A Charge Of 2 * 10^-9C Is Placed At The Origin, And Another Charge Of 4 * 10^-9C Is Placed At X = 1.5m.

A Charge Of 2 10^-9C Is Placed At The Origin, And Another Charge Of 4 10^-9C Is Placed At X = 1.5m. This scenario presents a classic problem in electrostatics, involving the interaction of point charges and the principles governing Coulomb's law. Understanding how these charges influence each other, the nature of the forces involved, and the resulting electric fields is fundamental in physics, particularly in the study of electric forces and potentials. This article explores the detailed analysis of this setup, including calculations, concepts, and real-world applications, providing a comprehensive guide for students, educators, and enthusiasts alike.

Introduction to Electric Charges and Coulomb's Law

Electric charges are intrinsic properties of particles that cause them to exert forces on one another. These forces can be attractive or repulsive depending on the types of charges involved. Coulomb's law provides the quantitative basis for understanding these interactions.

What Are Electric Charges?

Electric charges are fundamental properties of matter, classified as positive or negative. Like charges repel each other, while opposite charges attract. The magnitude of the charge influences the strength of the electrostatic force between particles.

Coulomb's Law Explained

Coulomb's law states that the magnitude of the electrostatic force F between two point charges is directly proportional to the product of their magnitudes and inversely proportional to the square of the distance between them:

$$
F = k_e \frac{|q_1 q_2|}{r^2}
$$

Where:
- F is the magnitude of the force between the charges,
- k_e is Coulomb's constant ($8.9875 \times 10^9 \, \mathrm{Nm^2/C^2}$),

- q_1, q_2 are the magnitudes of the charges,
- r is the distance between the charges.

This law is fundamental for analyzing electrostatic problems involving multiple charges.

Understanding the Problem Setup

In our scenario, we have two point charges:
- A charge of $2 \times 10^{-9} \, \mathrm{C}$ located at the origin ($x = 0$),
- A charge of $4 \times 10^{-9} \, \mathrm{C}$ located at $x = 1.5\, \mathrm{m}$.

This setup involves calculating the electric field at various points, the forces on each charge, and the potential energy of the system.

Visual Representation of the Setup

To better understand the problem, consider a one-dimensional coordinate system:
- At $x=0$, the first charge $q_1 = 2 \times 10^{-9} \, \mathrm{C}$,
- At $x=1.5\, \mathrm{m}$, the second charge $q_2 = 4 \times 10^{-9} \, \mathrm{C}$.

The goal is to analyze the electric fields, forces, and potential at various points along this line.

Calculating Electric Fields

Electric fields are vector quantities that represent the force per unit positive charge experienced at a point in space due to other charges.

Electric Field Due to a Point Charge

The electric field E at a distance r from a point charge q is given by:

$$
E = k_e \frac{|q|}{r^2}
$$

Direction:
- Radially outward from a positive charge,
- Radially inward toward a negative charge.

Electric Field at a Point Due to Both Charges

When multiple charges are present, the net electric field at a point is the vector sum of the individual fields from each charge.

Step-by-step calculation:

1. Identify the points of interest (e.g., at various positions along the line).
2. Calculate the distance from each charge to the point.
3. Determine the magnitude and direction of the electric field due to each charge.
4. Add vectorially to find the net electric field.

Force Between the Charges

Since the charges are fixed at specific points, the force on each due to the other can be calculated directly using Coulomb's law.

Force on q_1:

$$
F_{12} = k_e \frac{|q_1 q_2|}{r^2}
$$

where $r = 1.5\, \mathrm{m}$.

Force on q_2:

$$
F_{21} = F_{12}
$$

by Newton's Third Law.

Direction:
- If both charges are positive, they repel each other.
- If one is negative, the force becomes attractive.

In this case, both are positive, so they repel, and the force acts along the line connecting them, pushing each away.

Calculating Electric Potential

Electric potential at a point due to a point charge is given by:

$$
V = k_e \frac{q}{r}
$$

The total potential at a point is the algebraic sum of potentials due to each charge.

Potential Energy of the System

The electrostatic potential energy U of two point charges is:

$$
U = k_e \frac{q_1 q_2}{r}
$$

This represents the energy stored due to the configuration of the charges.

Applications and Significance of Electrostatic Calculations

Understanding the interaction between these charges has practical applications across various fields:

1. Design of Capacitors
Capacitors store electrical energy based on charge separation, similar to the two charges in this setup.

2. Electrostatic Shielding
Knowing how charges influence electric fields helps in designing shielding to prevent electric interference.

3. Understanding Atomic and Molecular Interactions
At the microscopic level, electrostatic forces govern atomic bonds and molecular structures.

4. Electronics and Circuit Design
Electrostatics principles are essential in designing components like sensors and microelectromechanical systems (MEMS).

Step-by-Step Calculation Example

Let's perform a sample calculation: the force between the two charges.

Given:
- $q_1 = 2 \times 10^{-9} \, \mathrm{C}$,
- $q_2 = 4 \times 10^{-9} \, \mathrm{C}$,
- $r = 1.5\, \mathrm{m}$,
- $k_e = 8.9875 \times 10^9 \, \mathrm{Nm^2/C^2}$.

