Use The Method Of Undetermined Coefficients To Solve The Given Nonhomogeneous System. = 2x + 3y - 8 Dx

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Introduction

Solving systems of differential equations is a fundamental aspect of advanced mathematics, particularly in fields such as engineering, physics, and applied mathematics. Among the various techniques available, the Method of Undetermined Coefficients stands out as a powerful method for finding particular solutions to nonhomogeneous linear differential equations with constant coefficients. This method simplifies the process of solving complex differential systems by guessing a particular solution form and then determining the coefficients through substitution.

In this article, we will explore how to use the Method of Undetermined Coefficients specifically for a nonhomogeneous system characterized by the differential equation:

$$\frac{dy}{dx} = 2x + 3y - 8$$

This equation represents a first-order linear nonhomogeneous differential equation, and understanding its solution is crucial for modeling real-world phenomena where systems are influenced by external forces or inputs. Our goal is to provide a comprehensive, step-by-step guide to solving this system, including the homogeneous solution, particular solution, and the overall general solution, all while optimizing for clarity and search engine visibility.

Understanding the Differential Equation

Before diving into the solution process, it’s essential to understand the structure of the given differential equation:

$$\frac{dy}{dx} = 2x + 3y - 8$$

This is a first-order linear nonhomogeneous differential equation in the form:

$$\frac{dy}{dx} + P(x) y = Q(x)$$

where:

- $P(x) = -3$
- $Q(x) = 2x - 8$

Rearranged, the equation becomes:

$$\frac{dy}{dx} - 3y = 2x - 8$$

Our task is to find the general solution to this differential equation, which involves two parts:

1. The homogeneous solution y_h, solving the associated homogeneous equation.
2. The particular solution y_p, addressing the nonhomogeneous part using the Method of Undetermined Coefficients.

Step 1: Find the Homogeneous Solution

The homogeneous equation associated with the original differential equation is:

$$ \frac{dy}{dx} - 3y = 0 $$

which can be rewritten as:

$$ \frac{dy}{dx} = 3y $$

Solving the Homogeneous Equation

This is a separable differential equation:

$$ \frac{dy}{y} = 3 dx $$

Integrating both sides:

$$ \int \frac{1}{y} dy = \int 3 dx $$

$$ \ln |y| = 3x + C $$

Exponentiating both sides:

$$ |y| = e^{3x + C} = e^{C} e^{3x} $$

Let $C' = e^{C}$, which is an arbitrary constant:

$$ y_h = C' e^{3x} $$

Thus, the homogeneous solution is:

$$ y_h = C e^{3x} $$

where C is an arbitrary constant.

Step 2: Find the Particular Solution Using the Method of Undetermined Coefficients

The method of undetermined coefficients involves assuming a particular solution y_p based on the form of the nonhomogeneous term $Q(x)$.

Analyzing the Nonhomogeneous Term

Given:

$$Q(x) = 2x - 8$$

This is a polynomial of degree 1 in x. The method suggests guessing a particular solution y_p that is also a polynomial of degree 1:

$$y_p = A x + B$$

where A and B are coefficients to be determined.

Deriving y_p'

Differentiate y_p:

$$y_p' = A$$

Substituting into the Differential Equation

Recall the differential equation in standard form:

$$y' - 3 y = 2x - 8$$

Substitute y_p and y_p':

$$A - 3 (A x + B) = 2x - 8$$

Simplify:

$$A - 3A x - 3B = 2x - 8$$

Group like terms:

$$(-3A) x + (A - 3B) = 2x - 8$$

Equate Coefficients

Matching coefficients of like powers of x:

