A Bullet Of Mass M Strikes A Block Of Mass M. The Bullet Remains Embedded In The Block. Find The Period

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Understanding the dynamics of collisions and oscillations is fundamental in physics, especially when analyzing systems involving elastic and inelastic collisions, momentum conservation, and simple harmonic motion. This article provides a comprehensive explanation of the problem where a bullet of mass M strikes a block of the same mass, embedding itself inside the block, and then determining the period of oscillation of the combined system. We will explore the concepts step-by-step, covering collision mechanics, conservation laws, and simple harmonic motion, to derive the period accurately.

Introduction to the Problem

The problem involves a projectile — a bullet — impacting a stationary block and becoming embedded within it. The key points are:

- Both the bullet and the block have the same mass, M.
- The bullet strikes the block with an initial velocity, v.
- The collision is perfectly inelastic, as the bullet remains embedded.
- After the collision, the combined mass (2M) swings or oscillates.
- The goal is to find the period of this oscillation.

Understanding this scenario requires knowledge of momentum conservation during collision, energy considerations during oscillation, and the physics of simple harmonic motion (SHM).

Step 1: Analyzing the Collision

Conservation of Momentum

In inelastic collisions where the objects stick together, the momentum before and after impact is conserved.

- Initial momentum: Only the bullet has velocity v; the block is stationary.

$$
p_{initial} = M \times v + M \times 0 = M v
$$

- Final momentum: The combined mass moves with velocity V.

$$
p_{final} = (M + M) \times V = 2 M V
$$

- Applying conservation of momentum:

$$
M v = 2 M V \Rightarrow V = \frac{v}{2}
$$

Result: The combined system moves with velocity $V = \frac{v}{2}$ immediately after impact.

Step 2: Post-Collision Motion — Oscillations of the Combined System

Once embedded, the block and bullet together can undergo oscillations if displaced from equilibrium. Typically, such a system resembles a mass-spring oscillator or a pendulum, depending on the context.

Assumption: The system is suspended or supported in a way that allows oscillation, for example, hanging from a massless, ideal string or in a frictionless horizontal setup.

Key idea: The embedded mass acts as a combined mass attached to a restoring force, leading to simple harmonic motion.

Understanding the Restoring Force

When displaced from equilibrium, the system experiences a restoring force proportional to the displacement:

$$
F = -k x
$$

where:

- k is the effective spring constant or stiffness of the system.
- x is the displacement from equilibrium.

In many practical problems, the restoring force could originate from gravity (for pendulums) or elastic deformation (for springs). The problem context should specify the nature of the restoring force, but for generality, we assume the combined system undergoes SHM due to a restoring force proportional to displacement.

Step 3: Deriving the Period of Oscillation

The period T of a simple harmonic oscillator is given by:

$$
T = 2 \pi \sqrt{\frac{m_{eff}}{k}}
$$

where:

- m_{eff} is the effective mass involved in oscillation (here, $2 M$).
- k is the effective spring constant or restoring force parameter.

Important: Since the problem doesn't specify the restoring force, common assumptions include:

- The block is attached to a spring with known k.
- The system is a pendulum with length L.
- Or the problem is a linear oscillation involving a known elastic restoring force.

Given the information, and for simplicity, assume the combined mass oscillates as a simple harmonic oscillator with an effective restoring force characterized by k.

Therefore:

$$
T = 2 \pi \sqrt{\frac{2 M}{k}}
$$

Step 4: Additional Considerations — Energy and

Velocity

The initial velocity of the combined system after impact, $V = \frac{v}{2}$, influences the amplitude of oscillation.

- If the initial displacement is small, the initial kinetic energy converts into potential energy at maximum displacement:

$$
\frac{1}{2} \times 2 M \times V^2 = \frac{1}{2} k A^2
$$

where A is the maximum displacement (amplitude).

- Solving for A:

$$
A = \frac{V}{\omega}
$$

where $\omega = \sqrt{\frac{k}{2 M}}$.

Summary of Key Results

- Velocity after impact:

$$
V = \frac{v}{2}
$$

- Amplitude of oscillation:

$$
A = \frac{V}{\omega} = \frac{v/2}{\sqrt{\frac{k}{2 M}}} = \frac{v}{2} \times \sqrt{\frac{2 M}{k}}
$$

- Period of oscillation:

$$
T = 2 \pi \sqrt{\frac{2 M}{k}}
$$

Practical Applications and Related Problems

Understanding this scenario is crucial in various fields, such as:

- Automotive safety: Analyzing impact absorption in crumple zones.
- Material science: Studying energy transfer during collisions.
- Seismology: Modeling oscillations resulting from impacts.
- Engineering: Designing systems with inelastic collision components.

Additional Factors to Consider

While the above derivation provides a fundamental understanding, real-world applications may involve additional factors:

- Energy losses: Friction and air resistance cause damping, affecting the period.
- Elasticity of materials: Not perfectly inelastic collisions may result in different dynamics.
- Nonlinear restoring forces: Large displacements may require nonlinear analysis.
- Damping effects: Over time, oscillations diminish, necessitating damping coefficients in calculations.

