(a) Derive The Expression For The Far Field Component Of A Monopole Antenna And Also Find Its Radiation

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A monopole antenna is one of the simplest and most widely used types of antennas, essential in various communication systems such as radio broadcasting, mobile communications, and radar. Understanding its radiation characteristics, especially in the far field region, is crucial for designing efficient antenna systems. This article aims to derive the mathematical expression for the far field component of a monopole antenna and analyze its radiation pattern and characteristics.

Introduction to Monopole Antennas

Before delving into the derivation, it is important to understand the fundamental structure and operation of a monopole antenna.

What Is a Monopole Antenna?

- A monopole antenna consists of a single conducting element, typically a quarter-wavelength long, mounted perpendicularly over a ground plane.
- The ground plane acts as a mirror, reflecting the electromagnetic waves, effectively creating a radiation pattern similar to a dipole.
- It is favored for its simplicity, ease of construction, and good radiation properties.

Operating Principle

- When an alternating current flows through the monopole, it creates an oscillating electric current and associated electromagnetic fields.
- The antenna radiates electromagnetic waves into the surrounding space, with the radiation pattern depending on the current distribution and geometry.

Theoretical Foundations for Far Field Analysis

The far field, also known as the Fraunhofer region, is the region where the angular field distribution becomes essentially independent of distance, and the fields behave as outward-propagating spherical waves.

Conditions for Far Field

- The observation point should be sufficiently far from the antenna, generally at distances greater than $2D^2 / \lambda$, where D is the largest dimension of the antenna and λ is the wavelength.
- In the far field, the electromagnetic fields can be approximated as spherical waves with a specific angular dependence.

Key Assumptions

- The antenna is assumed to be thin and small compared to the wavelength.
- The current distribution along the monopole is considered known, typically assumed to be sinusoidal or uniform for simplified models.

Derivation of the Far Field Expression

The process involves modeling the current distribution on the monopole, calculating the resulting vector potential, and then deriving the electric and magnetic fields in the far field.

Step 1: Model the Current Distribution

- For a monopole of length l ($l \approx \lambda/4$), the current distribution can be approximated as:

$$
I(z) = I_0 \sin \left[k \left(\frac{l}{2} - z \right) \right]
$$

where:

- z is the position along the monopole from the feed point,
- I_0 is the maximum current amplitude,
- $k = 2\pi / \lambda$ is the wave number.

- For simplicity, a uniform current distribution $I(z) = I_0$ is often assumed for a short monopole.

Step 2: Calculate the Vector Potential $\mathbf{A}$

The vector potential in the Lorentz gauge for a current distribution $\mathbf{J}$ is:

$$
\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4 \pi} \int \frac{\mathbf{J}(\mathbf{r}') e^{-j k |\mathbf{r}
$$

- \mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} dV'
\]

- For a thin monopole aligned along the z-axis, the current density simplifies to:

$$
\mathbf{J}(\mathbf{r}') = \hat{z} I(z') \delta(x') \delta(y')
$$

- Integrating over the volume reduces to integrating along the length of the monopole:

$$
\mathbf{A}(\mathbf{r}) = \hat{z} \frac{\mu_0}{4 \pi} \int_{-l/2}^{l/2} \frac{I(z') e^{-j k R}}{R} dz'
$$

where $R = |\mathbf{r} - \mathbf{r}'|$.

Step 3: Approximate R in the Far Field

- In the far-field approximation ($r \gg l$), $R \approx r - \hat{r} \cdot \mathbf{r}'$, and:

$$
\frac{e^{-j k R}}{R} \approx \frac{e^{-j k r}}{r} e^{j k \hat{r} \cdot \mathbf{r}'}
$$

- The vector potential becomes:

$$
\mathbf{A}(\mathbf{r}) \approx \hat{z} \frac{\mu_0}{4 \pi} \frac{e^{-j k r}}{r} \int_{-l/2}^{l/2} I(z') e^{j k \hat{r} \cdot \mathbf{r}'} dz'
$$

- Since $\hat{r} \cdot \mathbf{r}' = z' \cos \theta$, where θ is the angle between the observation point and the monopole axis:

$$
\mathbf{A}(\mathbf{r}) \approx \hat{z} \frac{\mu_0}{4 \pi} \frac{e^{-j k r}}{r} \int_{-l/2}^{l/2} I(z') e^{j k z' \cos \theta} dz'
$$

