What Is The Equation Of The Line That Passes Through The Given Point And Is Perpendicular To The Given

What Is The Equation Of The Line That Passes Through The Given Point And Is Perpendicular To The Given is a fundamental question in coordinate geometry that involves understanding the relationships between lines, their slopes, and how to find the equations of lines under specific conditions. This topic is essential for students and professionals working in mathematics, engineering, physics, and related fields, as it forms the basis for analyzing geometric relationships and solving real-world problems involving lines and angles.

In this comprehensive guide, we will explore the concepts needed to determine the equation of a line passing through a specific point and perpendicular to a given line. We will break down key definitions, step-by-step procedures, illustrative examples, and practical tips to help you master this topic efficiently.

Understanding the Foundations

Before delving into the process of finding the equation of the required line, it is crucial to understand some fundamental concepts related to lines on the coordinate plane.

1. The Equation of a Line

The most common form of a line's equation in the Cartesian plane is the slope-intercept form:

$$ y = mx + b $$

where:
- m is the slope of the line,
- b is the y-intercept (the point where the line crosses the y-axis).

Alternatively, the point-slope form is useful when a point on the line and the slope are known:

$$ y - y_1 = m(x - x_1) $$

where:
- (x_1, y_1) is a point on the line,
- m is the slope.

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2. Slope of a Line

The slope (m) measures the steepness and direction of a line:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

for two distinct points (x_1, y_1) and (x_2, y_2).

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3. Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at a right angle (90°). A key property is:

The slopes of two perpendicular lines are negative reciprocals of each other, provided neither is vertical or horizontal:

$$m_1 \times m_2 = -1$$

or equivalently:

$$m_2 = -\frac{1}{m_1}$$

This relationship allows us to find the slope of the line perpendicular to a given line once its slope is known.

Step-by-Step Approach to Find the Equation of the Perpendicular Line

Now, let's outline a systematic process to determine the equation of the line passing through a given point and perpendicular to a given line.

Step 1: Identify the Given Data

- The given point: (x_0, y_0)
- The given line: usually provided in slope-intercept form $y = m_1 x + b$, or in some other form from which the slope can be extracted.

Example:
Suppose the given point is $(3, 4)$, and the given line is $y = 2x + 1$.

Step 2: Find the Slope of the Given Line

Extract the slope m_1 from the given line.

Example:
From $y = 2x + 1$, the slope $m_1 = 2$.

Step 3: Determine the Slope of the Perpendicular Line

Use the negative reciprocal relationship:

$$m_2 = -\frac{1}{m_1}$$

Example:
Since $m_1 = 2$,

$$m_2 = -\frac{1}{2}$$

Step 4: Write the Equation of the Perpendicular Line Using Point-Slope Form

Apply the point-slope form with the point (x_0, y_0) and the new slope m_2:

$$y - y_0 = m_2 (x - x_0)$$

Example:
Plugging in $(3, 4)$ and $m_2 = -\frac{1}{2}$:

$$y - 4 = -\frac{1}{2} (x - 3)$$

Step 5: Simplify to Slope-Intercept Form or Standard Form

Expand and simplify to get the final equation.

Example:
$$y - 4 = -\frac{1}{2}x + \frac{3}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2} + 4$$
$$y = -\frac{1}{2}x + \frac{3}{2} + \frac{8}{2}$$
$$y = -\frac{1}{2}x + \frac{11}{2}$$

The equation of the line passing through $(3, 4)$ and perpendicular to $y = 2x + 1$ is:

$$ y = -\frac{1}{2}x + \frac{11}{2} $$

Special Cases and Additional Considerations

In some scenarios, the problem may involve lines in different forms or special cases. Here are some considerations to keep in mind.

1. When the Given Line Is Vertical or Horizontal

- If the given line is vertical, its equation is $x = a$.
- The slope is undefined.
- A line perpendicular to a vertical line is horizontal, with the form $y = k$.
- To find the perpendicular line passing through a point, simply use $y = y_0$.