Conclusion

In summary, when a bullet of mass M strikes a stationary block of the same mass and embeds itself, the combined system gains a velocity of $v/2$. Assuming it undergoes simple harmonic motion, the period of oscillation depends on the effective mass $2 M$ and the restoring force constant k:

$$
\boxed{
T = 2 \pi \sqrt{\frac{2 M}{k}}
}
$$

This formula encapsulates the physics of collision and oscillation, illustrating how conservation principles and harmonic motion principles combine to solve such problems. Understanding these concepts is vital for analyzing impact phenomena and designing systems that can withstand or utilize such collisions effectively.

References and Further Reading

- Halliday, Resnick, and Walker, Fundamentals of Physics, 10th Edition.
- Serway and Jewett, Physics for Scientists and Engineers.
- Giancoli, Physics: Principles with Applications.
- Online resources on conservation of momentum and simple harmonic motion.

Note: To accurately compute the period for a specific problem, additional information about the restoring force (spring constant k), the nature of the oscillation setup, or the initial velocity v is necessary. Always clarify these parameters before performing detailed calculations.

Frequently Asked Questions

How do you determine the period of oscillation after a bullet of mass M embeds in a block of the same mass?

First, calculate the combined velocity immediately after impact using conservation of momentum, then treat the embedded system as a pendulum or oscillating system to find the period using $T = 2\pi\sqrt{(I/mgd)}$ or similar formulas, depending on the setup.

What is the significance of the embedded bullet in calculating the period of the resulting oscillation?

The embedded bullet adds to the mass and affects the system's moment of inertia or mass distribution, which in turn influences the oscillation period, typically increasing it due to the increased mass.

If a bullet of mass M strikes and embeds in a block of mass M, what is the velocity of the combined system immediately after impact?

Using conservation of momentum: initial momentum = final momentum, so v = (initial velocity of the bullet) / 2, assuming the block is initially at rest.

How does the conservation of momentum apply in solving for the period of the embedded system?

Conservation of momentum allows you to find the velocity of the combined mass immediately after impact, which is necessary to analyze the subsequent oscillations and determine the period.

What assumptions are typically made when calculating the period of a block after a bullet embeds in it?

Assumptions often include no energy loss due to friction or air resistance, the impact is perfectly inelastic, the system behaves like a simple pendulum or harmonic oscillator, and the mass distribution is uniform.

Additional Resources

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Introduction

In classical mechanics, many problems revolve around understanding the conservation of momentum, energy transformations, and oscillatory motion. The scenario where a bullet strikes and lodges itself into a block combines concepts of inelastic collisions and simple harmonic motion (SHM). This problem is not only fundamental in physics education but also provides insights into real-world applications such as ballistic pendulums, impact analysis, and energy transfer during collisions.

This detailed review will explore the problem step-by-step, elucidate the underlying physics principles, and guide you through the process of determining the period of oscillation of the combined mass after the impact. We will examine the problem's assumptions, derive key equations, and discuss related concepts to foster a comprehensive understanding.

Scenario Description and Assumptions

Let's first restate the problem with clarity:

- A bullet of mass m strikes a stationary block of mass M (here both are denoted by M for simplicity, but typically, different variables are used. Clarifying notation is crucial for precision).
- The bullet is moving with an initial velocity u just before impact.
- The bullet remains embedded within the block after the collision, resulting in a perfectly inelastic collision.
- Post-collision, the combined system (bullet + block) oscillates about an equilibrium position, indicating SHM.
- The goal: Find the period T of this oscillation.

Note: For simplicity, the problem assumes:

- No external forces like friction or air resistance.
- The collision happens along a single straight line.

- The block is suspended or able to oscillate like a mass attached to a spring (or similar restoring force), enabling SHM.
- The impact is perfectly inelastic, with the bullet sticking in the block.

Fundamental Concepts Involved

To systematically analyze this problem, it’s essential to review the core physics principles involved:

1. Conservation of Momentum in Inelastic Collisions

In an inelastic collision where the bullet embeds into the block, the total momentum before and after impact is conserved:

$$
\boxed{
m u = (m + M) v
}
$$

where:

- m = mass of the bullet
- u = initial velocity of the bullet
- M = mass of the block
- v = velocity of the combined mass immediately after impact

This relation allows us to find the velocity with which the combined mass moves right after the collision.

2. Energy Considerations After Collision

Since the collision is inelastic, kinetic energy is not conserved during impact. The kinetic energy lost is converted into other forms of energy such as heat, deformation, or internal energy.

Post-collision kinetic energy:

$$
KE_{post} = \frac{1}{2} (m + M) v^2
$$

which is less than the initial kinetic energy of the bullet:

$$
KE_{initial} = \frac{1}{2} m u^2
$$

The energy difference quantifies the energy lost during impact.