Step 4: Evaluate the Integral for $\mathbf{A}$

- For a uniform current I_0:

\[
\mathbf{A}(\mathbf{r}) \approx \hat{z} \frac{\mu_0 I_0}{4 \pi} \frac{e^{-j k r}}{r} \int_{-

l/2}^{l/2} e^{j k z' \cos \theta} dz'
\]

- The integral yields:

$$
\int_{-l/2}^{l/2} e^{j k z' \cos \theta} dz' = \frac{2 \sin \left(\frac{k l}{2} \cos \theta \right)}{k \cos \theta}
$$

- Therefore:

$$
\mathbf{A}(\mathbf{r}) \approx \hat{z} \frac{\mu_0 I_0}{4 \pi} \frac{e^{-j k r}}{r} \times \frac{2 \sin \left(\frac{k l}{2} \cos \theta \right)}{k \cos \theta}
$$

Step 5: Derive the Electric and Magnetic Fields

- The far-field electric field $\mathbf{E}$ relates to the vector potential as:

$$
\mathbf{E} \approx j \omega \mathbf{A} \times \hat{r}
$$

- For a thin monopole aligned along z, the dominant radiated field components are transverse, perpendicular to the propagation direction.

- The magnetic field $\mathbf{H}$ can be derived from:

$$
\mathbf{H} = \frac{1}{\eta} \hat{r} \times \mathbf{E}
$$

where $\eta \approx 377 \ \Omega$ is the intrinsic impedance of free space.

Final Expression for the Far Field

Combining the above, the electric field component in the far field at a distance r and angle θ is:

$$
\mathbf{E}(\mathbf{r}) \approx \hat{\theta} \times \left(\frac{j \eta I_0}{2 \pi r} \right) \times \frac{\sin \left(\frac{k l}{2} \cos \theta \right)}{\cos \theta}
$$

or more explicitly,

$$
E_\theta(r, \theta) \approx \frac{j \eta I_0}{2 \pi r} \times \frac{\sin \left(\frac{k l}{2} \cos \theta \right)}{\sin \theta}
$$

This expression describes how the radiation varies with the angle θ, and its amplitude diminishes proportionally to $1/r$.

Radiation Pattern and Characteristics of a Monopole Antenna

Understanding the radiation pattern is essential for practical applications.

Radiation Pattern

- The pattern of a quarter-wavelength monopole resembles a doughnut shape, with maximum radiation perpendicular to the monopole ($\theta = 90^\circ$) and nulls along the axis ($\theta = 0^\circ, 180^\circ$).
- The directivity D of a monopole is approximately 1.64 (or 2.15 dBi).

Radiation Resistance

- The total power radiated can be calculated

Frequently Asked Questions

What is the expression for the electric field in the far field of a monopole antenna?

The far-field electric field of a monopole antenna is given by $E(\theta) = (\eta I_0 e^{-jkr} / 2\pi r) \cos(\theta)$, where η is the intrinsic impedance of free space, I_0 is the feed current amplitude, k is the wave number, r is the distance from the antenna, and θ is the elevation angle.

How is the radiation pattern of a monopole antenna derived in the far field?

The radiation pattern is derived by analyzing the current distribution on the monopole (typically assumed as a quarter-wavelength segment) and applying the antenna theory to compute the radiated fields, resulting in a cosine-shaped pattern with maximum gain in the broadside direction (θ=0).

What assumptions are made in deriving the far-field expression of a monopole antenna?

Assumptions include a thin, vertical monopole of length approximately $\lambda/4$, sinusoidal current distribution, observation point at large distances ($r >> \lambda$), and negligible near-field effects, allowing simplification of the electromagnetic fields into radiating components.

How does the monopole antenna's height affect its far-field radiation pattern?

The monopole's height (typically $\lambda/4$) determines the current distribution and the resulting radiation pattern; a quarter-wavelength monopole produces a broadside pattern with maximum radiation perpendicular to the antenna axis.

What is the significance of the intrinsic impedance η in the far-field expression?

The intrinsic impedance η (approximately 377 ohms in free space) relates the electric and magnetic fields in the far field, ensuring that the derived field expressions are consistent with electromagnetic wave propagation in free space.

