What Are The Coordinates Of The Image Of ABC After A Dilation With Center (0, 0) And A Scale Factor Of

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Understanding how geometric transformations work is fundamental in geometry, particularly when dealing with dilations. When a figure undergoes a dilation with a specific center and scale factor, its size changes proportionally, but its shape remains similar. One common question involves determining the new coordinates of the vertices of a figure after such a dilation. In this article, we will explore how to find the coordinates of the image of triangle ABC after a dilation centered at (0, 0) with a given scale factor.

Understanding Dilation in Coordinate Geometry

What Is Dilation?

Dilation is a transformation that enlarges or reduces a figure by a scale factor relative to a fixed point called the center of dilation. The key properties of dilation include:

- The shape remains similar to the original.
- The size changes according to the scale factor.
- The center of dilation remains fixed.

Center and Scale Factor

In our context:

- The center of dilation is at the point (0, 0).
- The scale factor (k) determines how much the figure is enlarged or reduced.

If $k > 1$, the figure enlarges; if $0 < k < 1$, it reduces; if $k = 1$, the figure remains unchanged.

Mathematical Formula for Dilation

Given a point $P(x, y)$, the image P' after dilation with center at (0, 0) and scale factor k has coordinates:

$$
P' (x', y') = (k \times x, k \times y)
$$

This simple formula applies because the center of dilation is at the origin, making the transformation straightforward.

Applying Dilation to Triangle ABC

Suppose the vertices of triangle ABC are:

- $A(x_A, y_A)$
- $B(x_B, y_B)$
- $C(x_C, y_C)$

To find the image of each point after dilation, apply the formula:

$$
A' = (k \times x_A, k \times y_A)
$$
$$
B' = (k \times x_B, k \times y_B)
$$
$$
C' = (k \times x_C, k \times y_C)
$$

The transformed triangle $A'B'C'$ is the dilation image of triangle ABC.

Step-by-Step Example

Let's consider a specific example for clarity.

Given Data

- Coordinates of triangle ABC:

- $A(2, 3)$
- $B(4, 1)$
- $C(1, -2)$

- Center of dilation: (0, 0)
- Scale factor: $k = 3$

Calculating the Image Coordinates

Applying the formula:

$$
A' = (3 \times 2, 3 \times 3) = (6, 9)
$$
$$
B' = (3 \times 4, 3 \times 1) = (12, 3)
$$
\[

$$
C' = (3 \times 1, 3 \times -2) = (3, -6)
$$

Thus, the image of triangle ABC after dilation is $A'(6, 9)$, $B'(12, 3)$, and $C'(3, -6)$.

Effects of Different Scale Factors

The scale factor significantly influences the size of the dilated figure:

- Scale factor greater than 1 ($k > 1$): The figure enlarges proportionally.
- Scale factor between 0 and 1 ($0 < k < 1$): The figure reduces in size.
- Negative scale factor ($k < 0$): The figure is scaled and reflected across the origin.

Example with a Negative Scale Factor

If $k = -2$, then:

$$
A' = (-2 \times 2, -2 \times 3) = (-4, -6)
$$
$$
B' = (-2 \times 4, -2 \times 1) = (-8, -2)
$$
$$
C' = (-2 \times 1, -2 \times -2) = (-2, 4)
$$

This results in a dilation that scales the figure by 2 and reflects it across the origin.

Special Cases and Considerations

When the Figure Is Located Near the Origin

If the vertices of ABC are close to (0, 0), dilation will proportionally increase or decrease their distances from the origin, magnifying or reducing the entire figure accordingly.

When the Center of Dilation Is Not at (0, 0)

In scenarios where the center is at a different point (say (h, k)), the transformation involves a different formula:

$$
P' = (h + k \times (x - h), \, k + k \times (y - k))

\]

But since our focus is on dilation centered at (0, 0), this more complex formula isn't necessary here.

Summary and Key Takeaways

- To find the image of a triangle after dilation centered at (0, 0), multiply each coordinate of its vertices by the scale factor.
- The formula $P' = (k \times x, k \times y)$ simplifies the process when the center is at the origin.
- The scale factor determines whether the figure enlarges, reduces, or reflects.
- Dilation preserves shape but alters size proportionally.

