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When exploring the fundamentals of linear equations in coordinate geometry, a common question is how to determine the equation of a line based on specific conditions. In this case, we are asked to find the equation of a line that crosses the x-axis at 36 and is perpendicular to another given line, Y = -4/9 + 5. Understanding this problem involves several steps, including interpreting the given information, calculating slopes, and applying the perpendicularity condition. This article will guide you through the process in detail, offering a comprehensive explanation suitable for learners at various levels.

Understanding the Components of the Problem

Before diving into calculations, it's essential to understand each component of the problem and what it requires.

What Does It Mean for a Line to Cross the X-axis at 36?

- The line crossing the x-axis at 36 indicates that the point (36, 0) lies on the line.
- This point is called the x-intercept, and it provides a critical coordinate for constructing the line's equation.

What Does It Mean for a Line to Be Perpendicular to Another Line?

- Perpendicular lines have slopes that are negative reciprocals of each other.
- If the original line has a certain slope (m), the perpendicular line will have a slope of -1/m.
- Recognizing this relationship allows us to determine the slope of the line we need to find.

Interpreting the Given Line Equation Y = -4/9 + 5

- The equation appears to be written in a form that might be confusing, but it likely represents a linear

function.
- Usually, a line's equation in slope-intercept form is written as $y = mx + b$.
- The given expression, $Y = -4/9 + 5$, simplifies to a constant, which suggests a horizontal line, but this may be a misinterpretation.
- Alternatively, the line could be expressed as $y = (-4/9)x + 5$, which makes more sense in context.

Note: For clarity, we will assume the original line is $y = (-4/9)x + 5$, since that aligns with standard linear form and the context of perpendicularity.

Step-by-Step Solution to Find the Required Line Equation

With a clear understanding of the problem components, let's proceed step-by-step.

1. Confirm the Equation of the Given Line

- The original line is $y = (-4/9)x + 5$.
- Its slope (m_1) is $-4/9$.

2. Determine the Slope of the Perpendicular Line

- The slope of the line perpendicular to $y = (-4/9)x + 5$ is:

$$m_2 = -1 / m_1 = -1 / (-4/9) = 9/4$$

- So, the slope of our target line is $9/4$.

3. Find the Equation of the Perpendicular Line Passing Through (36, 0)

- We know the point (36, 0) lies on the new line, and its slope is $9/4$.
- Using the point-slope form:

$$y - y_1 = m(x - x_1)$$

where $(x_1, y_1) = (36, 0)$, $m = 9/4$.

- Substituting:

$y - 0 = (9/4)(x - 36)$

$y = (9/4)(x - 36)$

4. Simplify the Equation

- Distribute the slope:

$y = (9/4)x - (9/4)\ 36$

- Calculate:

$(9/4)\ 36 = 9\ (36/4) = 9\ 9 = 81$

- Therefore, the equation becomes:

$y = (9/4)x - 81$

Final Equation of the Line

The line that crosses the x-axis at 36 and is perpendicular to $y = (-4/9)x + 5$ is:

$$\mathbf{y = (9/4)x - 81}$$

This line has the desired properties: it passes through (36, 0) and is perpendicular to the given line.

Additional Insights and Applications

Understanding how to derive such equations has practical applications in various fields, including physics, engineering, and computer graphics. For example:

- **Designing Perpendicular Structures:** Engineers often need to find lines perpendicular to existing structures at specific points.
- **Graphing Lines and Relationships:** Visualizing relationships between different variables in data analysis.

- **Solving Optimization Problems:** Finding maximum or minimum values along specific constraints represented by lines.

Tips for Solving Similar Problems

- Always identify the slope of the given line first.
- Remember that the slope of a perpendicular line is the negative reciprocal.
- Use the point-slope form for equations when a point and slope are known.
- Simplify the equation to slope-intercept form for easy interpretation.

Conclusion

In summary, to find the equation of a line crossing the x-axis at 36 and perpendicular to the line y = (-4/9)x + 5, follow these steps:

1. Recognize the slope of the given line as -4/9.
2. Calculate the perpendicular slope as 9/4.
3. Use the point (36, 0) and the slope 9/4 to write the equation in point-slope form.
4. Simplify to obtain the slope-intercept form: y = (9/4)x - 81.