- If the given line is horizontal, its equation is $y = b$.
- The slope is zero.
- The perpendicular line is vertical, with the form $x = x_0$.

Example:
Given line: $x = 5$, point: $(2, 3)$

- Perpendicular line: vertical, passing through $(2, 3)$, so:

$$ x = 2 $$

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2. When the Equation of the Given Line Is in Standard Form

If the line is given as $Ax + By + C = 0$:
- Find the slope as $m_1 = -\frac{A}{B}$ (assuming $B \neq 0$)
- Proceed with the negative reciprocal to find m_2.

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3. Graphical Interpretation

Understanding the geometric relationships enhances comprehension:
- The perpendicular line intersects the original line at a right angle.
- Visualizing the slopes as angles of inclination can help grasp the negative reciprocal relationship.

Practical Examples for Better Understanding

Let's explore a few more examples to solidify the concept.

Example 1: Perpendicular Line to a Non-Vertical, Non-Horizontal Line

Given point: $(1, 2)$

Given line: $y = -3x + 4$

Solution:
- Slope of the given line: $m_1 = -3$.
- Slope of the perpendicular line: $m_2 = -\frac{1}{-3} = \frac{1}{3}$.
- Equation of the perpendicular line passing through $(1, 2)$:

$$y - 2 = \frac{1}{3}(x - 1)$$

- Simplify:

$$y - 2 = \frac{1}{3}x - \frac{1}{3}$$
$$y = \frac{1}{3}x - \frac{1}{3} + 2$$
$$y = \frac{1}{3}x + \frac{5}{3}$$

Example 2: Perpendicular Line to a Vertical Line

Given point: $(0, 0)$

Given line: $x = 7$

Solution:
- The given line is vertical ($x = 7$), so slope is undefined.
- The perpendicular line is horizontal:

$$y = y_0 = 0$$

- Equation:

$$y = 0$$

Applications of Finding Perpendicular Line Equations

Understanding how to find the equation of a perpendicular line has numerous practical applications, including:

- Engineering and Architecture: Designing structures where perpendicular components are needed.
- Navigation and Robotics: Calculating paths that are perpendicular to existing trajectories.
- Physics: Analyzing forces or vectors that are at right angles.
- Computer Graphics: Rendering scenes involving perpendicular lines or surfaces.
- Mathematical Proofs: Establishing right angles and orthogonality in geometric proofs.

Tips for Accurate Calculation and Problem Solving

To ensure precision and efficiency, consider the following tips:

- **Always identify the form of the given line:** Whether it's slope-intercept, standard,

Frequently Asked Questions

How do I find the equation of a line passing through a specific point and perpendicular to a given line?

First, determine the slope of the given line, then find the negative reciprocal of that slope to get the perpendicular slope. Use the point-slope form with the given point and this perpendicular slope to find the equation.

What is the process to derive the equation of a line perpendicular to another line passing through a point?

Identify the slope of the original line, calculate its negative reciprocal for the perpendicular slope, then apply the point-slope formula using the given point to obtain the equation.

Can you provide an example of finding a perpendicular line

passing through a point?

Yes. For example, if the original line is $y = 2x + 3$ and the point is (4, 5), the perpendicular slope is -1/2. Using point-slope form: $y - 5 = -1/2(x - 4)$, which simplifies to $y = -1/2x + 7$.

What is the general formula to write the equation of a line perpendicular to a given line through a point?

If the given line has slope m, then the perpendicular line has slope $m_p = -1/m$. The equation is $y - y_1 = m_p(x - x_1)$, where (x_1, y_1) is the given point.

How do I determine the slope of the original line if only two points are given?

Use the slope formula: $m = (y_2 - y_1) / (x_2 - x_1)$. Once you have the slope, find its negative reciprocal for the perpendicular line.

