

(Score For Question 1: ____ Of 5 Points)What Is The Product Of $\frac{1}{2}x - \frac{1}{4}$ And $5x^2 - 2x + 6$? Write Your Answer

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Understanding how to multiply algebraic expressions is a fundamental skill in algebra. In this article, we will explore step-by-step how to find the product of the binomial $\left(\frac{1}{2}x - \frac{1}{4}\right)$ and the trinomial $(5x^2 - 2x + 6)$. We will break down the process, explain key concepts, and provide tips to ensure clarity and ease of understanding. Whether you're a student preparing for exams or someone looking to strengthen your algebra skills, this comprehensive guide will help you master the multiplication of binomials and trinomials.

Understanding the Problem

Before diving into the solution, it's essential to understand what the problem is asking.

Given Expressions

- Binomial: $\left(\frac{1}{2}x - \frac{1}{4}\right)$
- Trinomial: $(5x^2 - 2x + 6)$

Objective

- Find the product of these two algebraic expressions.
- Write the simplified form of the result.

Key Concepts in Polynomial Multiplication

Multiplying algebraic expressions involves applying specific rules and techniques. Here's a quick overview:

Distributive Property

- The foundation of polynomial multiplication.

- States that $(a \times (b + c) = a \times b + a \times c)$.

FOIL Method

- A technique mainly used for binomials.
- Stands for First, Outer, Inner, Last.
- Not directly used for a binomial times a trinomial, but understanding it helps with similar problems.

General Polynomial Multiplication

- Each term in one polynomial is multiplied by each term in the other.
- The results are then combined like terms to simplify the expression.

Step-by-Step Solution

Let's now proceed to multiply the binomial $(\frac{1}{2}x - \frac{1}{4})$ with the trinomial $(5x^2 - 2x + 6)$.

Step 1: Distribute Each Term of the Binomial

The binomial has two terms: $(\frac{1}{2}x)$ and $(-\frac{1}{4})$. We will multiply each of these by every term in the trinomial.

Multiply $(\frac{1}{2}x)$ by each term in the trinomial:

- $(\frac{1}{2}x \times 5x^2)$
- $(\frac{1}{2}x \times (-2x))$
- $(\frac{1}{2}x \times 6)$

Multiply $(-\frac{1}{4})$ by each term in the trinomial:

- $(-\frac{1}{4} \times 5x^2)$
- $(-\frac{1}{4} \times (-2x))$
- $(-\frac{1}{4} \times 6)$

Step 2: Calculate Each Product

Perform each multiplication carefully, paying attention to coefficients and variables.

Term	Calculation	Result
$\frac{1}{2}x \times 5x^2$	$\frac{1}{2} \times 5 \times x \times x^2$	$\frac{5}{2}x^3$
$\frac{1}{2}x \times -2x$	$\frac{1}{2} \times -2 \times x \times x$	$-1x^2$ or $-x^2$
$\frac{1}{2}x \times 6$	$\frac{1}{2} \times 6 \times x$	$3x$
$-\frac{1}{4} \times 5x^2$	$-\frac{1}{4} \times 5 \times x^2$	$-\frac{5}{4}x^2$
$-\frac{1}{4} \times -2x$	$-\frac{1}{4} \times -2 \times x$	$\frac{1}{2}x$
$-\frac{1}{4} \times 6$	$-\frac{1}{4} \times 6$	$-\frac{3}{2}$

Step 3: Write the Expression with All Terms

Now, sum all these products:

$$\frac{5}{2}x^3 - x^2 + 3x - \frac{5}{4}x^2 + \frac{1}{2}x - \frac{3}{2}$$

Step 4: Combine Like Terms

Identify terms with the same degree and combine:

- Cubic term: $\frac{5}{2}x^3$ (only one)
- Quadratic terms: $-x^2 - \frac{5}{4}x^2$
- Linear terms: $3x + \frac{1}{2}x$
- Constant term: $-\frac{3}{2}$

Let's combine them:

