(c) Differentiate The Following Two Functions: I. Y Ax+b/cx+d Ii. Y = E^2x^4(x^3+1) - Ln(2x+5) (d) Find

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Introduction

Differentiation is a fundamental concept in calculus, enabling us to understand the rate at which a function changes at any given point. Mastering the differentiation techniques for various types of functions is crucial for solving complex mathematical problems. In this article, we will focus on differentiating two specific functions:

1. The rational function $Y = \frac{Ax + b}{cx + d}$
2. The composite and logarithmic function $Y = e^{2x^4}(x^3 + 1) - \ln(2x + 5)$

We will explore step-by-step methods to differentiate these functions, compare their characteristics, and provide practical examples to solidify understanding. This comprehensive guide aims to enhance your calculus skills, whether you are a student or a professional preparing for exams or applying calculus in real-world scenarios.

Differentiating the Rational Function $Y = \frac{Ax + b}{cx + d}$

Understanding the Structure

The function $Y = \frac{Ax + b}{cx + d}$ is a rational function, which is a ratio of two linear functions. Here:

- A, B, C, D are constants
- x is the variable

This type of function often appears in algebra and calculus, and differentiating it requires the use of the quotient rule.

Applying the Quotient Rule

The quotient rule states that if

$$
Y = \frac{u(x)}{v(x)}
$$

then

$$
Y' = \frac{u'(x) v(x) - u(x) v'(x)}{[v(x)]^2}
$$

where:

- $u(x) = Ax + b$
- $v(x) = cx + d$

Step-by-Step Differentiation

1. Differentiate $u(x)$:

$$
u'(x) = A
$$

2. Differentiate $v(x)$:

$$
v'(x) = C
$$

3. Apply the quotient rule:

$$
\frac{dY}{dx} = \frac{A (cx + d) - (Ax + b) C}{(cx + d)^2}
$$

4. Simplify numerator:

$$
A(cx + d) - C(Ax + b) = A c x + A d - C A x - C b
$$

$$
= (A c x - C A x) + (A d - C b)
$$

$$
= x(A c - C A) + (A d - C b)
$$

5. Final derivative:

$$
\boxed{
\frac{dY}{dx} = \frac{x(A c - C A) + (A d - C b)}{(cx + d)^2}
}
$$

Special Cases and Observations

- If the constants satisfy particular relationships, the derivative simplifies further.
- The rational function may have asymptotes where the denominator equals zero.

Differentiating the Complex Function $Y = e^{2x^4}(x^3 + 1) - \ln(2x + 5)$

Understanding the Components

This function combines exponential, polynomial, and logarithmic components:

- e^{2x^4} is an exponential function with a polynomial exponent.
- $(x^3 + 1)$ is a polynomial.
- $\ln(2x + 5)$ is a logarithmic function.

Differentiating such a function involves multiple rules:

- Product rule
- Chain rule
- Derivative of exponential functions
- Derivative of logarithmic functions

Step-by-Step Differentiation

Step 1: Differentiate $e^{2x^4}(x^3 + 1)$

This is a product of two functions, so apply the product rule:

$$
\frac{d}{dx} \left[e^{2x^4} (x^3 + 1) \right] = \left(\frac{d}{dx} e^{2x^4} \right) (x^3 + 1) + e^{2x^4} \frac{d}{dx} (x^3 + 1)
$$

- Derivative of e^{2x^4}:

Apply the chain rule:

$$
\frac{d}{dx} e^{2x^4} = e^{2x^4} \times \frac{d}{dx} (2x^4) = e^{2x^4} \times 8x^3
$$

- Derivative of $x^3 + 1$:

$$
3x^2
$$

- Therefore:

$$
\frac{d}{dx} \left[e^{2x^4}(x^3 + 1) \right] = e^{2x^4} \times 8x^3 (x^3 + 1) + e^{2x^4} \times 3x^2
$$

Factoring e^{2x^4}:

$$
= e^{2x^4} \left[8x^3 (x^3 + 1) + 3x^2 \right]
$$

Step 2: Differentiate $- \ln(2x + 5)$

Recall that:

$$
\frac{d}{dx} \left[\ln(f(x)) \right] = \frac{f'(x)}{f(x)}
$$

So,

$$
\frac{d}{dx} \left[- \ln(2x + 5) \right] = - \frac{2}{2x + 5}
$$

since $\frac{d}{dx} (2x + 5) = 2$.

