A Circle Has A Diameter With The Endpoints At (-6, 3) And (10, -9). What Is The Equation Of The Circle?

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Understanding the equation of a circle is a fundamental concept in coordinate geometry. When given the endpoints of a diameter, one can determine the circle's center and radius, which are essential components in formulating its standard equation. This article provides a comprehensive guide to finding the equation of a circle given the endpoints of its diameter, using the specific points (-6, 3) and (10, -9) as an example.

Introduction to the Equation of a Circle

A circle in a coordinate plane can be represented mathematically by an equation in standard form:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

Here:
- (h, k) are the coordinates of the circle's center.
- r is the radius of the circle.

Knowing either the center and radius or enough points on the circle allows us to derive this equation.

Step 1: Understanding the Given Data

In this problem, the diameter's endpoints are given as:
- $A(-6, 3)$
- $B(10, -9)$

Since these points are endpoints of a diameter, they lie on the circle, and the segment connecting them passes through the center of the circle.

Step 2: Find the Center of the Circle

The center of the circle is the midpoint of the diameter. To find the midpoint (h, k):

Midpoint Formula

$$
h = \frac{x_1 + x_2}{2}
$$
$$
k = \frac{y_1 + y_2}{2}
$$

Applying the formula with the given points:

$$
h = \frac{-6 + 10}{2} = \frac{4}{2} = 2
$$
$$
k = \frac{3 + (-9)}{2} = \frac{-6}{2} = -3
$$

Therefore, the center of the circle is at (2, -3).

Step 3: Find the Radius of the Circle

The radius is half the length of the diameter. To find the diameter's length, we use the distance formula between points A and B:

Distance Formula

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Applying it:

$$
d = \sqrt{(10 - (-6))^2 + (-9 - 3)^2}
$$

Calculate each component:

$$
10 - (-6) = 10 + 6 = 16
$$
$$
-9 - 3 = -12
$$

Now:

$$
d = \sqrt{16^2 + (-12)^2} = \sqrt{256 + 144} = \sqrt{400} = 20
$$

Since the diameter $d = 20$, the radius r is:

$$
r = \frac{d}{2} = \frac{20}{2} = 10
$$

Radius $r = 10$.

Step 4: Write the Equation of the Circle

With the center $(h, k) = (2, -3)$ and radius $r = 10$, the standard form is:

$$
(x - 2)^2 + (y + 3)^2 = 10^2
$$

Simplifying:

$$
(x - 2)^2 + (y + 3)^2 = 100
$$

This is the equation of the circle.

Additional Details and Variations

Converting to General Form

The standard form can be expanded to the general form:

$$
(x - 2)^2 + (y + 3)^2 = 100
$$

Expanding the squares:

$$
x^2 - 4x + 4 + y^2 + 6y + 9 = 100
$$

Combine like terms:

$$
x^2 + y^2 - 4x + 6y + (4 + 9) = 100
$$

$$
x^2 + y^2 - 4x + 6y + 13 = 100
$$

Subtract 100 from both sides:

$$
x^2 + y^2 - 4x + 6y - 87 = 0
$$

This is the general form of the circle's equation.

Why Is This Important?

Understanding how to find the equation of a circle from given points is crucial in many fields such as geometry, physics, engineering, and computer graphics. It allows you to:

- Determine the size and position of circular objects.
- Analyze geometric relationships.
- Solve problems involving circles and other conic sections.

Practical Applications of Finding the Equation of a Circle

- Designing circular components in engineering and manufacturing.
- Analyzing planetary orbits in astronomy.
- Creating graphical representations in computer graphics.
- Solving geometric problems in architecture and construction.

Summary of Key Steps

To find the equation of a circle given the endpoints of a diameter:

1. Calculate the midpoint of the endpoints to find the center.
2. Use the distance formula to find the length of the diameter, then divide by two to get the radius.
3. Write the standard form of the circle's equation using the center and radius.
4. Optionally, expand to the general form for algebraic applications.