Quadratic:

$$-x^2 - \frac{5}{4}x^2 = -\left(1 + \frac{5}{4}\right)x^2$$

Convert 1 to $\frac{4}{4}$:

$$-\left(\frac{4}{4} + \frac{5}{4}\right)x^2 = -\frac{9}{4}x^2$$

Linear:

$$3x + \frac{1}{2}x = \left(3 + \frac{1}{2}\right)x$$

Express 3 as $\frac{6}{2}$:

$\frac{6}{2} + \frac{1}{2} = \frac{7}{2}$

$$\left(\frac{6}{2} + \frac{1}{2}\right) x = \frac{7}{2} x$$

Constants:

$$-\frac{3}{2}$$

Final Expression

Putting it all together, the product is:

$$\boxed{\frac{5}{2} x^3 - \frac{9}{4} x^2 + \frac{7}{2} x - \frac{3}{2}}$$

This is the simplified form of the product of the given binomial and trinomial.

Additional Tips for Polynomial Multiplication

- Always distribute each term carefully to avoid missing any products.
- Convert coefficients to common denominators when combining like terms.
- Remember that combining like terms involves adding coefficients of the same degree.
- Practice multiplying different combinations of polynomials to improve speed and accuracy.
- Use algebraic identities and factoring techniques when applicable to simplify complex expressions.

Real-World Applications of Polynomial Multiplication

Understanding how to multiply polynomials is not just an academic exercise; it has practical applications in various fields:

- Physics: Calculating areas and volumes involving quadratic and cubic functions.
- Engineering: Signal processing, control systems, and system modeling often involve polynomial equations.
- Economics: Modeling profit, cost, and revenue functions.
- Computer Science: Algorithms involving polynomial interpolation, coding theory, and cryptography.

Conclusion

Multiplying binomials and trinomials is a vital skill in algebra that lays the foundation for more advanced topics like polynomial division, factorization, and solving algebraic equations. By carefully distributing each term, converting coefficients to common denominators, and combining like terms, you can confidently find the product of complex algebraic expressions. Remember to practice with different types of polynomials to strengthen your understanding and proficiency.

In this guide, we demonstrated how to multiply $\left(\frac{1}{2}x - \frac{1}{4}\right)$ by $(5x^2 - 2x + 6)$, arriving at the simplified answer:

$$\boxed{\frac{5}{2}x^3 - \frac{9}{4}x^2 + \frac{7}{2}x - \frac{3}{2}}$$

Mastering this process empowers you to handle more complex algebraic problems with confidence and precision.

Frequently Asked Questions

What is the product of $\left(\frac{1}{2}x - \frac{1}{4}\right)$ and $5x^2 - 2x + 6$?

The product is $\frac{5}{2}x^3 - \frac{9}{4}x^2 + \frac{13}{4}x - \frac{3}{2}$.

How do you multiply $\left(\frac{1}{2}x - \frac{1}{4}\right)$ by $5x^2 - 2x + 6$?

Distribute each term of the first binomial to each term of the second polynomial and combine like terms, resulting in $\frac{5}{2}x^3 - \frac{9}{4}x^2 + \frac{13}{4}x - \frac{3}{2}$.

What is the simplified form of the product $\left(\frac{1}{2}x - \frac{1}{4}\right)$ and $5x^2 - 2x + 6$?

The simplified product is $\frac{5}{2}x^3 - \frac{9}{4}x^2 + \frac{13}{4}x - \frac{3}{2}$.

Can you explain the steps to multiply $\left(\frac{1}{2}x - \frac{1}{4}\right)$ by $5x^2 - 2x + 6$?

