A Circle Passes Through The Points (3, 2) And (7, 2) And Has Radius 22. Find The Two Possible Equations

A Circle Passes Through The Points (3, 2) And (7, 2) And Has Radius 22. Find The Two Possible Equations

Understanding the equations of circles passing through specific points is a fundamental topic in coordinate geometry. In this article, we will explore how to determine the equations of a circle that passes through the given points (3, 2) and (7, 2) and has a radius of 22. We will analyze the problem step by step, elucidate the geometric concepts involved, and derive the two possible equations systematically.

Introduction to Circles in Coordinate Geometry

A circle in the coordinate plane can be defined by its center (h, k) and its radius r. The standard form equation of a circle is:

```plaintext
(x - h)^2 + (y - k)^2 = r^2
```

Given the points through which the circle passes and its radius, the goal is to find the specific equation(s) that satisfy these conditions.

Problem Breakdown

The problem provides:

- Two points: (3, 2) and (7, 2)
- Radius: 22

The key observations are:

- Both points lie on the circle.
- The circle has radius 22 units.

From these, we can set up equations based on the circle's standard form and properties.

Step 1: Recognize the Geometric Setup

Since both points are on the circle, the distances from the circle's center (h, k) to each point must be equal to the radius:

```plaintext
Distance from (h, k) to (3, 2) = 22
Distance from (h, k) to (7, 2) = 22
```

Expressed mathematically:

```plaintext
√[(h - 3)^2 + (k - 2)^2] = 22
√[(h - 7)^2 + (k - 2)^2] = 22
```

Squaring both sides simplifies these to:

```plaintext
(h - 3)^2 + (k - 2)^2 = 484 ...(1)
(h - 7)^2 + (k - 2)^2 = 484 ...(2)
```

Step 2: Derive the Equation for the Center

Subtract equation (2) from equation (1):

```plaintext
[(h - 3)^2 + (k - 2)^2] - [(h - 7)^2 + (k - 2)^2] = 0
```

Simplify:

```plaintext
(h - 3)^2 - (h - 7)^2 = 0
```

Expand both squares:

```plaintext
[(h)^2 - 6h + 9] - [h^2 - 14h + 49] = 0
```

Simplify:

```plaintext
h^2 - 6h + 9 - h^2 + 14h - 49 = 0
```

Combine like terms:

```plaintext
( -6h + 14h ) + ( 9 - 49 ) = 0
```

```plaintext
8h - 40 = 0
```

Solve for h:

```plaintext
8h = 40
h = 5
```

Thus, the center's x-coordinate is h = 5.

Step 3: Find the Possible y-coordinates of the Center

Now, substitute h = 5 into one of the earlier equations, say equation (1):

```plaintext
(5 - 3)^2 + (k - 2)^2 = 484
```

Calculate:

```plaintext
(2)^2 + (k - 2)^2 = 484
```

```

```plaintext
4 + (k - 2)^2 = 484
```

Subtract 4 from both sides:

```plaintext
(k - 2)^2 = 480
```

Take square roots:

```plaintext
k - 2 = ±√480
```

Express √480 in simplest radical form:

```plaintext
√480 = √(16 × 30) = 4√30
```

Therefore:

```plaintext
k - 2 = ± 4√30
```

Finally, solve for k:

```plaintext
k = 2 ± 4√30
```

Thus, the two possible centers are:

- Center 1: (h, k) = (5, 2 + 4√30)
- Center 2: (h, k) = (5, 2 - 4√30)
```

Step 4: Write the Equations of the Circles

Using the standard form:

```plaintext
(x - h)^2 + (y - k)^2 = r^2
```

and substituting the known centers and radius, which is 22:

```plaintext
r^2 = 22^2 = 484
```

The two equations are:

Equation 1:

```plaintext
(x - 5)^2 + (y - (2 + 4√30))^2 = 484
```

Equation 2:

```plaintext
(x - 5)^2 + (y - (2 - 4√30))^2 = 484
```

Step 5: Expressing the Equations in Expanded Form

To make these equations more explicit, expand both.

