A Circle Passes Through The Point (2, 2) And Has Its Center At (1, -3). The Radius Of The Circle Is How

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Understanding the properties of circles is fundamental in geometry, especially when it comes to determining a circle's radius based on given points and centers. In this article, we will explore the step-by-step process to find the radius of a circle that passes through a specific point and has a known center. We will use the example of a circle passing through the point (2, 2) with its center at (1, -3), and guide you through the calculation, concepts involved, and practical applications.

Introduction to Circles and Their Properties

A circle is a set of all points in a plane that are equidistant from a fixed point called the center. The distance from the center to any point on the circle is called the radius. The fundamental equation of a circle with center at (h, k) and radius r is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Where:
- (h, k) are the coordinates of the center
- r is the radius
- (x, y) are the coordinates of any point on the circle

Given the center and a point on the circle, the radius can be calculated directly using the distance formula.

Understanding the Problem

In our case, we are told:
- The center of the circle is at $(1, -3)$
- The circle passes through the point $(2, 2)$

Our goal is to find the radius r.

Key concepts involved:
- Distance formula
- Coordinates of center and point
- Application of the distance formula to find the radius

Step-by-Step Calculation of the Radius

1. Recall the Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given by:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

This formula measures the straight-line distance between the two points.

2. Identify Known Coordinates

From the problem:
- Center: $(h, k) = (1, -3)$
- Point on the circle: $(x, y) = (2, 2)$

3. Apply the Distance Formula

Plugging the known points into the distance formula:

$$
r = \sqrt{(2 - 1)^2 + (2 - (-3))^2}
$$

Simplify the terms:

$$
r = \sqrt{(1)^2 + (2 + 3)^2}
$$
$$
r = \sqrt{1 + 5^2}
$$
\[

$$
r = \sqrt{1 + 25}
$$

$$
r = \sqrt{26}
$$

4. Final Result: The Radius

Therefore, the radius of the circle is:

$$
r = \sqrt{26}
$$

which is approximately:

$$
r \approx 5.10
$$

Implications and Applications

Knowing how to find a circle's radius given its center and a point on the circle has multiple applications in mathematics, engineering, and real-world problem-solving.

Practical Uses Include:

- Designing circular structures or components with specific point-based constraints
- Navigation and GPS calculations involving circular zones
- Computer graphics involving circles and their properties
- Robotics, where movement paths often involve circular arcs
- Physics problems related to fields and forces centered at specific points

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Additional Concepts Related to Circles

To deepen understanding, here are some related topics often encountered in circle problems:

1. Equation of a Circle

Given the center (h, k) and radius r, the standard form of the circle's equation is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Using our example:

$$
(x - 1)^2 + (y + 3)^2 = 26
$$

2. Finding the Center and Radius from an Equation

Conversely, if the equation of a circle is given, you can determine the center and radius by rewriting the equation in standard form.

3. Tangents and Chords

Understanding how tangents and chords relate to given points and centers can help solve more complex geometry problems.

Summary

- To find the radius of a circle passing through a point with a known center, apply the distance formula between the center and the point.
- In our example, the radius is $\sqrt{26}$, approximately 5.10 units.
- This fundamental approach is essential for solving many geometry problems involving circles.

Conclusion

Determining the radius of a circle based on its center and a point on the circle is a straightforward process rooted in the distance formula. By carefully substituting the known coordinates and simplifying, you can find the radius accurately. Whether you're solving academic problems or applying these concepts in practical scenarios, mastering this fundamental skill enhances your understanding of circle geometry.

Keywords for SEO Optimization:
- How to find the radius of a circle
- Circle passing through a point
- Center and radius of a circle
- Distance formula in geometry
- Geometry problems on circles
- Calculating circle radius from points
- Standard form of circle equation
- Coordinate geometry circle calculations

Frequently Asked Questions

How do you find the radius of a circle given its center and a point on the circle?

The radius is found by calculating the distance between the center and the point using the distance formula: $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$.

What is the radius of a circle with center at (1, -3) passing through the point (2, 2)?

The radius is $\sqrt{[(2 - 1)^2 + (2 - (-3))^2]} = \sqrt{[1^2 + 5^2]} = \sqrt{[1 + 25]} = \sqrt{26}$.

What is the approximate length of the radius for the given circle?

The radius is approximately $\sqrt{26} \approx 5.10$ units.

How is the equation of the circle written with the given center and radius?

The equation is $(x - 1)^2 + (y + 3)^2 = r^2$, where r is the radius, which is $\sqrt{26}$ in this case.

Why is the distance formula essential in determining the radius of a circle passing through a specific point?

Because the radius is the distance from the center to any point on the circle, and the distance formula calculates this length precisely.

Can the radius be determined without calculating the exact distance? Why or why not?

No, because the radius is specifically the distance from the center to the given point; without calculating or knowing that distance, the exact radius cannot be determined.