Practical Applications of Coordinate Dilation

Understanding coordinate dilation is essential in various fields such as:

- Computer graphics
- Engineering design
- Architecture
- Mathematical visualization

It allows for precise transformations and scaling of figures and models based on simple algebraic formulas.

Conclusion

In conclusion, finding the coordinates of the image of triangle ABC after a dilation with center (0, 0) and a specific scale factor is straightforward using the basic multiplication rule. By applying the formula to each vertex, you can accurately determine the new coordinates, enabling a clear understanding of how geometric figures transform under dilation. Mastery of this concept provides a strong foundation for exploring more complex geometric transformations and their applications in real-world scenarios.

Frequently Asked Questions

How do you find the coordinates of the image of point A after a dilation with center at the origin and a given scale factor?

To find the image of point A(x, y) after a dilation centered at (0, 0) with scale factor k, multiply both coordinates by k: the image will be at (kx, ky).

What is the significance of the scale factor in dilation transformations of a triangle like ABC?

The scale factor determines how much the figure is enlarged or reduced. A scale factor greater than 1 enlarges the figure, less than 1 shrinks it, and equal to 1 leaves it unchanged.

If point A has coordinates (2, 3) and the dilation scale factor is 4 with center at (0, 0), what are the new coordinates of A?

The new coordinates are (8, 12), obtained by multiplying each original coordinate by 4.

Are the angles of triangle ABC preserved after a dilation, and what about the side lengths?

Angles are preserved because dilation is a similarity transformation, but side lengths are scaled by the dilation's scale factor.

Can a dilation with center at (0, 0) change the shape of the image, and why?

No, because dilation with respect to the origin is a similarity transformation, which preserves the shape and angles but changes the size.

Additional Resources

What Are The Coordinates Of The Image Of ABC After A Dilation With Center (0, 0) And A Scale Factor Of

Understanding the transformation of geometric figures is fundamental in the study of coordinate geometry. Among these transformations, dilation (or scaling) is a pivotal concept that allows us to enlarge or reduce figures relative to a fixed point called the center of dilation. This article aims to comprehensively explore the process of determining the coordinates of an image of a triangle — specifically, triangle ABC — after a dilation centered at the origin (0, 0) with a given scale factor. Through detailed explanations, mathematical formulas, and illustrative examples, we will elucidate the steps necessary to find the transformed coordinates and analyze the implications of such transformations.

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1. Fundamentals of Dilation in Coordinate Geometry

What Is Dilation?

Dilation is a transformation that alters the size of a figure without changing its shape. When a figure undergoes dilation, it is either enlarged or reduced based on a specific scale factor, but its angles remain unchanged, preserving similarity. The transformation is defined relative to a fixed point called the center of dilation.

Key Characteristics:
- The shape of the figure remains similar to the original.
- The size of the figure changes proportionally based on the scale factor.
- The position of the figure relative to the center of dilation determines the location of the image.

Center of Dilation: The Origin (0, 0)

In many problems, the center of dilation is the origin (0, 0). This simplifies calculations because the transformation involves straightforward multiplication of coordinates. When the center is at (0, 0), each point (x, y) in the original figure maps to a new point (x', y') according to a simple rule involving the scale factor.

Scale Factor (k)

The scale factor (k) dictates how much the figure is enlarged or reduced:
- If |k| > 1, the figure enlarges.
- If |k| < 1, the figure reduces.
- If k = 1, the figure remains unchanged.
- If k = 0, the figure collapses to a single point at the center.

The sign of k also indicates orientation:
- Positive k maintains the original orientation.
- Negative k reflects the figure across the origin.

---

2. Mathematical Representation of Dilation at the Origin

Coordinate Transformation Formula

For a dilation centered at the origin with scale factor k, each point (x, y) in the original figure maps to a point (x', y') in the image as follows:

\[
x' = k \times x
\]
\[
y' = k \times y
\]

This simple formula stems from the fact that the distance of a point from the origin is scaled by k, and the direction remains the same.