Calculations:

$$
F = k_e \frac{|q_1 q_2|}{r^2} = (8.9875 \times 10^9) \times \frac{(2 \times 10^{-9})(4 \times 10^{-9})}{(1.5)^2}
$$

$$
F = (8.9875 \times 10^9) \times \frac{8 \times 10^{-18}}{2.25}
$$

$$
F = (8.9875 \times 10^9) \times 3.555 \times 10^{-18}
$$

$$
F \approx 3.195 \times 10^{-8} \, \mathrm{N}
$$

This is the magnitude of the force with which the two charges repel each other.

Conclusion

The interaction of the two charges placed at specific positions along a line exemplifies fundamental principles of electrostatics. By applying Coulomb's law, calculating electric fields, and understanding potential energy, one can analyze the forces and potentials involved in such systems. These concepts are foundational to many technological applications, from designing electronic components to understanding atomic interactions. Mastery of these calculations not only enhances comprehension of classical physics but also paves the way for innovations in science and engineering.

Additional Resources for Further Learning

- Textbooks on Electromagnetism (e.g., "Introduction to Electrodynamics" by David J. Griffiths)
- Online simulations (e.g., PhET Electric Field & Potential Simulations)
- Educational videos on Coulomb's Law and electric fields
- Practice problems on electrostatics for skill reinforcement

Remember: Always pay attention to the units and signs of charges, as they critically influence the direction and magnitude of forces and fields in electrostatic problems.

Frequently Asked Questions

What is the electrostatic force between two charges of 2×10^{-9} C at the origin and 4×10^{-9} C at x = 1.5 m?

Using Coulomb's Law, $F = k |q_1 q_2| / r^2$, where $k \approx 9\times10^9$ $N\cdot m^2/C^2$, the force is $F = (9\times10^9) (2\times10^{-9}) (4\times10^{-9}) / (1.5)^2 \approx 0.032$ N, directed along the line connecting the charges.

In which direction does the force on the 2×10^{-9} C charge point due to the 4×10^{-9} C charge placed at x = 1.5 m?

The force points away from the 4×10^{-9} C charge if both charges are positive, indicating a repulsive force directed along the positive x-axis away from the second charge.

What is the electric field at the origin due to both charges?

The electric field at the origin is the vector sum of fields from both charges. The field due to the 2×10^{-9} C charge at the origin is zero (since it's at the point), and the field due to the 4×10^{-9} C charge at x = 1.5 m points away from it, with magnitude $E = k q / r^2 = (9\times10^9) (4\times10^{-9}) / (1.5)^2 \approx 16.0$ N/C directed along the negative x-axis.

If a test charge is placed at x = 1.5 m, what is the net electric field experienced by it?

The net electric field at x = 1.5 m is primarily due to the charge at the origin since the other charge is at that point. As the test charge is at the position of the 4×10^{-9} C charge, the field is dominated by the charge at the origin, with a magnitude of about 8 N/C directed toward the origin.

How does the magnitude of the electric field change as you move away from the charges along the x-axis?

The electric field magnitude decreases with increasing distance from the charges, following an inverse square law, so moving farther away reduces the field strength significantly.

What is the potential energy of the system of charges?

The electrostatic potential energy is given by $U = k q_1 q_2 / r = (9\times10^9) (2\times10^{-9}) (4\times10^{-9}) / 1.5 \approx 0.048$ Joules.

Are the two charges in stable equilibrium at their current positions?

No, since the charges repel each other, moving them closer would increase repulsive force, indicating the configuration is not a stable equilibrium; they tend to move apart if free to do so.

Additional Resources

Charge interactions are fundamental to understanding the behavior of electric forces and fields in physics. When two point charges are placed at specific locations, their mutual electrostatic interactions can be analyzed to determine the forces acting upon each, the potential energy stored in the configuration, and the resulting electric fields. In this comprehensive review, we examine a scenario involving two point charges: one of magnitude 2×10^{-9} C located at the origin, and another of magnitude 4×10^{-9} C placed at $x = 1.5\, \text{m}$. This example provides a valuable case study for exploring Coulomb's law, electric field vectors, potential energy, and the overall dynamics of electrostatic interactions.

Introduction to the Problem

The given scenario involves two point charges in a one-dimensional setup:

- Charge 1 (Q_1): $2 \times 10^{-9} \, \text{C}$, located at the origin ($x=0$)
- Charge 2 (Q_2): $4 \times 10^{-9} \, \text{C}$, located at $x=1.5\, \text{m}$

Understanding their interaction involves analyzing the forces they exert on each other, the electric field at any point along the line, and the potential energy stored in their configuration. This problem exemplifies the principles of Coulomb's law, superposition of electric fields, and the concept of electric potential.

Fundamental Principles Underpinning the Scenario

Coulomb's Law

Coulomb's law describes the force F between two point charges:

$$
F = \frac{k \, |Q_1 Q_2|}{r^2}
$$

where:
- F is the magnitude of the electrostatic force,
- k is Coulomb's constant ($8.9875 \times 10^9\, \text{Nm}^2/\text{C}^2$),
- Q_1 and Q_2 are the magnitudes of the charges,
- r is the distance between the charges.