For Equation 1:

```plaintext
(x - 5)^2 + [ y - 2 - 4√30 ]^2 = 484
```

Expanding:

```plaintext
(x^2 - 10x + 25) + [ y^2 - 2 y (2 + 4√30) + (2 + 4√30)^2 ] = 484
```

Calculate the middle term:

```plaintext
- 2 y (2 + 4√30) = -2 y 2 - 2 y 4√30 = -4 y - 8√30 y
```

Calculate the square of (2 + 4√30):

```plaintext
(2)^2 + 224√30 + (4√30)^2 = 4 + 16√30 + 16 30
```

Since:

```plaintext
(4√30)^2 = 16 30 = 480
```

Adding:

```plaintext
4 + 16√30 + 480 = 484 + 16√30
```

Putting it all together:

```plaintext
x^2 - 10x + 25 + y^2 - 4 y - 8√30 y + 484 + 16√30 = 484
```

Combine constants:

```plaintext
(25 + 484) + x^2 - 10x + y^2 - 4 y - 8√30 y + 16√30 = 484
```

```plaintext
509 + x^2 - 10x + y^2 - 4 y - 8√30 y + 16√30 = 484
```

Subtract 484 from both sides:

```plaintext
(509 - 484) + x^2 - 10x + y^2 - 4 y - 8√30 y + 16√30 = 0
```

Simplify:

```plaintext
25 + x^2 - 10x + y^2 - 4 y - 8√30 y + 16√30 = 0
```

This is the expanded form of the first circle's equation.

Similarly, for Equation 2:

Replace $(2 - 4\sqrt{30})$ for k:

```plaintext
(x - 5)^2 + [ y - 2 + 4√30 ]^2 = 484
```

Expanding:

```plaintext
x^2 - 10x + 25 + y^2 - 2 y (2 - 4√30) + (2 - 4√30)^2 = 484
```

Calculate the middle term:

```plaintext
-2 y (2 - 4√30) = -4 y + 8√30 y
```

Calculate the square of $(2 - 4\sqrt{30})$:

```plaintext
4 - 16√30 + 480 = 484 - 16√30
```

Putting it all into the equation:

```plaintext
```

x^2 - 10x + 25 + y^2 - 4 y + 8√30 y + 484 - 16√30 = 484
```

Combine constants:

```plaintext
(25 + 484) + x^2 - 10x + y^2 - 4 y + 8√30 y - 16√30 = 484
```

Simplify:

```plaintext
509 + x^2 - 10x + y^2 - 4 y + 8√30 y - 16√30 = 484
```

Subtract 484:

```plaintext
(509 - 484) + x^2 - 10x + y^2 - 4 y + 8√30 y - 16√30 = 0
```

Simplify:

```plaintext
25 + x^2 -
```

Frequently Asked Questions

How do you find the equations of circles passing through points (3, 2) and (7, 2) with radius 22?

You set up the circle equation $(x - h)^2 + (y - k)^2 = 22^2$, then use the distance conditions that both points satisfy this equation, leading to a system to solve for h and k for the two possible centers.

What is the significance of the two possible solutions when finding the circle passing through (3, 2) and (7, 2) with radius 22?

The two solutions correspond to two different circle centers located above and below the line segment connecting the points, resulting in two distinct circles passing through both points with the given radius.
```

How do you derive the equations of the two possible circles in this problem?

First, find the midpoint of the points, then set up the circle equation with radius 22, and solve for the possible centers by considering the perpendicular bisector and the distance constraints, leading to two solutions for the center coordinates.

Can you provide the step-by-step process to find the equations of these circles?

Yes. Step 1: Find the midpoint between (3, 2) and (7, 2). Step 2: Determine the perpendicular bisector of the segment. Step 3: Use the distance formula to set the distance from the center to each point as 22. Step 4: Solve the resulting equations for the two possible centers, then write the circle equations.

What is the general form of the equations of the circles passing through the given points with radius 22?

The equations are $(x - h)^2 + (y - k)^2 = 484$, where (h, k) are the two possible centers found using the geometric constraints described.

How does the length of the segment between (3, 2) and (7, 2) affect the solution?