Implication for Triangle ABC

Suppose triangle ABC has vertices:
- $A(x_A, y_A)$
- $B(x_B, y_B)$
- $C(x_C, y_C)$

Applying the dilation with center at (0, 0) and scale factor k, the image triangle A'B'C' will have vertices:
- $A'(k \times x_A, k \times y_A)$
- $B'(k \times x_B, k \times y_B)$
- $C'(k \times x_C, k \times y_C)$

This straightforward multiplicative process makes the calculation of the new coordinates intuitive and efficient.

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3. Step-by-Step Process for Finding Image Coordinates

Step 1: Identify the Coordinates of Original Triangle ABC

Begin by noting the coordinates of the original triangle’s vertices:
- $A(x_A, y_A)$
- $B(x_B, y_B)$
- $C(x_C, y_C)$

Ensure these are correctly identified, as errors here propagate through subsequent calculations.

Step 2: Determine the Scale Factor (k)

Obtain the value of the scale factor provided in the problem. The scale factor may be a positive or negative real number, which affects the size and orientation of the image.

Step 3: Apply the Dilation Formula to Each Vertex

Multiply each coordinate by the scale factor:

$$
A'(x'_A, y'_A) = (k \times x_A, k \times y_A)
$$
$$
B'(x'_B, y'_B) = (k \times x_B, k \times y_B)
$$
$$
C'(x'_C, y'_C) = (k \times x_C, k \times y_C)
$$

This step yields the coordinates of the transformed vertices.

Step 4: Verify and Visualize

Plot the original and transformed points to visualize the dilation effect. Confirm that the image maintains the same shape and proportions, scaled according to the scale factor.

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4. Illustrative Example

Original Coordinates of Triangle ABC

Suppose triangle ABC has vertices:
- $A(2, 3)$
- $B(4, 1)$
- $C(1, -2)$

Given Scale Factor

Let’s assume the scale factor $k = 3$.

Calculating the Image Coordinates

Applying the dilation formula:

$$
A' = (3 \times 2, 3 \times 3) = (6, 9)
$$
$$
B' = (3 \times 4, 3 \times 1) = (12, 3)
$$
$$
C' = (3 \times 1, 3 \times -2) = (3, -6)
$$

The image triangle A'B'C' has vertices:
- $A'(6, 9)$
- $B'(12, 3)$
- $C'(3, -6)$

This enlarged triangle is three times the size of the original, maintaining similar angles and shape.

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5. Special Cases and Considerations

Negative Scale Factors

A negative scale factor reflects the figure across the origin, in addition to scaling its size. For example, if $k = -2$, the points are multiplied by -2, which not only doubles the size but also flips the figure to the opposite side of the origin.

Example:
- $A(2, 3)$ becomes $A'(-4, -6)$.

Scale Factor of Zero

When $k=0$, all points collapse to the center of dilation, which is (0, 0), transforming the entire figure into a single point.

Implications for Geometric Properties

- The dilation preserves angles and shape, making the image similar to the original.
- The ratios of corresponding segments are equal to the scale factor.
- The distances from the center are scaled by $|k|$.

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6. Applications and Significance in Real-world Contexts

Engineering and Design

Understanding how to scale figures accurately is crucial in engineering drawings, architectural plans, and product design, where precise transformations are necessary.

Computer Graphics and Animation

Dilation principles are fundamental in computer graphics for zooming effects, object scaling, and transformations within virtual environments.

Mathematical Education and Problem Solving

Mastering dilation helps students grasp similarity, proportional reasoning, and the properties of geometric figures, laying the groundwork for advanced mathematical concepts.

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7. Conclusion: The Power of Coordinate-Based Dilation

In summary, determining the coordinates of the image of triangle ABC after a dilation centered at (0, 0) with a given scale factor involves a straightforward application of multiplication to each coordinate. This transformation preserves the shape's similarity while adjusting its size proportionally, making it a vital concept in both theoretical mathematics and practical applications. Understanding the precise steps and implications of dilation enhances one's ability to analyze geometric figures effectively, bridging the gap between abstract concepts and real-world visualizations.

Whether enlarging a design or reducing a model, the principles outlined in this article serve as a foundational tool in the mathematician's or engineer's toolkit, demonstrating the elegance and utility of coordinate transformations in geometry.

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