The final answer is:

y = (9/4)x - 81

This equation precisely describes the line that meets the specified conditions, illustrating fundamental principles of linear equations and their geometric relationships. Mastering these concepts enables you to solve a wide range of problems involving lines, slopes, and points in the coordinate plane.

Frequently Asked Questions

What is the slope of the line that is perpendicular to the line y = -4/9x + 5?

The slope of the given line is -4/9, so the perpendicular line's slope is the negative reciprocal, which is 9/4.

How do I find the equation of a line that crosses the x-axis at 36?

Since the line crosses the x-axis at 36, its x-intercept is 36, so the point is (36, 0). Using the slope of the perpendicular line, you can write the equation in point-slope form.

What is the y-intercept of the line perpendicular to y = -4/9x + 5 that crosses x at 36?

The y-intercept can be found after determining the line's equation. First, find the slope (9/4), then use the point (36, 0) to find the y-intercept.

Can you provide the equation of the line that crosses x=36 and is perpendicular to y=-4/9x+5?

Yes. The line has a slope of 9/4 and passes through (36, 0). Its equation in point-slope form is y - 0 = (9/4)(x - 36). Simplified, y = (9/4)x - 81.

What is the slope of the line crossing x=36 and perpendicular to y=-4/9x+5?

The slope is 9/4, as it is the negative reciprocal of the original line's slope, which is -4/9.

How do I verify that the line y = (9/4)x - 81 is perpendicular to y = -4/9x + 5?

Check the slopes: the original line has slope -4/9, and the new line has slope 9/4. Since 9/4 is the negative reciprocal of -4/9, the lines are perpendicular.

What is the y-value when x=36 for the line y=(9/4)x - 81?

Substituting x=36, y = (9/4)(36) - 81 = (9/4)(36) - 81 = 81 - 81 = 0.

Is the line y = (9/4)x - 81 the only line crossing x=36 and perpendicular to y=-4/9x+5?

Yes. The line passing through (36, 0) with slope 9/4 is unique and perpendicular to the original line.

What is the general form of the line crossing x=36, perpendicular to

y=-4/9x+5?

The specific equation is y = (9/4)x - 81, which can also be written in standard form as 9x - 4y = 324.

Additional Resources

Equation of a line that crosses the x-axis at 36 and is perpendicular to the line y = -4/9 x + 5 is a fascinating problem in coordinate geometry that combines understanding of slope-intercept form, perpendicular lines, and the x-intercept. This question requires a thorough exploration of the properties of lines, how to determine their equations, and the relationships between their slopes and intercepts. Whether you're a student brushing up on algebra skills or a math enthusiast exploring the geometric relationships between lines, this problem offers a comprehensive learning experience.

In this article, we will break down the problem into manageable steps, analyze the key concepts involved, and derive the required equation systematically. We will also discuss related concepts such as the slope-intercept form, the significance of the x-intercept, and the criteria for perpendicularity. By the end of this discussion, you'll have a clear understanding of how to approach similar problems and a solid grasp of the underlying mathematical principles.

Understanding the Problem

Breaking Down the Question

The problem asks us to find the equation of a line with two specific characteristics:

1. It crosses the x-axis at 36
2. It is perpendicular to the line y = -4/9 x + 5

These conditions give us critical information:

- The line's x-intercept is at x = 36.
- The original line's slope is -4/9, and the new line must be perpendicular to it, meaning it will have a slope that is the negative reciprocal of -4/9.

Key Concepts Needed

Slope-Intercept Form of a Line

The general equation of a straight line is:

$$ y = mx + b $$

where:

- m is the slope of the line
- b is the y-intercept, the point where the line crosses the y-axis

Given the x-intercept, we can find the point on the line where it crosses the x-axis, which is at $(36, 0)$. This point is crucial for determining the line's equation.