Yes. Multiply $\left(\frac{1}{2}x\right)$ by each term: $\left(\frac{1}{2}x\right) 5x^2 = \frac{5}{2}x^3$, $\left(\frac{1}{2}x\right) (-2x) = -x^2$, $\left(\frac{1}{2}x\right) 6 = 3x$. Then multiply $\left(-\frac{1}{4}\right)$ by each term: $\left(-\frac{1}{4}\right) 5x^2 = -\frac{5}{4}x^2$, $\left(-\frac{1}{4}\right) (-2x) = \frac{1}{2}x$, $\left(-\frac{1}{4}\right) 6 = -\frac{3}{2}$. Add all results: $\frac{5}{2}x^3 + (-x^2 - \frac{5}{4}x^2) + (3x + \frac{1}{2}x) - \frac{3}{2}$. Simplify to get $\frac{5}{2}x^3 - \frac{9}{4}x^2 + \frac{13}{4}x - \frac{3}{2}$.

What is the degree of the resulting polynomial after multiplying $(\frac{1}{2})x - \frac{1}{4}$ and $5x^2 - 2x + 6$?

The degree of the resulting polynomial is 3.

Is the product of $(\frac{1}{2})x - \frac{1}{4}$ and $5x^2 - 2x + 6$ a polynomial? Why?

Yes, because multiplying two polynomials results in another polynomial.

What coefficients appear in the final product of $(\frac{1}{2})x - \frac{1}{4}$ and $5x^2 - 2x + 6$?

The coefficients are $\frac{5}{2}$, $-\frac{9}{4}$, $\frac{13}{4}$, and $-\frac{3}{2}$.

How can I verify the product of $(\frac{1}{2})x - \frac{1}{4}$ and $5x^2 - 2x + 6$?

You can expand both expressions and simplify, or use polynomial multiplication techniques to confirm the result.

What is the importance of multiplying polynomials like $(\frac{1}{2})x - \frac{1}{4}$ and $5x^2 - 2x + 6$?

Multiplying polynomials is fundamental in algebra for simplifying expressions, solving equations, and modeling real-world problems.

Additional Resources

Score For Question 1: ___ Of 5 Points is a phrase that indicates the grading criteria for a specific math problem, particularly one involving algebraic expressions and polynomial multiplication. This type of question is common in algebra assessments, testing students' understanding of multiplication of fractions and quadratic expressions. In this review, we will analyze the problem's structure, the mathematical concepts involved, step-by-step solution strategies, common pitfalls, and educational value. The goal is to provide a comprehensive overview suitable for students, educators, or anyone interested in algebraic problem-solving.

Understanding the Problem

Breaking Down the Question

The problem asks: "What is the product of $(\frac{1}{2}x - \frac{1}{4})$ and $(5x^2 - 2x + 6)$?" The phrase "Write Your

Answer" suggests that the student is expected to perform the multiplication and present the simplified result.

Key components:

- First expression: $(\frac{1}{2}x - \frac{1}{4})$
- Second expression: $(5x^2 - 2x + 6)$

The task involves multiplying a binomial by a trinomial, which is a common algebraic operation. Understanding how to correctly handle fractions and polynomial multiplication is essential here.

Mathematical Concepts Involved

1. Multiplying Fractions and Algebraic Expressions

- Recognizing that $(\frac{1}{2}x)$ can be viewed as $(\frac{1}{2})x$, emphasizing the importance of understanding coefficients and variables.
- Handling subtraction of fractions: ensuring common denominators are considered.
- Applying distributive property (FOIL method for binomials, and extended for binomials times trinomials).

2. Polynomial Multiplication

- Distributing each term in the first expression across all terms in the second.
- Combining like terms after multiplication.
- Simplifying fractions within the expression.

3. Simplification of the Result

- Reducing fractions where applicable.
- Combining like terms to arrive at the simplest form of the product.

Step-by-Step Solution Strategy

Step 1: Rewrite the expressions clearly

Express $(\frac{1}{2}x - \frac{1}{4})$ as:

$$- (\frac{1}{2})x - \frac{1}{4}$$

Express the second polynomial as:

$$- 5x^2 - 2x + 6$$

Step 2: Distribute each term in the first binomial across the second polynomial

The multiplication involves:

$$- (\frac{1}{2})x (5x^2 - 2x + 6)$$

$$- (-\frac{1}{4}) (5x^2 - 2x + 6)$$

Handle each separately.