What should I do if the original line is vertical or horizontal when finding a perpendicular line?

If the original line is vertical ($x = a$), the perpendicular line is horizontal ($y = b$). If the original is horizontal ($y = b$), the perpendicular line is vertical ($x = a$).

Is there a quick way to write the equation of a perpendicular line through a point without finding the slope explicitly?

Only if the original line's equation is known in slope-intercept form and the perpendicular line is vertical or horizontal. Otherwise, calculating the slope and using point-slope form is recommended.

How does the concept of negative reciprocal relate to perpendicular lines in coordinate geometry?

Two lines are perpendicular if their slopes are negative reciprocals of each other—that is, their slopes multiply to -1.

Can I use the slope-intercept form directly to find the perpendicular line through a point?

It's easier to first determine the perpendicular slope and then use the point-slope form to write the equation. The slope-intercept form can be obtained after applying the point-slope formula.

What common mistakes should I avoid when finding the equation of a perpendicular line through a point?

Avoid mixing up the original slope and its negative reciprocal, forgetting to use the correct point in the point-slope formula, and assuming the line is horizontal or vertical without checking the original line's orientation.

Additional Resources

What Is The Equation Of The Line That Passes Through The Given Point And Is Perpendicular To The Given

Understanding the equation of a line that passes through a specific point and is perpendicular to a given line is a fundamental concept in coordinate geometry. This topic not only forms a core part of algebra but also has practical applications in fields such as engineering, physics, computer graphics, and navigation systems. The process involves deciphering the slope of the original line, determining the negative reciprocal for the perpendicular line, and then formulating the new line's equation based on the given point. This comprehensive guide aims to demystify this process through detailed explanations, structured sections, and analytical insights.

Foundations of Line Equations in Coordinate Geometry

Understanding the Slope-Intercept Form

The most common way to represent a line in coordinate geometry is through the slope-intercept form:

$$ y = mx + b $$

where:
- m is the slope of the line, indicating its steepness and direction.
- b is the y-intercept, the point where the line crosses the y-axis.

The slope m can be calculated if two points on the line are known, using:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

This form is particularly useful for deriving the equation of a line when given a point and a slope.

The Point-Slope Form

When a specific point (x_1, y_1) and a slope m are known, the point-slope form provides a direct way to write the line's equation:

$$ y - y_1 = m(x - x_1) $$

This form is instrumental when dealing with perpendicular lines, as it allows straightforward insertion of the point and the relevant slope.

Standard Form of a Line

Another common form is the standard form:

$$ Ax + By + C = 0 $$

which can be useful in certain algebraic manipulations, especially when dealing with perpendicular lines.

Understanding Perpendicular Lines in Geometry

What Does It Mean for Lines to Be Perpendicular?

Two lines are perpendicular if they intersect at a right angle (90 degrees). Mathematically, the slopes of perpendicular lines are negative reciprocals of each other. If one line has slope m, the perpendicular line will have a slope $m_{\perp}$ such that:

$$m_{\perp} = -\frac{1}{m}$$

This negative reciprocal relationship is the cornerstone for constructing the equation of the perpendicular line.

Why Are Perpendicular Lines Important?

Perpendicular lines are fundamental in various geometric constructions and proofs. They are used to establish right angles, define coordinate axes, and solve real-world problems involving orthogonality, such as designing roads, architectural layouts, and signal pathways.

Visualizing Perpendicularity

Graphically, if you plot a line and its perpendicular, you'll see that the angles between them are exactly 90 degrees. When working with equations, understanding the slopes helps in predicting how lines will intersect and at what angles.

Step-by-Step Procedure for Finding the Equation of the Perpendicular Line

To determine the line passing through a specific point (x_0, y_0) and perpendicular to a given line, follow these structured steps:

Step 1: Identify the Equation of the Given Line

The process begins with the original line's equation, which can be in any form. For example:

$$ y = m x + b $$

or

$$ A x + B y + C = 0 $$

If the line is given in standard form, convert it into slope-intercept form for easier analysis:

$$ y = -\frac{A}{B} x - \frac{C}{B} $$

From this, determine the slope m.

