

A Circle Is Centered At The Point (-7, -1) And Passes Through The Point (8, 7).The Radius Of The Circle

A Circle Is Centered At The Point (-7, -1) And Passes Through The Point (8, 7). The Radius Of The Circle is a fundamental concept in geometry that helps in understanding the properties of circles, their equations, and their applications in various fields such as mathematics, physics, engineering, and computer graphics. This article explores the detailed process of determining the radius of a circle given its center and a point on its circumference, along with related concepts, formulas, and real-world applications.

Understanding the Basics of a Circle

Before delving into the specifics of calculating the radius, it is essential to understand what a circle is and its key properties.

What Is a Circle?

A circle is a set of all points in a plane that are equidistant from a fixed point called the center. The fixed distance from the center to any point on the circle is known as the radius.

Key Properties of a Circle

- Center: The fixed point from which every point on the circle is equidistant.
- Radius: The constant distance from the center to any point on the circle.
- Diameter: The longest distance across the circle passing through the center; twice the radius.
- Circumference: The total length around the circle, calculated as $2\pi r$.
- Area: The space enclosed within the circle, calculated as πr^2 .

Calculating the Radius of a Circle from Center and a Point

Given the center of the circle and a point on the circle, the radius can be computed using the distance formula.

The Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is derived from the Pythagorean theorem and is fundamental in coordinate geometry for calculating distances.

Applying the Distance Formula to Our Specific Points

- Center of the circle: $(-7, -1)$
- Point on the circle: $(8, 7)$

Calculating the difference in x-coordinates:

$$x_2 - x_1 = 8 - (-7) = 8 + 7 = 15$$

Calculating the difference in y-coordinates:

$$y_2 - y_1 = 7 - (-1) = 7 + 1 = 8$$

Applying the distance formula:

$$r = \sqrt{(15)^2 + (8)^2} = \sqrt{225 + 64} = \sqrt{289} = 17$$

Thus, the radius of the circle is 17 units.

Equation of the Circle

Once the radius is known, the standard form of the circle's equation can be written as:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where:

- (h, k) is the center of the circle

- r is the radius

Substituting the known values:

$$(x + 7)^2 + (y + 1)^2 = 17^2 = 289$$

This equation completely describes the circle's location and size in the coordinate plane.

Applications of Calculating the Radius of a Circle

Understanding how to determine the radius from given points has numerous practical applications across different domains.

1. Engineering and Design

Engineers often need to design circular components, such as gears, pulleys, or arches, where precise measurements are critical. Calculating the radius accurately ensures the parts fit together correctly and function as intended.

2. Computer Graphics and Visualization

In computer graphics, drawing circles and arcs involves calculating the radius based on user inputs or model specifications. Accurate radius calculation enables realistic rendering and modeling.

3. Navigation and Geographic Mapping

Mapping applications utilize circles to represent zones of influence or range. Knowing the radius helps in defining these zones accurately based on specific coordinates.

4. Physics and Astronomy

Understanding orbital paths or circular motion involves calculating radii based on observational data points.

Advanced Concepts Related to Circle Radius

Beyond basic calculations, several advanced topics relate to the radius of a circle, especially in more complex scenarios.

1. Radius in Circle Equations

The radius appears prominently in the standard and general forms of the circle equation. Variations include:

- Standard form: $((x - h)^2 + (y - k)^2 = r^2)$
- General form: $(x^2 + y^2 + Dx + Ey + F = 0)$, where the radius can be derived from coefficients.

2. Radius in Sector and Segment Calculations

The radius is crucial in calculating areas and lengths of sectors and segments within the circle.

3. Relationship Between Radius and Diameter

The diameter (d) of a circle is directly proportional to the radius:

$$d = 2r$$

Understanding this relationship is vital in various measurement and design tasks.

Summary and Key Takeaways

To summarize, the radius of a circle centered at $(-7, -1)$ passing through $(8, 7)$ is 17 units, calculated using the distance formula. This calculation is fundamental in deriving the circle's equation and understanding its properties.