How is the total radiated power of a monopole antenna calculated from the far-field component?

The total radiated power is obtained by integrating the power flux (Poynting vector) over a spherical surface in the far field, often resulting in $P = (|I_0|^2 \eta / 12\pi) (\cos^2(\theta))$, integrated over the solid angle to find total radiated power.

What is the directivity of a monopole antenna based on its far-field pattern?

The directivity of a monopole antenna is approximately 1.64 (or 2.15 dBi), derived from its cosine-shaped radiation pattern, indicating how concentrated the radiation is in the main direction compared to an isotropic radiator.

How does the wavelength influence the far-field expression of a monopole antenna?

The wavelength λ determines the antenna length (ideally $\lambda/4$), influences the wave number $k = 2\pi/\lambda$, and affects the phase variation in the far field, thereby shaping the radiation pattern and the magnitude of the radiated fields.

What are the practical applications of deriving the far-field component of a monopole antenna?

Understanding the far-field component helps in designing antennas with desired radiation patterns, calculating link budgets in communication systems, optimizing antenna placement, and ensuring efficient radiation for applications like broadcasting, mobile communications, and radar.

Additional Resources

Derive The Expression For The Far Field Component Of A Monopole Antenna And Also Find Its Radiation Pattern

Understanding the far field component of a monopole antenna and its radiation pattern is fundamental to antenna theory and design. These concepts are crucial for engineers and enthusiasts aiming to optimize antenna performance, understand signal propagation, and develop efficient wireless communication systems. In this article, we will explore the derivation of the far field expression for a monopole antenna, analyze its radiation characteristics, and provide a comprehensive guide to its behavior in space.

Introduction to Monopole Antennas

A monopole antenna is one of the simplest and most widely used antenna types, primarily because of its straightforward construction and effective radiation properties. It consists of a single metallic conductor, typically a quarter-wavelength long, mounted perpendicularly over a ground plane which acts as a reflective surface. The ground plane effectively doubles the length of the monopole in terms of radiation, creating a radiation pattern similar to that of a dipole antenna.

Why Focus on the Far Field?

The far field region, also known as the Fraunhofer region, is the area sufficiently far from the antenna where the electromagnetic fields behave predictably as radiating waves. Understanding the far field component is essential because it determines how the antenna radiates energy into space, affecting coverage, gain, and directivity.

Fundamental Concepts and Assumptions

Before diving into the derivation, it’s important to clarify some foundational concepts and assumptions:

- Time-Harmonic Fields: The analysis assumes sinusoidal steady-state electromagnetic fields with time dependence $e^{j \omega t}$.

- Thin Wire Approximation: The monopole is considered a thin wire, meaning its radius a is much smaller than its length l ($a \ll l$).

- Uniform Current Distribution: For simplicity, the current along the monopole is assumed to be sinusoidal and approximately uniform, especially near the feed point.

- Large Distance Approximation: The far field is considered at distances $r \gg \lambda$, where λ is the wavelength, allowing us to neglect near-field effects.

Step 1: Modeling the Monopole Antenna as a Current Element

The fundamental starting point for deriving the radiation pattern is modeling the monopole as a current distribution. Since the monopole is a quarter-wavelength wire, it can be approximated as a linear current element with a sinusoidal current distribution:

$$
I(z) = I_0 \sin \left[k \left(\frac{l}{2} - z \right) \right]
$$

where:
- I_0 is the maximum current at the feed point,
- z is the coordinate along the wire from the feed point,
- l is the length of the monopole (approximately $\lambda/4$),
- $k = 2\pi / \lambda$ is the wave number.

However, for the purposes of the far-field derivation, the current distribution can be approximated as a sinusoidal distribution along the wire, with the understanding that the current peaks at the feed point and tapers off toward the end.

Step 2: Deriving the Vector Potential

The vector potential $\mathbf{A}$ is central to electromagnetic radiation analysis, as it relates directly to the radiated electric and magnetic fields. For a thin linear current element aligned along the z-axis, the vector potential at a point $\mathbf{r}$ in space (assuming $\mathbf{r}$ is in the far field) is given by:

$$
\mathbf{A} (\mathbf{r}) = \hat{\mathbf{z}} \frac{\mu_0}{4\pi} \int I(z) \frac{e^{-j k R}}{R} dz
$$

where:
- R is the distance from the current element at position z to the observation point,
- $\hat{\mathbf{z}}$ indicates the direction of current flow.

