pi^2 m}{T^2}
$$

Plugging in the known values:

$$
k = \frac{4 \pi^2 \times 0.4}{(2)^2}
$$

Calculating step-by-step:

- $4 \pi^2 \approx 4 \times (3.1416)^2 \approx 4 \times 9.8696 \approx 39.4784$
- Numerator: $39.4784 \times 0.4 \approx 15.79136$
- Denominator: $2^2 = 4$

Thus,

$$
k \approx \frac{15.79136}{4} \approx 3.947 \, \text{N/m}
$$

Result: The spring's stiffness, or spring constant, is approximately 3.95 N/m.

Interpreting the Results: How Much Does the Spring Stretch?

Understanding the spring constant allows us to explore the static displacement of the spring under the weight of the mass—i.e., how much the spring stretches due to gravity when the system is at rest.

Static Equilibrium and Hooke's Law

At equilibrium (no oscillation), the weight of the mass is balanced by the spring's restoring force:

$$
mg = kx
$$

Where:

- $g \approx 9.8\, \text{m/s}^2$ is acceleration due to gravity,
- x is the static extension of the spring.

Rearranged to find x:

$$
x = \frac{mg}{k}
$$

Plugging in the known values:

$$
x = \frac{0.4 \times 9.8}{3.95} \approx \frac{3.92}{3.95} \approx 0.99\, \text{m}
$$

Interpretation: The spring stretches approximately 0.99 meters under the weight of the 0.4 kg mass.

Note: This is a significant extension, indicating a relatively soft spring or a system designed for large amplitude oscillations.

Considering Oscillation Amplitude and Energy

While the static extension provides insight into the spring's properties, the actual oscillations involve dynamic energy exchanges between kinetic and potential energy.

Amplitude and Maximum Displacement

- The amplitude A is the maximum displacement from equilibrium during oscillation.
- The energy in the system oscillates between kinetic and potential forms, dictated by the amplitude.

In ideal simple harmonic motion, the amplitude can be any value, but for practical purposes, the amplitude is constrained by the physical setup.

Energy Considerations

Total mechanical energy E of the oscillating body:

$$
E = \frac{1}{2} k A^2
$$

This energy remains constant if no damping occurs.

Practical Implications and Applications

Understanding the parameters of such a system has broader implications:

- Designing Mechanical Systems: Engineers can design oscillatory systems (e.g., suspension systems, sensors) with specific periods and displacements.
- Vibration Analysis: In machinery, knowing how mass and spring stiffness influence oscillation helps prevent resonance and potential failure.
- Educational Demonstrations: Such setups serve as tangible demonstrations of fundamental physics principles.

Limitations and Assumptions

The analysis above assumes:

- The spring obeys Hooke's Law perfectly within the oscillation range.
- Damping effects (air resistance, internal friction) are negligible.
- The oscillations are small enough for SHM approximation to hold.

In real-world systems, factors like damping and non-linear spring behavior can alter the motion, requiring more complex models.

Conclusion: The Interplay of Mass, Spring Stiffness, and Oscillation Period

In summary, a 0.4 kg mass suspended from a spring oscillates with a period of 2 seconds, implying a spring constant of approximately 3.95 N/m. The static extension of the spring due to the weight is nearly 1 meter, illustrating the relationship between gravitational force and elastic restoring force.

These insights exemplify how fundamental physics principles—Hooke's Law, harmonic motion equations, and energy conservation—interconnect to describe oscillatory systems. Whether in designing precision instruments, analyzing vibrations, or simply understanding the motion of everyday objects, grasping these concepts is essential.

By exploring the mathematics and physics underlying this simple yet profound system, learners and engineers alike can better appreciate the harmony governing oscillatory phenomena in nature and technology.

A Body Whose Mass Is 0 4 Kg Is Suspended From A Spring And Oscillates With A Period Of 2 S By How Much

A Body Whose Mass Is 0 4 Kg Is Suspended From A Spring And Oscillates With A Period Of 2 S By How Much

Related Articles

- 2- The Figure Below Shows The State Transition Diagram For The Markov Chain. In This Diagram, There Are
- What Will Be The Current Amplitude At An Angular Frequency Of 405 Rad/s ? Express Your Answer With The
- Water And Alcohol Is Mixed In The Ratio 1:4.find The Density Of The Mixture If The Density Of Water Is

Back to Home