

Using The Bernoulli Equation, Show That For An Incompressible Substance, The Pressure Must Decrease In

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Introduction

The Bernoulli equation is a fundamental principle in fluid dynamics that describes the conservation of energy in a flowing fluid. It provides a powerful tool for analyzing how fluid velocity, pressure, and elevation relate to each other in idealized conditions. One notable application of the Bernoulli equation is demonstrating that, for an incompressible fluid, an increase in flow velocity corresponds to a decrease in pressure. This principle underpins many practical phenomena, from airplane lift to the operation of Venturi tubes. In this article, we explore how the Bernoulli equation leads to the conclusion that pressure must decrease as the velocity of an incompressible fluid increases.

Understanding the Bernoulli Equation

Formulation of the Bernoulli Equation

The Bernoulli equation for a steady, incompressible, non-viscous flow along a streamline is expressed as:

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

where:

- P = pressure at a point in the fluid
- ρ = density of the fluid (constant for incompressible fluids)
- v = fluid velocity at that point
- g = acceleration due to gravity
- h = elevation height relative to a reference point

This equation states that the sum of the static pressure energy, the kinetic energy per unit volume,

and the potential energy per unit volume remains constant along a streamline.

Physical Interpretation

- Static pressure (P): Energy stored in the fluid due to its pressure.
- Dynamic pressure ($\frac{1}{2} \rho v^2$): Energy associated with the fluid's motion.
- Hydrostatic pressure ($\rho g h$): Potential energy related to elevation in a gravitational field.

By analyzing these terms, we can understand how changes in one form of energy affect the others.

Applying the Bernoulli Equation to Incompressible Fluids

Assumptions for Incompressibility

Incompressible fluids have a constant density (ρ) regardless of pressure or temperature changes. Liquids like water are often treated as incompressible for engineering calculations, which simplifies the Bernoulli equation's application.

Key assumptions include:

- No change in fluid density (ρ is constant).
- Flow is steady (does not change with time).
- No viscosity or energy losses.
- Flow is along a streamline.

Implication of Incompressibility on Pressure and Velocity

Because density remains constant, any variation in velocity must be accompanied by an inverse variation in pressure and/or elevation to satisfy the Bernoulli equation. Specifically, an increase in velocity results in a decrease in pressure, assuming elevation remains unchanged.

Demonstrating the Pressure Decrease with Increasing Velocity

Consider Two Points in a Steady Flow

Suppose fluid flows through a pipe with two points, Point 1 and Point 2, aligned along a streamline. The properties at these points are:

Property	Point 1	Point 2
Pressure	P_1	P_2
Velocity	v_1	v_2
Height	h_1	h_2

Assuming no significant elevation difference ($h_1 \approx h_2$), the Bernoulli equation simplifies to:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

Rearranged as:

$$P_2 - P_1 = \frac{1}{2} \rho (v_1^2 - v_2^2)$$

Analyzing the Relationship Between Pressure and Velocity

If the flow accelerates ($v_2 > v_1$), then:

$$v_2^2 > v_1^2 \Rightarrow v_1^2 - v_2^2 < 0$$

which implies:

$$P_2 - P_1 < 0$$

or

$$P_2 < P_1$$

This confirms that an increase in velocity ($v_2 > v_1$) leads to a decrease in static pressure ($P_2 < P_1$).

Case Study: The Venturi Effect

Principle Demonstration

The Venturi tube is a classic example illustrating the pressure-velocity relationship. When fluid flows through a constricted section of pipe, its velocity increases in that section due to the reduced cross-sectional area.

Key observations:

- The velocity in the narrow section increases.
- Bernoulli's principle predicts a corresponding drop in static pressure.
- The pressure difference can be measured and validated experimentally.

Mathematical Explanation

Using the Bernoulli equation between the wider section (point 1) and the throat (point 2):

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

Given the conservation of mass:

$$A_1 v_1 = A_2 v_2$$

where A_1 and A_2 are cross-sectional areas at points 1 and 2, respectively.

Since $A_2 < A_1$:

$$v_2 = \frac{A_1}{A_2} v_1 > v_1$$

Thus, $(v_2 > v_1)$, leading to:

$$P_2 < P_1$$

indicating the pressure decreases in the constricted section.

Physical Implications and Real-World Applications

Aircraft Lift

The Bernoulli principle is often invoked to explain how air flowing faster over the curved upper surface of an airplane wing results in lower pressure above the wing, generating lift.

Flow Measurement Devices

Devices like Venturi meters and orifice plates utilize the pressure-velocity relationship to measure fluid flow rates accurately.

Hydraulic and Plumbing Systems

Understanding how pressure decreases with increased flow velocity helps in designing piping systems to prevent cavitation and ensure efficient operation.

Limitations of the Bernoulli Equation

While the Bernoulli equation provides valuable insights, it has limitations:

- It assumes inviscid flow; real fluids have viscosity.
- It neglects energy losses due to turbulence and friction.
- It applies strictly along a streamline; different streamlines can have different total energies.
- It assumes steady flow; unsteady flows require more complex analysis.