Additional Resources

A Circle Passes Through The Point (2, 2) And Has Its Center At (1, -3). The Radius Of The Circle Is How is a classic problem in coordinate geometry that offers a comprehensive illustration of how geometric principles and algebraic methods intertwine. This problem is both instructive and engaging, providing insights into the fundamental concepts of circles, distances, and equations in a Cartesian plane. Through detailed analysis and step-by-step calculations, we can explore how to determine the radius of a circle given its center and a specific point through which it passes. This article aims to dissect the problem thoroughly, offering clarity and depth for students, educators, and mathematics enthusiasts alike.

Understanding the Problem Statement

At its core, the problem involves identifying the radius of a circle based on two key pieces of information:

- The center of the circle, located at point (1, -3).
- A point (2, 2) that lies on the circle's circumference.

The question posed is: How is the radius of the circle determined given these points? This is a typical problem used to reinforce the understanding of the relationship between a circle's center, a point on its circumference, and its radius.

Fundamental Concepts in Circle Geometry

Before delving into calculations, it’s essential to revisit some foundational ideas:

Definition of a Circle

A circle is the set of all points in a plane that are equidistant from a fixed point called the center. The constant distance from the center to any point on the circle is the radius.

Equation of a Circle

In the coordinate plane, the standard form of a circle's equation is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where (h, k) is the center, and r is the radius.

Relationship Between Center, Point, and Radius

Given the center (h, k) and a point (x, y) on the circle, the radius can be found by calculating the distance between these two points using the distance formula:

$$
r = \sqrt{(x - h)^2 + (y - k)^2}
$$

Calculating the Radius: Step-by-Step Approach

Applying the fundamental concepts to our problem:

Step 1: Identify the Center and Point

- Center: $(h, k) = (1, -3)$
- Point on the circle: $(x, y) = (2, 2)$

Step 2: Use the Distance Formula

The radius r is the distance between the center and the given point:

$$
r = \sqrt{(x - h)^2 + (y - k)^2}
$$

Substituting the known values:

$$
r = \sqrt{(2 - 1)^2 + (2 - (-3))^2}
$$

Step 3: Simplify the Expression

Calculate each component:

$$
(2 - 1)^2 = (1)^2 = 1
$$
$$
(2 - (-3))^2 = (2 + 3)^2 = (5)^2 = 25
$$

Add these together:

$$
r = \sqrt{1 + 25} = \sqrt{26}
$$

Step 4: Final Radius

The radius of the circle is:

$$
r = \sqrt{26}
$$

which is approximately 5.10 units when expressed as a decimal.

Implications and Significance of the Result

The computed radius $\sqrt{26}$ not only provides the specific measurement but also exemplifies the practical utility of algebraic methods in geometric problems. It demonstrates how the distance formula serves as a bridge between algebra and geometry, allowing us to solve for unknowns in a coordinate plane efficiently.

Features of the Calculation:
- Direct application of the distance formula makes the process straightforward.
- Use of coordinate geometry principles simplifies what could otherwise be a complex spatial problem.
- Exact radical form ensures precision, while the decimal approximation offers practical understanding.

Pros:
- Clear, step-by-step methodology enhances comprehension.
- Reinforces fundamental concepts relevant across various math topics.
- Can be adapted for more complex problems involving circles and distances.

Cons:
- Assumes familiarity with coordinate systems and algebra.
- Does not explore the derivation of the circle's full equation, which could be a next step.
- Limited to two-dimensional Euclidean space; more complex geometries require advanced techniques.

Additional Considerations and Extensions

While this problem focuses on a simple calculation, it opens doors to numerous extensions:

1. Deriving the Equation of the Circle

Using the center $(1, -3)$ and the radius $\sqrt{26}$, the circle's equation is:

$$
(x - 1)^2 + (y + 3)^2 = 26
$$

This formulation allows for graphing and further analysis.

2. Verifying Other Points

One might test whether additional points lie on this circle by substituting their coordinates into the equation to verify.

3. Exploring the Geometric Properties

Calculating the circumference ($2\pi r$) or area (πr^2) can deepen understanding of the circle's size and spatial properties.

4. Generalization to Other Coordinates

The method used here can be generalized for any circle given its center and a point on its circumference, making it a versatile tool in geometry.

Conclusion

The question, "A circle passes through the point (2, 2) and has its center at (1, -3). The radius of the circle is how?", encapsulates a fundamental geometric principle that is both accessible and profound. Through the application of the distance formula, we've determined that the radius is \(\sqrt{26}\), providing a precise measure of the circle's size. This problem exemplifies the power of algebraic techniques in solving geometric problems and highlights the elegance of coordinate geometry. Whether for academic exercises or real-world applications, understanding how to find a circle's radius from given points is an essential skill that underpins much of higher mathematics and geometry.

In summary, the process is straightforward yet rich with conceptual significance, illustrating how simple formulas can unlock complex spatial relationships. This problem serves as a cornerstone in the study of circles and coordinate geometry, reinforcing the interconnectedness of algebra and geometry in mathematical reasoning.

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