In the far field, $R \approx r - z \cos \theta$, where:
- r is the distance from the antenna to the observation point,
- θ is the angle between the antenna axis and the observation point.

Applying the far-field approximation $R \approx r$, and noting that $e^{-j k R}$ simplifies to $e^{-j k r} e^{j k z \cos \theta}$, the vector potential simplifies to:

$$
\mathbf{A} (\mathbf{r}) \approx \hat{\mathbf{z}} \frac{\mu_0 e^{-j k r}}{4\pi r} \int_{-l/2}^{l/2} I(z) e^{j k z \cos \theta} dz
$$

Step 3: Calculating the Radiation Field

The electric field in the far zone can be derived from the vector potential using:

$$
\mathbf{E} = j \omega \mathbf{A} \times \hat{\mathbf{r}}
$$

or, more directly, the radiation electric field is proportional to the transverse component of $\mathbf{A}$.

Assuming a sinusoidal current distribution $I(z) = I_0 \sin \left[k \left(\frac{l}{2} - z \right) \right]$, the integral becomes:

$$
\int_{-l/2}^{l/2} I(z) e^{j k z \cos \theta} dz
$$

which evaluates to an expression involving the sine function, leading to the classic antenna factor:

$$
F(\theta) = \frac{\cos \left(\frac{k l}{2} \cos \theta \right) - \cos \left(\frac{k l}{2} \right)}{\sin \theta}
$$

This function describes how the amplitude of the radiated field varies with the angle θ.

Step 4: Final Expression for the Far Field

Combining the above steps, the far field electric field component of a monopole antenna can be expressed as:

$$
\mathbf{E}_\text{far} (\mathbf{r}) = \hat{\boldsymbol{\theta}} \, \eta \frac{I_0 l}{2 \pi r} \, \frac{\cos \left(\frac{k l}{2} \cos \theta \right) - \cos \left(\frac{k l}{2} \right)}{\sin \theta} e^{-j k r}
$$

where:
- $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\, \Omega$ is the intrinsic impedance of free space,
- r is the distance to the observation point,
- θ is the angle from the monopole axis.

This expression captures the radiation pattern and amplitude of the far field.

Step 5: Radiation Pattern and Directivity

The radiation pattern of a monopole antenna is characterized by the angular dependence of $|\mathbf{E}_\text{far}|$. The normalized radiation pattern is given by:

$$
F(\theta) = \frac{\cos \left(\frac{k l}{2} \cos \theta \right) - \cos \left(\frac{k l}{2} \right)}{\sin \theta}
$$

Key observations:

- For a quarter-wavelength monopole ($l = \lambda/4$), the pattern resembles a donut shape, with maximum radiation in the horizontal plane ($\theta = 90^\circ$).

- The directivity D of a monopole over an infinite ground plane is approximately 1.64 (or 2.15 dBi), because the monopole effectively doubles the radiation of a simple dipole in free space.

- The gain depends on losses, ground conditions, and antenna quality, but the pattern remains predominantly omnidirectional in the azimuthal plane.

Additional Considerations

Ground Plane Influence

The ground plane acts as a mirror, reflecting the electromagnetic waves and effectively creating an image monopole. This mirror effect enhances the radiation in the horizontal plane and influences the overall pattern.

Effect of Length Variations

Changing the length l alters the radiation pattern:

- For $l = \lambda/4$, the pattern is broad and omnidirectional.

- For other lengths, the pattern becomes more directional with multiple lobes.

Practical Implications

- Design Optimization: Adjusting the monopole length and ground plane size can tailor the radiation pattern for specific applications.

- Impedance Matching: Ensuring the antenna's input impedance aligns with the transmitter improves power transfer, often requiring matching networks.

Conclusion

Deriving the expression for the far field component of a monopole antenna provides critical insights into its radiation behavior. By modeling the antenna as a current distribution, applying the principles of electromagnetic theory, and simplifying under far-field conditions, we arrive at a comprehensive formula that describes the amplitude and angular dependence of its radiated electric field.

This understanding not only explains the fundamental radiation pattern of monopole antennas but also guides practical

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