Despite these limitations, the Bernoulli equation remains a foundational tool for qualitative and quantitative analysis of fluid behavior.

Conclusion

Using the Bernoulli equation, we have demonstrated that for an incompressible substance, an increase in flow velocity must be accompanied by a decrease in static pressure, provided elevation

remains constant. This inverse relationship underpins many fluid dynamic phenomena and engineering applications. The mathematical derivation, supported by practical examples such as the Venturi effect, confirms that the pressure in a flowing incompressible fluid decreases as the velocity increases. Understanding this relationship is crucial for designing efficient fluid systems, predicting flow behavior, and explaining natural phenomena.

References

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Frequently Asked Questions

How does the Bernoulli equation demonstrate that pressure decreases in an incompressible fluid when velocity increases?

The Bernoulli equation states that along a streamline, the sum of pressure energy, kinetic energy, and potential energy remains constant. When the fluid velocity increases, the kinetic energy term rises, which must be balanced by a decrease in the pressure energy term, leading to a pressure decrease.

Why does an increase in fluid speed cause a drop in pressure according to Bernoulli's principle?

Because the Bernoulli equation conserves energy, an increase in kinetic energy (due to higher speed) requires a corresponding decrease in static pressure energy, resulting in lower pressure in regions of higher velocity.

Can Bernoulli's equation be used to explain pressure drops in incompressible flow through a narrowing pipe?

Yes, Bernoulli's equation shows that as the cross-sectional area decreases, the fluid speeds up, and the static pressure decreases, illustrating the pressure drop in the constricted section.

What assumptions are necessary for Bernoulli's equation to show that pressure decreases in an incompressible fluid?

The assumptions include steady, incompressible, non-viscous flow along a streamline, with no energy added or lost due to work or heat transfer, ensuring energy conservation and valid application of Bernoulli's principle.

Does Bernoulli's equation account for viscosity and energy losses in real fluids?

No, Bernoulli's equation assumes ideal, inviscid flow. In real fluids with viscosity, additional energy losses occur, modifying the pressure distribution compared to the ideal case.

How does the concept of conservation of energy relate to pressure decrease in an incompressible fluid flow?

Conservation of energy implies that as kinetic energy increases (due to higher velocity), static pressure must decrease to maintain total energy constant along a streamline, demonstrating the inverse relationship.

What physical situations illustrate the pressure decrease in incompressible fluids predicted by Bernoulli's equation?

Examples include flow over an airplane wing (lift generation), flow through Venturi tubes, and water flowing faster through a narrowed pipe section, all showing a pressure decrease where velocity increases.

How is the pressure decrease in an incompressible fluid related to the fluid's velocity according to Bernoulli's equation?

The equation indicates an inverse relationship: as velocity increases, static pressure decreases, maintaining the total energy constant along the streamline.

Why is Bernoulli's equation particularly useful in analyzing incompressible flow scenarios?

Because it simplifies the analysis by relating pressure, velocity, and elevation without considering viscosity, making it ideal for understanding pressure variations in steady, incompressible, inviscid flows.

Can Bernoulli's equation be used to predict pressure drops in real-world applications involving incompressible fluids?

Yes, within its assumptions, Bernoulli's equation effectively predicts pressure drops in practical situations like pipe flows and aerodynamic lift, although real-world factors like viscosity may require additional considerations.

Additional Resources

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Introduction

Fluid dynamics, a cornerstone of classical physics and engineering, examines how fluids (liquids and gases) behave in various conditions. Among the fundamental tools in this field is the Bernoulli equation, a principle derived from the conservation of energy for flowing fluids. This equation provides critical insights into the relationship between a fluid's velocity, pressure, and height, particularly under idealized conditions. A key implication of the Bernoulli equation is its prediction that, for an incompressible fluid, an increase in flow velocity corresponds to a decrease in pressure along the streamline. This phenomenon underpins many practical applications, from airplane lift generation to fluid transport systems. This article delves into the mathematical derivation of Bernoulli's principle, demonstrating how it leads to the conclusion that pressure must decrease when fluid speed increases in an incompressible medium.

Background and Fundamentals of Bernoulli's Equation

The Principle of Conservation of Energy in Fluids

At its core, Bernoulli's equation is a manifestation of the conservation of energy principle tailored for fluid flow. It states that within a steady, incompressible, non-viscous, and irrotational flow, the sum of three forms of energy per unit volume remains constant along a streamline:

- Pressure energy: Related to the static pressure exerted by the fluid.
- Kinetic energy: Associated with the fluid's velocity.
- Potential energy: Due to elevation in the gravitational field.

Mathematically, Bernoulli's equation can be expressed as:

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

where:

- P = static pressure
- ρ = fluid density
- v = flow velocity
- g = acceleration due to gravity
- h = elevation height

This compact relation encapsulates the energy conservation principle, linking observable quantities such as pressure and velocity.