3. Oscillatory Motion of the Combined Mass

Post-impact, the combined mass is assumed to oscillate about an equilibrium point. This is indicative of a simple harmonic oscillator, typically modeled as a mass attached to a spring, with the restoring force proportional to displacement:

$$
F = -k x
$$

where:

- k = spring constant
- x = displacement from equilibrium

The period T of oscillation for such a system is:

$$
\boxed{
T = 2 \pi \sqrt{\frac{m_{eff}}{k}}
}
$$

where m_{eff} is the effective mass involved in oscillation (here, $M + m$).

Connecting the Concepts: Step-by-Step Solution

The problem involves two key stages:

1. Determining the velocity of the combined mass immediately after impact
2. Relating that velocity to the amplitude of oscillation and then calculating the period

Let's analyze each stage in detail.

Step 1: Calculating the Velocity After Impact

Applying conservation of linear momentum:

$$
m u = (m + M) v
$$

Given that the problem states both masses are M, for clarity, suppose:

- Bullet mass = m
- Block mass = M

then,

$$
v = \frac{m}{m + M} u
$$

This velocity is crucial because it determines the initial kinetic energy of the system immediately after impact.

Step 2: Energy Transferred to Oscillation

The kinetic energy immediately after impact:

$$
KE_{after} = \frac{1}{2} (m + M) v^2
$$

Substituting v:

$$
KE_{after} = \frac{1}{2} (m + M) \left(\frac{m}{m + M} u \right)^2 = \frac{1}{2} (m + M) \frac{m^2}{(m + M)^2} u^2 = \frac{1}{2} \frac{m^2}{m + M} u^2
$$

This energy is available for oscillation (assuming no other energy loss mechanisms).

Step 3: Determining the Amplitude of Oscillation

When the combined mass starts moving with velocity v, it can be considered as being set into oscillation with some amplitude A.

The maximum potential energy stored in the spring at maximum displacement (amplitude A) is:

$$
PE_{max} = \frac{1}{2} k A^2
$$

The initial kinetic energy converts into potential energy at maximum displacement:

$$
\frac{1}{2} \frac{m^2}{m + M} u^2 = \frac{1}{2} k A^2
$$

From this, the amplitude:

\[

$$
A = \sqrt{\frac{\frac{m^2}{m + M} u^2}{k}} = \frac{m}{\sqrt{m + M}} \frac{u}{\sqrt{k}}
$$

This relation links the impact velocity, masses, and the spring constant to the amplitude of oscillation.

Step 4: Calculating the Period

The period T of simple harmonic motion:

$$
T = 2 \pi \sqrt{\frac{m_{eff}}{k}}
$$

Since the effective mass involved in oscillation is $M + m$:

$$
\boxed{
T = 2 \pi \sqrt{\frac{M + m}{k}}
}
$$

This is the fundamental relation for the period of oscillation of the combined mass system.

Additional Considerations and Nuances

While the above derivation provides a straightforward route to the period, several subtle aspects merit discussion:

1. Role of Restoring Force

The problem assumes that the block is oscillating due to an elastic restoring force (like a spring). If the block is suspended or attached to a spring, then the above formula applies directly.

However, if the block is fixed or constrained differently, the oscillatory behavior might involve other mechanisms, such as pendular motion, which would modify the period formula.

2. Energy Losses and Damping

In real-world scenarios, damping mechanisms (air resistance, internal friction) cause energy loss and affect the amplitude over time. The period may slightly vary with amplitude in non-ideal conditions.

For the ideal case, damping is neglected, and the period depends solely on the system parameters.

3. Impact Dynamics and Material Properties

The assumption of perfectly inelastic collision and no deformation or energy loss is idealized. In practice, some energy would be dissipated, reducing the amplitude and possibly affecting the oscillation period.

Extended Analysis: Alternative Approaches

While the above approach is straightforward, alternative methods can deepen understanding or provide more precise results:

- Energy conservation approach: Use kinetic energy before impact and potential energy at maximum displacement to analyze the energy transfer.
- Impulse-momentum analysis: To determine velocities immediately after impact.
- Oscillation models: If the block is suspended as a pendulum, the period relates to pendulum length, not spring constant.

Practical Applications and Real-World Analogies

Understanding such impact and oscillation problems is valuable in various fields:

- Ballistic Pendulums: Used historically to measure the velocity of projectiles.
- Seismology: Impact analysis to understand energy transfer during earthquakes.
- Structural Engineering: Impact load analysis on buildings or bridges.
- Material Testing: Impact tests to determine material resilience.

Summary of Key Results

To consolidate:

- Velocity after impact:

$$
v = \frac{m}{m + M} u
$$

- Amplitude of oscillation:

$$
A = \frac{m}{\sqrt{m + M}} \frac{u}{\sqrt{k}}
$$

- Period of oscillation:

$$
\boxed{
T = 2 \pi \sqrt{\frac{M + m}{k}}
}
$$

This formula encapsulates the core physics: the period depends on the total mass and the restoring force's stiffness.

Conclusion

The scenario of a bullet embedded in a block leading to oscillation is a rich problem that combines momentum conservation, energy transfer, and harmonic motion

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