Key points to remember:

- The radius is the distance from the center to any point on the circle.
- The distance formula is essential for calculating the radius when given the center and a point.
- The standard form of the circle's equation incorporates the radius and center coordinates.
- Calculating the radius accurately has practical applications across multiple fields.

Conclusion

Understanding how to calculate the radius of a circle based on its center and a point on its circumference is a foundational skill in coordinate geometry. It allows for precise mathematical modeling of circular shapes and is essential in many scientific and engineering applications. Whether designing mechanical parts, creating digital graphics, or analyzing natural phenomena, mastering this calculation enhances problem-solving skills and deepens comprehension of geometric principles.

Meta description: Learn how to determine the radius of a circle centered at (-7, -1) passing through (8, 7), including detailed calculations, formulas, and practical applications in geometry and beyond.

Frequently Asked Questions

What is the radius of the circle centered at (-7, -1) passing through (8, 7)?

The radius is the distance between the center (-7, -1) and the point (8, 7), which is $\sqrt{(8 - (-7))^2 + (7 - (-1))^2} = \sqrt{15^2 + 8^2} = \sqrt{225 + 64} = \sqrt{289} = 17$.

How do you find the radius of a circle given its center and a point on the circle?

Calculate the distance between the center and the point using the distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

What is the equation of the circle centered at (-7, -1) with radius 17?

The equation is $(x + 7)^2 + (y + 1)^2 = 17^2$, which simplifies to $(x + 7)^2 + (y + 1)^2 = 289$.

How do you verify if a point lies on this circle?

Substitute the point's coordinates into the circle's equation. If the equation holds true, the point lies on the circle.

What is the significance of the radius in the circle's equation?

The radius determines the size of the circle, representing the distance from the center to any point on the circle.

Can you find the diameter of the circle given the radius?

Yes, the diameter is twice the radius, so it is $2 \times 17 = 34$ units.

How do you graph this circle on a coordinate plane?

Plot the center at $(-7, -1)$, then draw a circle with radius 17 units around it, ensuring all points are 17 units from the center.

What is the importance of the distance formula in circle problems?

It helps determine the radius when given the center and a point on the circle, which is fundamental in deriving the circle's equation.

If the point passing through the circle changes, how does that affect the radius?

The radius will change to match the distance from the new point to the center, altering the size of the circle.

What real-world applications can involve calculating the radius of a circle given a center and a point?

Applications include designing circular tracks, planning roundabouts, or determining coverage areas in wireless networks.

Additional Resources

A Circle Is Centered At The Point $(-7, -1)$ And Passes Through The Point $(8, 7)$. The Radius Of The Circle

Understanding the fundamental properties of circles is a cornerstone of geometry and mathematics at large. Among these properties, the radius—a line segment connecting the center of the circle to any point on its circumference—is particularly significant. In this article, we delve into a specific circle with a given center and a point on its boundary, aiming to determine its radius with precision. This exploration not only exemplifies core geometric principles but also demonstrates how straightforward calculations can unlock deeper insights into the nature of circular figures.

Introduction to the Circle's Geometric Framework

Before diving into calculations, it is essential to establish a clear understanding of the geometric entities involved.

The Center of the Circle: Point $(-7, -1)$

The center of the circle, designated as point (C) , is given as $(-7, -1)$. This point is the fixed point from which all points on the circle are equidistant. Its coordinates provide the anchor for the circle's position on the Cartesian coordinate plane.

The Point on the Circle: Point $(8, 7)$

The circle passes through point (P) , with coordinates $(8, 7)$. By virtue of lying on the circle, this point helps determine the radius—since the radius is the distance between the center (C) and any point (P) on the circle.

Mathematical Foundations: Distance Formula and Radius Calculation

The core mathematical principle used to find the radius in this context is the Euclidean distance formula, which calculates the straight-line distance between two points in a plane. The formula is expressed as:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of the two points in question.