Final Derivative

Putting it all together:

$$
\boxed{
\frac{dY}{dx} = e^{2x^4} \left[8x^3 (x^3 + 1) + 3x^2 \right] - \frac{2}{2x + 5}
}
$$

Comparing the Differentiation Techniques

Key Differences

Aspect	Rational Function $\frac{Ax + b}{cx + d}$	Complex Function $e^{2x^4}(x^3 + 1) - \ln(2x + 5)$
Main Rules Used	Quotient rule	Product rule, chain rule, logarithmic derivative
Complexity	Moderate	High, due to chain and product rules
Types of functions involved	Rational functions	Exponential, polynomial, logarithmic functions
Common Mistakes	Forgetting to square the denominator	Missing chain rule application for exponential

Practical Tips for Differentiation

- Always identify the form of the function before choosing the rule.
- For rational functions, use the quotient rule.
- For products, apply the product rule carefully.
- When exponential or logarithmic functions are involved, remember the chain rule.
- Simplify derivatives where possible to avoid errors.

Practical Examples and Applications

Example 1: Differentiating a Rational Function

Suppose $A = 2, B = 3, C = 1, D = 4$, then:

$$
Y = \frac{2x + 3}{x + 4}
$$

Derivative:

$$
\frac{dY}{dx} = \frac{x(2 \times 1 - 1 \times 2) + (2 \times 4 - 1 \times 3)}{(x + 4)^2} = \frac{x(2 - 2) + (8 - 3)}{(x + 4)^2} = \frac{0 + 5}{(x + 4)^2} = \frac{5}{(x + 4)^2}
$$

Example 2: Differentiating the Complex Function

Calculate the derivative at $x = 1$:

$$
Y = e^{2(1)^4} (1^3 + 1) - \ln(2 \times 1 + 5) = e^{2} \times 2 - \ln(7)
$$

Derivative at $x = 1$:

$$
\frac{dY}{dx} \bigg|_{x=1} = e^{2} \left[8 \times 1^3 (1^3 + 1) + 3 \times 1^2 \right] - \frac{2}{2 \times 1 + 5} = e^{2} (8 \times 1 \times 2 + 3) - \frac{2}{7}
$$

$$
= e^{2} (16 + 3) - \frac{2}{7} = 19 e^{2} - \frac{2}{7}
$$

Conclusion

Differentiating various types of functions is a vital skill

Frequently Asked Questions

How do you differentiate the function $Y = (Ax + b) / (cx + d)$?

To differentiate $Y = (Ax + b) / (cx + d)$, use the quotient rule: $d/dx [u/v] = (v du/dx - u dv/dx) / v^2$. Here, $u = Ax + b$ and $v = cx + d$. Differentiating, we get: $Y' = [(cx + d)(A) - (Ax + b)(c)] / (cx + d)^2$.

What is the derivative of the function $Y = e^{2x^4}(x^3+1) - \ln(2x+5)$?

The derivative is found by applying the product rule to $e^{2x^4}(x^3+1)$ and the chain rule to $\ln(2x+5)$. Specifically, $d/dx [e^{2x^4}(x^3+1)] = e^{2x^4} [8x^3(x^3+1) + 3x^2]$, and $d/dx [\ln(2x+5)] = 2 / (2x+5)$. The overall derivative is: $e^{2x^4}[8x^3(x^3+1) + 3x^2] - 2 / (2x+5)$.

How is the quotient rule applied to differentiate $(Ax + b) / (cx + d)$?

The quotient rule states that $d/dx [u/v] = (v du/dx - u dv/dx) / v^2$. For $u = Ax + b$ and $v = cx + d$, this becomes $Y' = [(cx + d) A - (Ax + b) c] / (cx + d)^2$.

What steps are involved in differentiating $Y = e^{2x^4}(x^3+1) - \ln(2x+5)$?