- For x:

$$-3A = 2 \Rightarrow A = -\frac{2}{3}$$

- For constants:

$$A - 3B = -8$$

Substitute $A = -\frac{2}{3}$:

$$-\frac{2}{3} - 3B = -8$$

Solve for B:

$$-3B = -8 + \frac{2}{3}$$

Express -8 as $-\frac{24}{3}$:

$$-3B = -\frac{24}{3} + \frac{2}{3} = -\frac{22}{3}$$

Divide both sides by -3:

$$B = \frac{22}{3} \times \frac{1}{3} = \frac{22}{9}$$

Particular Solution

Thus, the particular solution is:

$$y_p = -\frac{2}{3} x + \frac{22}{9}$$

Step 3: Write the General Solution

The overall solution combines the homogeneous and particular solutions:

$$y(x) = y_h + y_p$$

$$y(x) = C e^{3x} - \frac{2}{3} x + \frac{22}{9}$$

where C is an arbitrary constant determined by initial conditions if provided.

Step 4: Verify the Solution (Optional but Recommended)

Verification ensures that the derived solution satisfies the original differential equation.

Compute y':

$$ y' = 3 C e^{3x} - \frac{2}{3} $$

Substitute into the original equation:

$$ y' - 3 y = 2x - 8 $$

$$ \left(3 C e^{3x} - \frac{2}{3} \right) - 3 \left(C e^{3x} - \frac{2}{3} x + \frac{22}{9} \right) $$

Simplify:

$$ 3 C e^{3x} - \frac{2}{3} - 3 C e^{3x} + 2 x - \frac{66}{9} $$

$$ (3 C e^{3x} - 3 C e^{3x}) + (- \frac{2}{3}) + 2 x - \frac{66}{9} $$

$$ 0 + (- \frac{2}{3}) + 2 x - \frac{66}{9} $$

Express $\frac{66}{9}$ as $\frac{22}{3}$:

$$ - \frac{2}{3} + 2 x - \frac{22}{3} = 2x - \frac{2}{3} - \frac{22}{3} = 2x - \frac{24}{3} = 2x - 8 $$

which matches the nonhomogeneous term, confirming the solution’s correctness.

Additional Considerations

Handling More Complex Nonhomogeneous Terms

While the above method works efficiently for polynomial nonhomogeneous terms, more complicated functions such as exponentials, sines, and cosines require different ansatz forms. For example:

- For exponential functions e^{ax}, try $y_p = K e^{ax}$.
- For sines and cosines, try $y_p = A \sin bx + B \cos bx$.

Limitations of the Method of Undetermined Coefficients

This method is most effective when the nonhomogeneous term is a polynomial, exponential, sine, or cosine. It is not suitable for arbitrary functions, where the Variation of Parameters method might be necessary.

Conclusion

Applying the Method of Undetermined Coefficients to solve nonhomogeneous differential equations streamlines the process of finding particular solutions, especially when the nonhomogeneous term is a polynomial, exponential, or sinusoidal function. The key steps involve:

1. Solving the homogeneous equation to find the complementary solution.
2. Guessing a particular solution based on the form of the nonhomogeneous term.
3. Determining the coefficients by substitution.
4. Combining solutions to obtain the general solution.

For the specific differential equation:

$$\frac{dy}{dx} = 2x + 3y - 8$$

we successfully derived the general solution:

$$y(x) = C e^{3x} - \frac{2}{3} x + \frac{22}{9}$$

This comprehensive approach not only provides the solution but also enhances understanding of the underlying principles, making it a valuable technique for solving a wide range of linear differential equations in applied mathematics and engineering.

Keywords for SEO Optimization

- Method of Undetermined Coefficients
- Nonhomogeneous Differential Equations
- Solving Differential Equations Step-by

Frequently Asked Questions

What is the method of undetermined coefficients used for in solving differential systems?

The method of undetermined coefficients is used to find particular solutions to nonhomogeneous differential equations or systems with constant coefficients by guessing a form of the particular solution and solving for its unknown parameters.

How do you set up the particular solution when using the method of undetermined coefficients for a system like Dx = 2x + 3y - 8?

You assume a particular solution with an appropriate form (e.g., polynomials, exponentials, or combinations) based on the nonhomogeneous term, then substitute into the system to solve for the undetermined coefficients.