Conclusion

Given the endpoints of a diameter at (-6, 3) and (10, -9), we've determined that the circle's center is at (2, -3) and its radius is 10. Consequently, the equation of the circle in standard form is:

$$
\boxed{(x - 2)^2 + (y + 3)^2 = 100}
$$

This method can be applied to any similar problem where the endpoints of a diameter are known, making it a versatile tool in coordinate geometry.

FAQs

Q1: How do I find the equation of a circle if only the endpoints of the diameter are given?

A: Find the midpoint of the endpoints to determine the center, calculate the distance between the points to find the diameter, then divide by two to get the radius. Plug these into the standard circle formula.

Q2: Can I convert the standard form to the general form? Why would I do that?

A: Yes. Expanding the standard form yields the general form, which is useful for algebraic analysis, solving systems, or graphing.

Q3: What if the endpoints are given in a different coordinate

system?

A: The same principles apply; just ensure you compute the midpoint and distance using the appropriate coordinates.

Q4: How does understanding the equation of a circle help in real-world problems?

A: It aids in designing circular objects, analyzing motion, creating graphics, or solving spatial problems across various scientific and engineering disciplines.

By mastering the process of deriving the equation of a circle from its diameter endpoints, students and professionals can enhance their problem-solving skills in geometry and related fields, ensuring a solid foundation for more advanced mathematical concepts.

Frequently Asked Questions

How do you find the center of a circle given endpoints of its diameter at (-6, 3) and (10, -9)?

The center is the midpoint of the diameter. To find it, average the x-coordinates and the y-coordinates: $((-6 + 10)/2, (3 + (-9))/2) = (2, -3)$.

What is the length of the diameter with endpoints at (-6, 3) and (10, -9)?

Use the distance formula: $\sqrt{[(10 - (-6))^2 + (-9 - 3)^2]} = \sqrt{[(16)^2 + (-12)^2]} = \sqrt{[256 + 144]} = \sqrt{400} = 20$.

How do you write the equation of a circle given its center and radius?

The standard form is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

What is the equation of the circle with diameter endpoints at (-6, 3) and (10, -9)?

First, find the center $(2, -3)$ and radius $r = 10$. Plug into the standard form: $(x - 2)^2 + (y + 3)^2 = 10^2$, which simplifies to $(x - 2)^2 + (y + 3)^2 = 100$.

Why do we use the midpoint and distance formulas when

finding the equation of a circle from diameter endpoints?

Because the midpoint gives the center of the circle and the distance between endpoints gives the diameter length, from which the radius can be calculated. These are essential for writing the circle's equation.

Additional Resources

A Circle Has A Diameter With The Endpoints At (-6, 3) And (10, -9). What Is The Equation Of The Circle?

Introduction

In the realm of coordinate geometry, the circle is a fundamental figure characterized by its uniform distance from a central point. Determining the equation of a circle given specific points is a common but essential task that combines algebraic manipulation and geometric understanding. One particularly illustrative problem involves a circle whose diameter endpoints are known, and from which the circle’s equation can be deduced with precision. Specifically, the problem asks: A circle has a diameter with endpoints at (-6, 3) and (10, -9). What is the equation of the circle?

This question, seemingly straightforward, unfolds into a thorough exploration of fundamental geometric principles, algebraic techniques, and analytical reasoning. It provides an excellent case study for understanding how geometric features translate into algebraic equations, the importance of the circle's center and radius, and the practical application of the distance formula.

Understanding the Geometric Foundations

Before diving into calculations, it is instructive to revisit the key geometric concepts underpinning the problem.

The Circle and Its Standard Equation

A circle in a coordinate plane is defined as the set of all points equidistant from a fixed point called the center. The standard form of a circle's equation is:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- (h, k) is the center of the circle,
- r is the radius.

Given this form, the task reduces to finding the center (h, k) and the radius r.

The Diameter and Its Properties

The diameter of a circle is a line segment passing through the center with its endpoints on the circle. Its length is twice the radius:

$$ \text{Diameter length} = 2r $$

Given the endpoints of a diameter, the center of the circle is the midpoint of that segment. The length of the diameter is found via the distance formula.