Since the segment length is 4, which is less than the diameter (44), there are exactly two circles with radius 22 passing through both points, corresponding to centers lying on the perpendicular bisector at specific distances.

What is the perpendicular bisector of the segment connecting (3, 2) and (7, 2), and how is it used here?

The perpendicular bisector is the vertical line $x = 5$, since the segment is horizontal. It helps locate the centers of the circles, which lie somewhere along this line at a certain distance from the midpoint.

What are the final equations of the two circles passing through (3, 2) and (7, 2) with radius 22?

The centers are at $(5, y_1)$ and $(5, y_2)$, where y_1 and y_2 are found by solving $(x - 5)^2 + (y - k)^2 = 484$ with the distance constraints. The equations are $(x - 5)^2 + (y - y_1)^2 = 484$ and $(x - 5)^2 + (y - y_2)^2 = 484$, with specific y-coordinates obtained from the calculations.

Additional Resources

A Circle Passes Through The Points (3, 2) And (7, 2) And Has Radius 22. Find The Two Possible Equations

Understanding the geometric problem of determining the equations of a circle passing through two specific points with a fixed radius is fundamental in coordinate geometry. This problem not only reinforces concepts related to circles but also enhances problem-solving skills involving distance formulas, midpoints, and algebraic manipulations. The challenge lies in deriving the equations of the possible circles that satisfy the given conditions, which involves a systematic approach combining geometric insights and algebraic calculations.

Introduction to the Problem

The problem states that a circle passes through the points (3, 2) and (7, 2) and has a radius of 22. The goal is to find the equations of the two possible circles that satisfy these conditions.

Key points:
- The circle passes through two points: (3, 2) and (7, 2).
- The radius of the circle is fixed at 22.
- The problem implies there are two possible circles because, given two points and a radius, the circle's center can lie on two different positions.

Understanding this problem involves recalling properties of circles, the distance formula, and the geometric interpretation of the circle's center relative to the given points.

Geometric Foundations and Approach

Understanding the Circle's Center

Given two points on a circle, the center of the circle must lie somewhere that maintains equal distance (the radius) from both points. Specifically:

- The distance from the center (h, k) to each point is 22.
- This leads to two equations:

$$
(h - 3)^2 + (k - 2)^2 = 22^2
$$

$$
(h - 7)^2 + (k - 2)^2 = 22^2
$$

Since the points (3, 2) and (7, 2) lie on the same horizontal line ($y=2$), the centers of the circles are located somewhere along the perpendicular bisector of the segment joining these points.

Perpendicular Bisector of the Chord

The segment joining (3, 2) and (7, 2) is horizontal, with length:

$$
\text{Distance} = |7 - 3| = 4
$$

The midpoint of this segment is at:

$$
\left(\frac{3 + 7}{2}, \frac{2 + 2}{2}\right) = (5, 2)
$$

The perpendicular bisector of this segment is a vertical line passing through (5, 2), i.e., $x = 5$.

Key insight:
Since the centers of the circles must be equidistant from both points, and both points are on the circle, the centers must lie somewhere along the perpendicular bisector $x=5$.

Deriving the Equations of the Circles

Step 1: Center Coordinates

Let the center be at (h, k). Because of the symmetry, the x-coordinate of the centers must be 5:

$$
h = 5
$$

Now, the centers are at (5, k), where k is an unknown coordinate to be determined.

Step 2: Use the Distance Formula

Since (5, k) is the center and (3, 2) lies on the circle:

$$
(5 - 3)^2 + (k - 2)^2 = 22^2
$$

Simplify:

$$
(2)^2 + (k - 2)^2 = 484
$$

$$
4 + (k - 2)^2 = 484
$$

$$
(k - 2)^2 = 480
$$

Similarly, check with the other point (7, 2):

$$
(5 - 7)^2 + (k - 2)^2 = 484
$$

$$
(-2)^2 + (k - 2)^2 = 484
$$

$$
4 + (k - 2)^2 = 484
$$

This matches the previous equation, confirming the assumption that centers lie along x=5.