What is the role of the homogeneous solution in solving the system using the method of undetermined coefficients?

The homogeneous solution solves the associated system with zero on the right-hand side, providing the complementary solution. The particular solution, found via undetermined coefficients, accounts for the nonhomogeneous part. The general solution is the sum of both.

How do you handle the nonhomogeneous term $2x + 3y - 8$ when applying the method of undetermined coefficients?

Since the nonhomogeneous term is a polynomial, you typically assume a polynomial form for the particular solution with unknown coefficients and then determine these by substituting back into the system.

Can the method of undetermined coefficients be used for all types of nonhomogeneous systems?

No, it works best when the nonhomogeneous term is of a form (e.g., polynomial, exponential, sine, cosine) that matches the form of the assumed particular solution. For other types, variation of parameters may be more appropriate.

What are the steps involved in applying the method of undetermined coefficients to the given system?

First, solve the homogeneous system to find the complementary solution. Second, assume a form for the particular solution based on the nonhomogeneous term. Third, substitute into the original system and solve for the unknown coefficients. Finally, combine the solutions for the general solution.

How do you verify the correctness of the particular solution obtained

through the method of undetermined coefficients?

Substitute the particular solution back into the original nonhomogeneous system to check if the equations are satisfied, ensuring the solution correctly accounts for the nonhomogeneous part.

What challenges might arise when applying the method of undetermined coefficients to a system like Dx = 2x + 3y - 8?

Challenges include choosing the correct form for the particular solution, especially if the nonhomogeneous term is complex or involves functions outside typical polynomial or exponential forms, and solving for multiple unknown coefficients simultaneously.

How does the method of undetermined coefficients relate to solving real-world problems modeled by differential systems?

It provides a systematic way to find particular solutions to systems with specific nonhomogeneous inputs, enabling accurate modeling of phenomena in engineering, physics, and other sciences where such systems frequently arise.

Additional Resources

Use The Method Of Undetermined Coefficients To Solve The Given Nonhomogeneous System. = 2x + 3y - 8 Dx

In the realm of differential equations, the method of undetermined coefficients stands out as a powerful and systematic approach for solving certain classes of nonhomogeneous linear differential systems. When faced with a system characterized by constant coefficients and a nonhomogeneous term—such as the one given by the equation $\frac{dy}{dx} = 2x + 3y - 8$—this method offers a structured pathway to uncover the particular solution, completing the overall general solution. This article explores the process in detail, providing clarity on each step, and illustrating how to apply the method effectively to such systems.

Understanding the System and Its Components

Before diving into the solution process, it’s vital to understand the structure of the differential system at hand. Although the initial statement appears somewhat cryptic, it can be interpreted as a first-order linear differential equation with a nonhomogeneous term. For clarity, consider the system in the standard form:

$$
\frac{dy}{dx} + P(x)y = Q(x)
$$

where:

- $P(x)$ is a coefficient function associated with y,
- $Q(x)$ is the nonhomogeneous term—here, $2x - 8$,
- and the system involves the variable y.

Given the expression, it appears that the differential equation can be rewritten as:

$$
\frac{dy}{dx} - 3y = 2x - 8
$$

which is a linear first-order ODE with constant coefficients.

Note: If the original problem involves a system with multiple equations or variables, the same principles extend with matrices, but for simplicity, this discussion assumes a single-equation context.

Step 1: Solving the Homogeneous Equation

The first step in the method of undetermined coefficients is solving the associated homogeneous equation:

$$
\frac{dy}{dx} - 3y = 0
$$

This is straightforward:

$$
\frac{dy}{dx} = 3y
$$

which can be separated:

$$
\frac{dy}{y} = 3\, dx
$$

Integrating both sides:

$$
\ln|y| = 3x + C
$$

Exponentiating:

$$
y_h = Ce^{3x}
$$

\]

where C is an arbitrary constant. This homogeneous solution forms the basis for the general solution once the particular solution is found.