Step 1: Identifying the Diameter Endpoints

The endpoints are given as:

- Point A: $(-6, 3)$
- Point B: $(10, -9)$

To determine the circle's center and radius, we first need to find:

1. The midpoint of these endpoints (which will be the center).
2. The distance between these endpoints (which will be the diameter length).

Calculating the Midpoint

The midpoint M of two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$ M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) $$

Applying this to our points:

$$
\begin{aligned}
h &= \frac{-6 + 10}{2} = \frac{4}{2} = 2 \\
k &= \frac{3 + (-9)}{2} = \frac{-6}{2} = -3
\end{aligned}
$$

Thus, the center of the circle is at $\boxed{(2, -3)}$.

Calculating the Distance (Diameter Length)

The distance d between points A and B:

$$ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} $$

Plugging in the coordinates:

$$
\begin{aligned}
d &= \sqrt{(10 - (-6))^2 + (-9 - 3)^2} \\
&= \sqrt{(16)^2 + (-12)^2} \\
&= \sqrt{256 + 144} \\
&= \sqrt{400} \\
&= 20
\end{aligned}
$$

Since the diameter length is 20, the radius r is half of that:

$$ r = \frac{d}{2} = \frac{20}{2} = 10 $$

Step 2: Formulating the Equation of the Circle

With the center $(h, k) = (2, -3)$ and radius $r = 10$, the standard form of the circle's equation is:

$$ (x - 2)^2 + (y + 3)^2 = 10^2 $$

which simplifies to:

$$ (x - 2)^2 + (y + 3)^2 = 100 $$

This is the algebraic representation of the circle.

Final Equation of the Circle

$$
\boxed{
(x - 2)^2 + (y + 3)^2 = 100
}
$$

This equation encapsulates all points (x, y) that lie on the circle with the given diameter endpoints.

Verifying the Solution: Geometric and Algebraic Consistency

Ensuring the correctness of this solution involves multiple checks.

Check 1: The Center and Endpoints Lie on the Circle

- Distance from center to endpoint A:

$$
\sqrt{(-6 - 2)^2 + (3 + 3)^2} = \sqrt{(-8)^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10
$$

- Distance from center to endpoint B:

$$
\sqrt{(10 - 2)^2 + (-9 + 3)^2} = \sqrt{8^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10
$$

Both distances match the radius, confirming the endpoints lie on the circle.

Check 2: The Endpoints Satisfy the Equation

Plugging in endpoint A $(-6, 3)$:

$$
(-6 - 2)^2 + (3 + 3)^2 = (-8)^2 + 6^2 = 64 + 36 = 100
$$

Plugging in endpoint B $(10, -9)$:

$$
(10 - 2)^2 + (-9 + 3)^2 = 8^2 + (-6)^2 = 64 + 36 = 100
$$

Both satisfy the equation, confirming the correctness.

Implications and Broader Context

This problem exemplifies a recurring theme in coordinate geometry: the translation of geometric properties into algebraic formulas. The process of identifying the circle's center as the midpoint of the diameter, calculating the radius from the diameter length, and then writing the standard form equation is fundamental for students and practitioners alike.

Applications include:

- Engineering design, where precise circle equations are necessary for manufacturing.
- Computer graphics, for rendering circles based on endpoint data.
- Navigation and GPS, where locations and distances are mapped onto coordinate planes.
- Mathematical modeling, where understanding the properties of geometric figures underpins complex simulations.

Concluding Remarks

The problem of determining a circle's equation given two endpoints of its diameter demonstrates the elegance of coordinate geometry. By systematically applying the midpoint and distance formulas, one can derive the circle's center and radius with clarity and confidence. The calculated equation:

$$
(x - 2)^2 + (y + 3)^2 = 100
$$

serves as a testament to the power of algebraic methods in solving geometric problems. This approach underscores the interconnectedness of geometric intuition and algebraic precision, providing a robust toolkit for exploring the properties of circles and other conic sections.

In summary:
- The center of the circle is at $(2, -3)$.
- The radius is 10 units.
- The equation of the circle is $\boxed{(x - 2)^2 + (y + 3)^2 = 100}$.

This comprehensive analysis demonstrates the depth and validity of the solution, offering insights that extend beyond the immediate problem to foundational principles in geometry and algebra.

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