Conclusion:
The centers are at:

$$
(5, k) \quad \text{where} \quad (k - 2)^2 = 480
$$

Hence,

$$
k - 2 = \pm \sqrt{480}
$$

and

$$
k = 2 \pm \sqrt{480}
$$

Note that:

$$
\sqrt{480} = \sqrt{16 \times 30} = 4 \sqrt{30}
$$

So,

$$
k = 2 \pm 4 \sqrt{30}
$$

Final Equations of the Circles

Centers and Radii

The two possible centers are:

$$
(5, 2 + 4 \sqrt{30}) \quad \text{and} \quad (5, 2 - 4 \sqrt{30})
$$

The general equation of a circle with center (h, k) and radius r is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Substituting the known centers and radius:

$$
(x - 5)^2 + \left(y - \left(2 + 4 \sqrt{30} \right) \right)^2 = 484
$$

and

$$
(x - 5)^2 + \left(y - \left(2 - 4 \sqrt{30} \right) \right)^2 = 484
$$

These are the two possible equations of the circles satisfying the conditions.

Explicit Equations of the Circles

First circle:

$$
(x - 5)^2 + \left(y - 2 - 4 \sqrt{30} \right)^2 = 484
$$

Second circle:

$$
(x - 5)^2 + \left(y - 2 + 4 \sqrt{30} \right)^2 = 484
$$

Expanding these equations for clarity:

First circle:

$$
x^2 - 10x + 25 + y^2 - 2 y (2 + 4 \sqrt{30}) + (2 + 4 \sqrt{30})^2 = 484
$$

which simplifies to:

$$
x^2 - 10x + y^2 - 2 y (2 + 4 \sqrt{30}) + \left(2 + 4 \sqrt{30} \right)^2 = 484
$$

Similarly for the second circle.

Features, Pros, and Cons of the Approach

Features:

- Utilizes symmetry and geometric properties to simplify the problem.
- Demonstrates the power of algebraic and geometric methods working together.
- Clearly identifies the locus of possible centers, simplifying the solution process.

Pros:

- Straightforward once the perpendicular bisector is identified.
- Reduces the problem to solving quadratic equations, which are manageable.
- Provides explicit equations for both possible circles without ambiguity.

Cons:

- Assumes the points are distinct and not coincident; special cases may require different approaches.
- The calculation involves square roots of non-perfect squares, leading to irrational numbers, which might be less intuitive.
- For more complex configurations, this method may not be as direct and could require numerical methods.

Conclusion and Summary

The problem of finding the equations of two circles passing through points (3, 2) and (7, 2) with radius 22 exemplifies the interplay between geometric intuition and algebraic formalism. By recognizing the symmetry and the perpendicular bisector as the locus of potential centers, the solution simplifies dramatically. The key steps involve:

- Computing the midpoint and understanding the symmetry.
- Using the distance formula to find possible center coordinates.
- Formulating the equations of the circles based on these centers and the given radius.

The resulting equations are:

$$
\boxed{
\begin{aligned}
(x - 5)^2 + \left(y - 2 - 4 \sqrt{30} \right)^2 &= 484 \\
(x - 5)^2 + \left(y - 2 + 4 \sqrt{30} \right)^2 &= 484
\end{aligned}
}
$$

This problem highlights how geometric properties can be exploited to solve what might initially seem like a purely algebraic challenge. It offers a comprehensive illustration of circle properties, symmetry, and the use of coordinate formulas, making it an excellent example for students and enthusiasts of mathematics seeking to deepen their understanding of circle equations and geometric problem-solving.

A Circle Passes Through The Points 3 2 And 7 2 And Has Radius 22 Find The Two Possible Equations

A Circle Passes Through The Points 3 2 And 7 2 And Has Radius 22 Find The Two Possible Equations

Related Articles

- What Will Be The Nominal Rate Of Return On A Perpetual Preferred Stock With A $100 Par Value, A Stated
- [8] What Is The Difference Between Photosynthesis Rate And Net Photosynthesis Rate? Which Rate Are You
- What Factors Precipitated The Development Of The Illinois Juvenile Court Act Of 1899? Discuss How Thus

Back to Home