First, apply the product rule to differentiate $e^{2x^4}(x^3+1)$. Then, differentiate $\ln(2x+5)$ using the chain rule. Sum the results, keeping track of derivatives of exponential and polynomial functions.

Can you simplify the derivative of the function $Y = (Ax + b) / (cx + d)$?

Yes. After applying the quotient rule, the derivative simplifies to $Y' = [(cx + d)A - (Ax + b)c] / (cx + d)^2$, which can be expanded and combined for specific constants A, B, C, D.

What is the significance of differentiating $Y = e^{2x^4}(x^3+1) - \ln(2x+5)$?

Differentiating this function helps in understanding its rate of change, analyzing its increasing/decreasing intervals, and finding critical points for graphing or optimization purposes.

How do you handle the natural logarithm when differentiating functions like $\ln(2x+5)$?

Use the chain rule: $d/dx [\ln(2x+5)] = 1 / (2x+5)$ derivative of the inside, which is 2. So, the derivative is $2 / (2x+5)$.

What are common mistakes to avoid when differentiating these types of functions?

Common mistakes include neglecting the chain rule in composite functions, incorrect application of the quotient rule, and forgetting to differentiate all parts of a product or sum. Carefully applying rules step-by-step prevents errors.

Additional Resources

(c) Differentiate The Following Two Functions: I. $Y = (Ax + b) / (Cx + d)$ II. $Y = e^{2x^4}(x^3 + 1) - \ln(2x + 5)$ (d) Find

Understanding how to differentiate functions is fundamental in calculus, providing insights into the behavior of functions, such as increasing or decreasing trends, points of inflection, and local extrema. Today, we delve into two distinct types of functions: a rational function and a combination of exponential and logarithmic functions. We aim to not only differentiate these functions but also elucidate the techniques involved in their derivatives, offering clarity for students, educators, and enthusiasts alike.

Introduction

In the realm of calculus, functions often fall into different categories—rational functions, exponential functions, logarithmic functions, and their combinations. Differentiation techniques vary depending on the structure of the function. The first function we examine is a rational function, specifically a quotient of two linear functions, which calls for the quotient rule. The second function is more complex, involving exponential and logarithmic components, requiring the application of both the product rule and the chain rule.

Understanding the differentiation of these functions involves recognizing their structure and choosing the appropriate rules. Mastery over these techniques enables us to analyze real-world phenomena, from physics to economics, where rates of change are pivotal.

Differentiating a Rational Function: $Y = \frac{A x + b}{C x + d}$

The Structure of Rational Functions

A rational function is expressed as the ratio of two polynomials. In our case, both numerator and denominator are linear functions:

$$
Y = \frac{A x + b}{C x + d}
$$

where A, b, C, d are constants, and x is the variable.

The Quotient Rule: A Primer

The quotient rule is essential when differentiating functions of the form $\frac{u(x)}{v(x)}$. It states:

$$
\frac{d}{dx} \left(\frac{u(x)}{v(x)} \right) = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{\left(v(x) \right)^2}
$$

This rule ensures an accurate differentiation by accounting for the change in both numerator and denominator.

Applying the Quotient Rule

For our function:

- $u(x) = A x + b$
- $v(x) = C x + d$

Calculating derivatives:

- $u'(x) = A$
- $v'(x) = C$

Plugging into the quotient rule:

$$
Y' = \frac{(C x + d) \cdot A - (A x + b) \cdot C}{(C x + d)^2}
$$

Simplify numerator:

$$
Y' = \frac{A (C x + d) - C (A x + b)}{(C x + d)^2}
$$

Expanding both terms:

$$
Y' = \frac{A C x + A d - C A x - C b}{(C x + d)^2}
$$

Notice $A C x$ and $- C A x$ cancel each other out:

$$
Y' = \frac{A d - C b}{(C x + d)^2}
$$

Final derivative:

\[

\boxed{
\frac{d}{dx} \left(\frac{A x + b}{C x + d} \right) = \frac{A d - C b}{(C x + d)^2}
}
\]

Interpretation

The derivative simplifies to a constant numerator over the squared denominator. This indicates that the rate of change of this rational function depends linearly on the constants \(A, b, C, d\). If \(A d - C b = 0\), the derivative becomes zero, implying the function is constant, which makes sense because the numerator and denominator are proportional.