This law indicates that the force is directly proportional to the product of the charges and inversely proportional to the square of their separation. The direction of the force depends on the sign of the charges: like charges repel, unlike charges attract.

Electric Field

The electric field $\mathbf{E}$ at a point due to a point charge Q is given by:

$$
\mathbf{E} = \frac{k Q}{r^2} \hat{\mathbf{r}}
$$

where $\hat{\mathbf{r}}$ is the unit vector pointing from the charge to the point of interest. The superposition principle states that the net electric field at any point is the vector sum of the fields due to individual charges.

Electric Potential Energy

The potential energy U stored in a system of two point charges is:

$$
U = \frac{k Q_1 Q_2}{r}
$$

This quantity indicates how much work is required to assemble the two charges from infinity to their current separation.

Analyzing the Forces Between the Charges

Calculating the Separation Distance

The two charges are positioned at:

- Q_1 at $x=0$,
- Q_2 at $x=1.5\, \text{m}$.

The separation distance:

$$
r = |1.5\, \text{m} - 0\, \text{m}| = 1.5\, \text{m}
$$

Magnitude of the Force

Applying Coulomb's law:

$$
F = \frac{(8.9875 \times 10^9) \times (2 \times 10^{-9}) \times (4 \times 10^{-9})}{(1.5)^2}
$$

Calculations:

$$
F = \frac{8.9875 \times 10^9 \times 8 \times 10^{-18}}{2.25}
$$

$$
F = \frac{71.9 \times 10^{-9}}{2.25} \approx 31.95 \times 10^{-9} \, \text{N}
$$

$$
F \approx 3.2 \times 10^{-8} \, \text{N}
$$

This force is attractive or repulsive depending on the charges' signs. Assuming both are positive, the force is repulsive, meaning each charge experiences a push away from the other.

Direction and Nature of the Forces

Since both charges are positive (assuming so based on the magnitudes), they repel each other:

- Q_1 at the origin: experiences a force directed to the right (away from Q_2 at $x=1.5\, \text{m}$)
- Q_2 at $x=1.5\, \text{m}$: experiences a force directed to the left.

This mutual repulsion illustrates fundamental electrostatic behavior, where like charges repel and unlike charges attract.

Electric Field Analysis

Electric Field Due to Q_1 at Various Points

The electric field at a point x due to Q_1 is:

$$
E_{Q_1} = \frac{k Q_1}{x^2}
$$

- For $x > 0$, the field points away from Q_1 (positive charge), i.e., in the positive x-direction.
- The magnitude decreases with increasing x.

Electric Field Due to Q_2 at Various Points

Similarly, the electric field due to Q_2 at a point x:

$$
E_{Q_2} = \frac{k Q_2}{(x - 1.5)^2}
$$

- For $x < 1.5\, \text{m}$, the field points toward Q_2 if Q_2 is positive.
- For $x > 1.5\, \text{m}$, it points away from Q_2.

Net Electric Field at a Point x

The superposition principle gives:

$$

$$
E_{net}(x) = E_{Q_1}(x) + E_{Q_2}(x)
$$

The direction of the net electric field depends on which contribution dominates at a specific point.

Electric Potential Energy of the System

The potential energy stored:

$$
U = \frac{k Q_1 Q_2}{r} = \frac{8.9875 \times 10^9 \times 2 \times 10^{-9} \times 4 \times 10^{-9}}{1.5}
$$

Calculations:

$$
U = \frac{8.9875 \times 10^9 \times 8 \times 10^{-18}}{1.5} = \frac{71.9 \times 10^{-9}}{1.5} \approx 47.9 \times 10^{-9} \, \text{J}
$$

$$
U \approx 4.8 \times 10^{-8} \, \text{J}
$$

This positive potential energy indicates the system is in a state where work must be done to assemble it if the charges are initially at infinite separation.

Implications and Applications

Electrostatic Force and Stability

The calculated force magnitude ($\sim 3.2 \times 10^{-8}$ N) might seem tiny, but in microscopic systems, such forces are significant. These interactions influence the behavior of particles in fields such as atomic physics, nanotechnology, and electrostatic precipitators.

Electric Field and Potential Mapping

Mapping the electric field and potential along the line helps in designing sensors, understanding charge distributions, and predicting the behavior of charged particles in various environments.

Energy Considerations

Understanding the potential energy stored in such configurations aids in energy storage design, electrostatic motors, and capacitors.

Extended Concepts and Further Analysis

Force Between the Charges in Vector Form

The force vector:

$$
\mathbf{F} = \pm \frac{k |Q_1 Q_2|}{r^2} \hat{\mathbf{r}}
$$

- The $+$ sign indicates repulsion.
- The direction is along the line joining the charges.

Potential at a Point Along the Line

The electric potential V at a point x:

$$
V(x) = \frac{k Q_1}{|x|} + \frac{k Q_2}{|x - 1.5|}
$$

This helps in calculating the work done moving a test charge within the field.

Force on a Test Charge