Understanding X-Intercept

The x-intercept is the point where $y = 0$. For the line crossing at 36, the x-intercept point is $(36, 0)$. Knowing this point allows us to write the line's equation if we know its slope.

Perpendicular Lines and Their Slopes

Lines are perpendicular if the product of their slopes is -1. If a line has slope m_1, then a perpendicular line has slope:

$$ m_2 = -\frac{1}{m_1} $$

Applying this to our problem:

- The original line has slope $-\frac{4}{9}$
- The perpendicular line's slope will be:

$$ m_{perp} = -\frac{1}{-\frac{4}{9}} = \frac{9}{4} $$

Step-by-Step Solution

Step 1: Identify the slope of the perpendicular line

From the original line $y = -\frac{4}{9} x + 5$:

- Slope $m_1 = -\frac{4}{9}$

Perpendicular slope:

$$m_{perp} = \frac{9}{4}$$

This is the slope of the line we need to find.

Step 2: Use the x-intercept to find the point on the perpendicular line

The x-intercept at 36 means the line crosses the x-axis at $(36, 0)$. Since our line passes through this point:

$$(x_0, y_0) = (36, 0)$$

Step 3: Write the equation of the line with known point and slope

Using the point-slope form:

$$y - y_0 = m (x - x_0)$$

Substitute:

$$y - 0 = \frac{9}{4} (x - 36)$$

which simplifies to:

$$y = \frac{9}{4} (x - 36)$$

Final Equation of the Line

Expanding the expression:

$$ y = \frac{9}{4} x - \frac{9}{4} \times 36 $$

Calculate:

$$ \frac{9}{4} \times 36 = 9 \times 9 = 81 $$

So, the equation becomes:

$$ y = \frac{9}{4} x - 81 $$

This is the equation of the line that crosses the x-axis at 36 and is perpendicular to the given line.

Features and Analysis of the Resulting Line

Key Features

- Slope: $\frac{9}{4}$, positive and relatively steep.
- X-intercept: At $(36, 0)$, as specified.
- Y-intercept: At $y = -81$, found by substituting $x=0$:

$$ y = \frac{9}{4} \times 0 - 81 = -81 $$

- Perpendicularity: Confirmed by the negative reciprocal relationship with the original line's slope.

Pros and Cons of the Equation

Pros:

- Clearly defined by two points: the x-intercept and the perpendicularity condition.
- Straightforward to derive using basic algebra.
- The slope-intercept form makes it easy to interpret features like intercepts.

Cons:

- Assumes familiarity with the concept of perpendicular slopes, which might be confusing for beginners.
- The calculation of the y-intercept requires careful arithmetic to avoid errors.

Alternative Approaches and Considerations

Using Point-Slope Form:
Instead of directly expanding to slope-intercept form, other forms like point-slope could be used, especially if the point and slope are known.

Graphical Interpretation:
Plotting the original line and the new line can help visualize the perpendicularity and intercepts, reinforcing understanding.

Checking the Perpendicularity:
Verify the slopes' product:

$$ \left(-\frac{4}{9} \right) \times \left(\frac{9}{4} \right) = -1 $$

which confirms the lines are perpendicular.

Conclusion

The equation of the line that crosses the x-axis at 36 and is perpendicular to the line $y = -\frac{4}{9} x + 5$ is:

$$ y = \frac{9}{4} x - 81 $$

This problem exemplifies key concepts in coordinate geometry, including the relationships between slopes, intercepts, and perpendicularity. By systematically applying the principles of algebra and geometry, we can derive the required equation efficiently. Such problems enhance our understanding of how lines interact in the coordinate plane and improve our problem-solving skills for more complex geometric tasks.

Summary of Key Points

- The x-intercept provides a specific point on the line: $(36, 0)$.
- The slope of the perpendicular line is the negative reciprocal of the original line's slope.
- Using the point-slope form simplifies the derivation.
- The final line's equation is $y = \frac{9}{4} x - 81$.

Through this exploration, we've demonstrated a comprehensive approach to solving line equations under given conditions, combining algebraic manipulation with geometric insights.

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