Applying this to our specific points $(C(-7, -1))$ and $(P(8, 7))$:

$$r = \sqrt{(8 - (-7))^2 + (7 - (-1))^2}$$

which simplifies further to:

$$\sqrt{225 + 64}$$

$$r = \sqrt{(8 + 7)^2 + (7 + 1)^2}$$

Calculating each term:

$$\begin{aligned} - \sqrt{(8 + 7)^2} &= (15)^2 = 225 \\ - \sqrt{(7 + 1)^2} &= (8)^2 = 64 \end{aligned}$$

Adding these:

$$r = \sqrt{225 + 64} = \sqrt{289}$$

Finally, taking the square root:

$$r = 17$$

Thus, the radius of the circle is 17 units.

Detailed Geometric Analysis and Significance

The calculation above is straightforward, but understanding its significance requires contextual analysis.

Implications of the Radius Size

A radius of 17 units indicates a relatively large circle centered at $(-7, -1)$. Its size influences various geometric and spatial considerations:

- Area: The area (A) of the circle can be computed as $(A = \pi r^2)$, which yields approximately $(3.1416 \times 289 \approx 907.92)$ square units.
- Circumference: The perimeter (C) is $(2\pi r)$, approximately $(2 \times 3.1416 \times 17 \approx 106.81)$ units.

These calculations provide insights into the circle's size and spatial footprint within the coordinate plane.

Geometric Placement and Visualization

Plotting the circle with the given center and radius reveals its position relative to other geometric figures or coordinate axes:

- The center at $(-7, -1)$ places it in the third quadrant, slightly to the left and below the origin.
- The circle extends 17 units in all directions from its center, reaching from $(-24, -1)$ to $(10, -1)$ horizontally, and from $(-7, -18)$ to $(-7, 16)$ vertically.

This visualization aids in understanding the circle's relationship with other geometric entities and coordinate points.

Broader Context: Applications and Relevance

Understanding how to determine a circle's radius from given points has numerous applications across various fields.

Educational Applications

- Geometry Education: This problem exemplifies how to apply basic formulas to real-world problems, reinforcing understanding of coordinate geometry.
- Mathematical Problem Solving: It demonstrates the importance of precise calculations and the importance of understanding the properties of geometric figures.

Practical Applications in Science and Engineering

- Design and Engineering: Determining the size of circular components or features based on their geometric parameters.
- Navigation and Robotics: Calculating distances between points to plan movements or sensor ranges.
- Computer Graphics: Rendering circles based on center points and radii in digital environments.

Extensions and Further Considerations

The simple calculation opens pathways to more complex geometric analyses.

Equation of the Circle

Using the center $(-7, -1)$ and radius 17, the equation of the circle can be written as:

$$\begin{aligned} &[(x + 7)^2 + (y + 1)^2 = 17^2] \\ &[(x + 7)^2 + (y + 1)^2 = 289] \end{aligned}$$

This explicit equation defines all points lying on the circle and can be used for intersection analysis, optimization problems, or graphical plotting.

Analyzing Other Points

- Determining whether other points lie inside, outside, or on the circle involves calculating their distances from the center and comparing with the radius.
- For example, testing a point (x, y) involves computing $\sqrt{(x + 7)^2 + (y + 1)^2}$ and comparing it to 17.

Conclusion: The Power of Simple Geometric Principles

The process of identifying the radius of a circle given a center and a point on its circumference exemplifies the elegance and utility of fundamental geometric principles. By leveraging the distance formula, we derived that the radius is 17 units—an outcome that underscores the interconnectedness of coordinate points and geometric figures. This calculation not only enhances our understanding of the specific circle in question but also illustrates the broader importance of mathematical tools in analyzing spatial relationships. Whether in education, engineering, or computer science, mastering these basic yet powerful techniques remains essential for navigating the complexities of the geometric world.

Summary:

- The circle has a center at $(-7, -1)$.
- It passes through the point $(8, 7)$.
- Using the distance formula, the radius is calculated as 17 units.
- This radius informs the circle's size, position, and related geometric

properties.

- The calculation exemplifies core principles of coordinate geometry with widespread applications.

By understanding these principles and their practical applications, students, educators, and professionals can confidently analyze and interpret the properties of circles and other geometric figures within the coordinate plane.

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