Step 2: Identifying the Form of the Particular Solution

The core of the method of undetermined coefficients involves guessing a form for the particular solution y_p, based on the nonhomogeneous term $Q(x)$. Since $Q(x) = 2x - 8$ is a polynomial of degree 1, the standard approach suggests trying a polynomial of the same degree:

$$
y_p = Ax + B
$$

where A and B are coefficients to be determined.

Important note: If the nonhomogeneous term involved exponential, sine, cosine, or more complicated functions, the guess for y_p would be adjusted accordingly. But polynomials are used when the nonhomogeneous part is polynomial.

Step 3: Computing the Derivative of the Guess

Next, differentiate y_p:

$$
\frac{dy_p}{dx} = A
$$

Substitute y_p and $\frac{dy_p}{dx}$ into the original differential equation:

$$
A - 3(Ax + B) = 2x - 8
$$

which simplifies to:

$$
A - 3A x - 3B = 2x - 8
$$

Rearranged:

$$
-3A x + (A - 3B) = 2x - 8
$$

Matching coefficients of like terms:

- For x:

$$
-3A = 2 \Rightarrow A = -\frac{2}{3}
$$

- For the constant term:

$$
A - 3B = -8
$$

Substitute $A = -\frac{2}{3}$:

$$
-\frac{2}{3} - 3B = -8
$$

Solve for B:

$$
-3B = -8 + \frac{2}{3}
$$

Expressing -8 as $-\frac{24}{3}$:

$$
-3B = -\frac{24}{3} + \frac{2}{3} = -\frac{22}{3}
$$

Divide both sides by -3:

$$
B = \frac{22}{9}
$$

Therefore, the particular solution:

$$
y_p = -\frac{2}{3} x + \frac{22}{9}
$$

Step 4: Constructing the General Solution

The general solution combines the homogeneous and particular solutions:

$$
y(x) = y_h + y_p = C e^{3x} - \frac{2}{3} x + \frac{22}{9}
$$

where C is an arbitrary constant determined by initial conditions or boundary values.

Applying the Method of Undetermined Coefficients: Key Takeaways

The process highlighted above encapsulates the core principles of the method of undetermined coefficients:

- Identify the type of nonhomogeneous term: Polynomial, exponential, sinusoidal, etc.
- Guess an appropriate form for the particular solution: Based on the nonhomogeneous term.
- Differentiate and substitute into the original equation to determine unknown coefficients.
- Solve the algebraic equations for these coefficients.
- Combine with the homogeneous solution to form the general solution.

This method is especially effective for linear differential equations with constant coefficients and nonhomogeneous parts that are polynomials, exponentials, or sinusoids.

Extensions and Practical Considerations

While the method of undetermined coefficients offers a straightforward solution path for many problems, it does have limitations and extensions:

- When the nonhomogeneous term overlaps with the homogeneous solution, the guessed form must be multiplied by x to account for duplication.
- For more complex nonhomogeneous terms, alternative methods such as variation of parameters may be preferred.
- Systems of equations: The method extends to systems by guessing vector solutions with similar forms, often involving matrices and eigenvalues.

In practical applications, this method is invaluable in engineering, physics, and applied mathematics, where solving differential systems accurately and efficiently is essential.

Conclusion

The method of undetermined coefficients provides a systematic framework for solving nonhomogeneous linear differential equations with constant coefficients. In the specific case of the equation $\frac{dy}{dx} = 2x + 3y - 8$, the process involves solving the homogeneous part, guessing an appropriate form for the particular solution based on the nonhomogeneous term, and then determining the unknown coefficients through substitution and algebraic solving. The resulting general solution combines the exponential homogeneous solution with the polynomial particular solution, offering a complete picture of the system's behavior. Mastery of this approach equips mathematicians and engineers with a reliable tool for tackling a broad class of differential problems, ensuring precise modeling and analysis of dynamic systems.

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