Differentiating a Complex Function: \(Y = e^{2x^4}(x^3 + 1) - \ln(2x + 5)\)

Dissecting the Function

This is a more intricate function involving:

- An exponential component \(e^{2x^4}\)
- A polynomial component \(x^3 + 1\)
- A logarithmic component \(\ln(2x + 5)\)

The function combines product and difference operations, requiring multiple differentiation rules.

Strategy Overview

To differentiate:

\. Apply the product rule to \(e^{2x^4} \cdot (x^3 + 1)\)
\. Differentiate \(- \ln(2x + 5)\) using the chain rule

Differentiating the Exponential-Polynomial Product

The Product Rule

The product rule states:

\[
\frac{d}{dx} \left(u(x) v(x) \right) = u'(x) v(x) + u(x) v'(x)
\]

Here:

- \(u(x) = e^{2x^4}\)
- \(v(x) = x^3 + 1\)

Derivatives of Components

1. $u(x) = e^{2x^4}$

Applying the chain rule:

$$
u'(x) = e^{2x^4} \cdot \frac{d}{dx} (2x^4) = e^{2x^4} \cdot 8x^3
$$

2. $v(x) = x^3 + 1$

$$
v'(x) = 3x^2
$$

Applying the Product Rule

$$
\frac{d}{dx} \left(e^{2x^4}(x^3 + 1) \right) = e^{2x^4} \cdot 8x^3 (x^3 + 1) + e^{2x^4} \cdot 3x^2
$$

Factor out e^{2x^4}:

$$
= e^{2x^4} \left(8x^3 (x^3 + 1) + 3x^2 \right)
$$

Expanding the term inside brackets:

$$
8x^3 \cdot x^3 = 8x^6
$$
$$
8x^3 \cdot 1 = 8x^3
$$

So:

$$
Y'_{exp\text{-}poly} = e^{2x^4} (8x^6 + 8x^3 + 3x^2)
$$

Differentiating the Logarithmic Term

$$
- \ln(2x + 5)
$$

The derivative of $- \ln(2x + 5)$ involves the chain rule:

$$
\frac{d}{dx} \left(- \ln(2x + 5) \right) = - \frac{1}{2x + 5} \cdot \frac{d}{dx} (2x + 5) = - \frac{1}{2x + 5} \cdot 2 = - \frac{2}{2x + 5}
$$

Combining All Parts

The total derivative:

$$
Y' = e^{2x^4} (8x^6 + 8x^3 + 3x^2) - \frac{2}{2x + 5}
$$

This expression encapsulates the rate of change of the original function at any point x.

Practical Implications and Applications

Differentiating such functions isn't purely academic; it has practical relevance across sciences and engineering. For example:

- The rational function derivative helps in optimization problems involving ratios, such as efficiency calculations or cost-to-benefit ratios.
- The exponential-log function derivative finds use in modeling growth processes, radioactive decay, or financial computations involving compound interest and logarithmic scales.

Summary and Key Takeaways

- The quotient rule is essential for rational functions, simplifying the differentiation of ratios of polynomials.
- The product rule combined with the chain rule enables differentiation of complex exponential-polynomial functions.
- Logarithmic functions require the chain rule for their derivatives, especially when composed inside other functions.

Mastering these techniques enhances one's ability to analyze a wide array of functions, unlocking a deeper understanding of their behavior. As calculus continues to be a cornerstone of scientific inquiry, proficiency in differentiation remains invaluable.

Final Thoughts

Differentiation, while sometimes intricate, becomes manageable when approached systematically. Recognizing the structure of functions guides the choice of the correct rules, ensuring accurate derivatives. Whether dealing with simple rational functions or complex exponential-logarithmic combinations, the core principles of calculus serve as reliable tools to explore the dynamic world of mathematical functions.

By practicing these differentiation techniques regularly and understanding their applications, learners can develop a robust foundation for advanced studies in mathematics, physics, engineering, and beyond.

C Differentiate The Following Two Functions I Y Ax B Cx D Ii Y E 2x 4 X 3 1 Ln 2x 5 D Find

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