Assumptions Underlying Bernoulli's Equation

The derivation and application of Bernoulli's equation rest on several key assumptions:

- Incompressibility: The fluid's density remains constant regardless of pressure or temperature changes.
- Steady flow: Fluid properties at a point do not change over time.

- Non-viscous (Ideal) fluid: Viscous effects such as friction are neglected.
- Irrotational flow: The flow has zero vorticity, meaning it is free of rotational motion.
- Along a streamline: The equation holds true along a single streamline, not necessarily across different streamlines.

While these assumptions simplify the mathematics and help derive the relation, real-world scenarios often involve deviations, especially concerning viscosity and compressibility. Nonetheless, for many liquids and slow-moving flows, the approximation provides powerful insights.

Mathematical Derivation and Analytical Framework

Starting Point: The Bernoulli Equation

Consider a steady, incompressible, inviscid flow along a streamline between two points, labeled 1 and 2. The Bernoulli equation states:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

This relation asserts that the total mechanical energy per unit volume remains constant between these two points.

Isolating Pressure Differences

Rearranging the equation to express the pressure difference:

$$P_2 - P_1 = \frac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (h_1 - h_2)$$

This form clearly demonstrates the relationship between pressure and velocity changes, as well as elevation differences.

Focusing on Horizontal Flow

In many practical applications, flow occurs horizontally, and the change in height ($h_1 - h_2$) is negligible:

$$h_1 \approx h_2 \Rightarrow \rho g (h_1 - h_2) \approx 0$$

Thus, the simplified Bernoulli equation reduces to:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

or equivalently,

$$P_2 - P_1 = \frac{1}{2} \rho (v_1^2 - v_2^2)$$

Demonstration of Pressure Decrease with Increasing Velocity

The Core Concept

From the simplified Bernoulli equation, it becomes evident that an increase in velocity ($v_2 > v_1$) must be accompanied by a decrease in pressure ($P_2 < P_1$) to satisfy the energy conservation:

$$P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2)$$

Given that $v_2 > v_1$, the term $(v_1^2 - v_2^2)$ is negative, leading to:

$$P_2 < P_1$$

This inequality proves that higher flow velocities are associated with lower static pressures within an incompressible fluid.

Physical Interpretation

This inverse relationship can be intuitively understood through the analogy of energy transfer:

- When a fluid accelerates, kinetic energy increases.
- Since the total energy per unit volume remains constant (per Bernoulli's principle), an increase in kinetic energy must be balanced by a reduction in other energy forms—in this case, static pressure.

This insight explains phenomena such as the lift generated on an airplane wing, where the airflow over the curved upper surface accelerates, decreasing pressure and creating lift.

Practical Implications and Applications

Aerodynamics and Lift Generation

One of the most celebrated applications of Bernoulli's principle is in aerodynamics. The design of airplane wings exploits the shape that causes air to accelerate over the top surface, leading to a pressure difference that produces lift. The decrease in pressure over the curved upper surface, relative to the lower surface, is directly attributable to the increase in velocity over the top, as predicted by Bernoulli's equation.

Venturi Effect and Fluid Measurement

The Venturi tube, a device used in flow measurement, demonstrates the pressure decrease in a constricted section of a pipe. As the cross-sectional area narrows, the fluid velocity increases, causing a drop in pressure. This pressure differential is then used to calculate flow rates, exemplifying the practical relevance of Bernoulli's principle.

Hydraulic and Plumbing Systems

In pipelines, understanding the pressure-velocity relationship guides the design of efficient systems. For example, in water distribution networks, areas of high velocity are carefully managed to prevent excessive pressure drops, which could lead to cavitation or pipe failure.

Limitations and Real-World Considerations

While Bernoulli's equation provides a powerful theoretical framework, real-world applications often encounter deviations due to:

- Viscous Effects: Frictional forces dissipate energy, leading to pressure losses not accounted for in the idealized Bernoulli equation.
- Compressibility: For gases or high-speed flows approaching Mach numbers, density changes become significant, invalidating the incompressibility assumption.
- Turbulence: Irregular, chaotic flow patterns introduce complexities beyond the laminar assumptions underlying Bernoulli's derivation.
- Flow Unsteadiness: Fluctuations over time can cause local variations in pressure and velocity.

Engineers and physicists often incorporate correction factors or utilize more comprehensive equations—such as the Navier-Stokes equations—to address these limitations.

Conclusion

Through a detailed examination of Bernoulli's equation, it is clear that, within the framework of an incompressible, steady, and non-viscous flow, an increase in fluid velocity necessarily results in a decrease in static pressure along the streamline. This inverse relationship is rooted in the conservation of mechanical energy, manifesting physically in various natural and engineered systems. Recognizing the conditions and assumptions behind Bernoulli's principle enables practitioners to harness its insights effectively, while also understanding its limitations in complex, real-world scenarios. Ultimately, the elegant simplicity of Bernoulli's equation continues to illuminate fundamental aspects of fluid behavior, reinforcing its status as a cornerstone of fluid mechanics.

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