Step 3: Multiply $(\frac{1}{2})x$ with each term in the second polynomial

$$- (\frac{1}{2})x 5x^2 = (\frac{1}{2}) 5 x x^2 = (\frac{5}{2}) x^3$$

$$- (\frac{1}{2})x (-2x) = (\frac{1}{2}) (-2) x x = (-\frac{2}{2}) x^2 = -1 x^2$$

$$- (\frac{1}{2})x 6 = (\frac{1}{2}) 6 x = 3x$$

Step 4: Multiply $(-\frac{1}{4})$ with each term in the second polynomial

$$- (-\frac{1}{4}) 5x^2 = (-\frac{5}{4}) x^2$$

$$- (-\frac{1}{4}) (-2x) = (\frac{1}{2}) x$$

$$- (-\frac{1}{4}) 6 = -3$$

Step 5: Combine all products

Resulting expression:

$$(\frac{5}{2}) x^3 + (-1) x^2 + 3x + (-\frac{5}{4}) x^2 + (\frac{1}{2}) x - 3$$

Step 6: Combine like terms

$$- \text{For } x^3: (\frac{5}{2}) x^3 \text{ (only one term)}$$

$$- \text{For } x^2: (-1) x^2 - (\frac{5}{4}) x^2 = (-\frac{4}{4}) x^2 - (\frac{5}{4}) x^2 = (-\frac{9}{4}) x^2$$

$$- \text{For } x: 3x + (\frac{1}{2}) x = (\frac{6}{2}) x + (\frac{1}{2}) x = (\frac{7}{2}) x$$

$$- \text{Constant term: } -3$$

Final simplified expression:

$$\left(\frac{5}{2}\right)x^3 - \left(\frac{9}{4}\right)x^2 + \left(\frac{7}{2}\right)x - 3$$

Educational Analysis and Insights

Pros of the Problem

- Reinforces the understanding of distributing multiplication across polynomial terms.
- Promotes careful handling of fractions and variables.
- Encourages students to simplify algebraic expressions systematically.
- Combines multiple algebraic skills in one problem, fostering comprehensive mastery.

Cons or Challenges

- Fractions can be confusing; students may struggle with common denominators.
- Potential for errors in distribution or combining like terms.
- The problem requires attention to detail; oversight can lead to incorrect answers.
- May be intimidating for beginners unfamiliar with polynomial multiplication involving fractions.

Features of the Problem

- Combines binomial and trinomial multiplication.
- Involves fractional coefficients, adding complexity.
- Demonstrates the importance of organizing work and simplifying step-by-step.

Educational Tips

- Emphasize the importance of rewriting expressions clearly.
- Practice distributing each term carefully, perhaps with visual aids.
- Encourage students to write fractions explicitly to avoid mistakes.
- Stress the importance of combining like terms accurately.

Potential Variations and Extensions

- Changing the coefficients to integers or decimal fractions.
- Extending to multiplication involving larger polynomials.
- Introducing variables with exponents or coefficients involving radicals.
- Applying similar methods to division of polynomials.

Conclusion and Final Thoughts

The problem "What is the product of $(\frac{1}{2}x - \frac{1}{4})$ and $(5x^2 - 2x + 6)$?" serves as an excellent exercise in algebraic multiplication, fraction handling, and polynomial simplification. Its complexity lies in juggling fractional coefficients and ensuring meticulous distribution and combination of like terms. The detailed step-by-step approach outlined above provides clarity and reinforces core algebraic principles, making it an invaluable learning tool.

For educators, incorporating such problems in assessments can effectively gauge students' understanding of polynomial multiplication and their ability to manipulate fractions within algebraic contexts. For students, practicing similar problems enhances problem-solving skills and mathematical confidence. Overall, this type of question exemplifies the beauty and challenge of algebra, encouraging precision, patience, and systematic thinking.

Note: The final answer, in simplified form, is:

$$(\frac{5}{2})x^3 - (\frac{9}{4})x^2 + (\frac{7}{2})x - 3$$

which can be written as a single expression for submission or further analysis.

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