Step 2: Find the Slope of the Perpendicular Line

Once the slope m of the original line is known, calculate the slope $m_{\perp}$ of the perpendicular line as:

$$ m_{\perp} = -\frac{1}{m} $$

Special considerations:
- If the original line is vertical ($x = c$), its slope is undefined, and the perpendicular line will be horizontal ($y = y_0$).
- If the original line is horizontal ($y = c$), the perpendicular line will be vertical ($x = x_0$).

Step 3: Use the Point-Slope Form to Write the Equation

Using the point (x_0, y_0) through which the new line passes and the slope $m_{\perp}$, write:

$$y - y_0 = m_{\perp} (x - x_0)$$

This is the fundamental formula from which the line's explicit equation can be derived.

Step 4: Convert to Slope-Intercept or Standard Form (Optional)

Depending on the context, you might prefer to express the line in slope-intercept form:

$$y = m_{\perp} (x - x_0) + y_0$$

or in standard form by rearranging:

$$m_{\perp} x - y + (y_0 - m_{\perp} x_0) = 0$$

This provides a flexible way to present the line's equation for various applications.

Illustrative Examples and Analytical Insights

Example 1: Line Passing Through a Point and Perpendicular to a Known Line

Suppose the original line is:

$$ y = 2x + 3 $$

and the point through which the perpendicular line passes is $(4, 1)$.

Step 1: Find the slope of the original line:

$$ m = 2 $$

Step 2: Compute the perpendicular slope:

$$ m_{\perp} = -\frac{1}{2} $$

Step 3: Write the equation using point-slope form:

$$ y - 1 = -\frac{1}{2}(x - 4) $$

Step 4: Convert to slope-intercept form:

$$ y - 1 = -\frac{1}{2}x + 2 $$

$$ y = -\frac{1}{2}x + 3 $$

Result: The equation of the perpendicular line passing through $(4,1)$ is:

$$ y = -\frac{1}{2}x + 3 $$

Example 2: Handling Vertical and Horizontal Lines

Suppose the original line is:

$$x = 5$$ (a vertical line)
and the point is $(2, 3)$.

Analysis:
- The original line's slope is undefined (vertical).
- The perpendicular line must be horizontal, which has the form:

$$y = y_0$$

- Since the line passes through $(2, 3)$, the perpendicular line is:

$$y = 3$$

Similarly, if the original line is:

$$y = 4$$ (horizontal line)
and the point is $(1, 2)$, then:

$$x = x_0 = 1$$

This is the vertical line passing through $(1, 2)$.

Applications and Practical Significance

The ability to find the equation of a line perpendicular to a

given line through a specific point has diverse applications:

- Engineering Design: Ensuring structures are orthogonal for stability and aesthetic purposes.
- Navigation and Mapping: Calculating paths that are perpendicular to existing routes for efficiency.
- Computer Graphics: Rendering perpendicular lines or features in digital images.
- Physics: Analyzing forces or trajectories orthogonal to existing vectors.
- Mathematical Problem Solving: Solving complex geometric problems involving right angles and orthogonality.

Advanced Considerations and Limitations

While the process described is straightforward for most cases, some complexities may arise:

- Lines with Zero or Infinite Slope: Vertical and horizontal lines require special treatment, as their slopes are undefined or zero.
- Numerical Precision: When working with irrational slopes or decimals, maintaining precision is critical.
- Coordinate Transformations: Sometimes, shifting or rotating coordinate axes simplifies the problem but requires additional transformations.
- Three-Dimensional Extensions: In 3D space,

perpendicularity involves dot products and vector equations, adding complexity beyond 2D analysis.

Conclusion

The task of deriving the equation of a line that passes through a given point and is perpendicular to another line is rooted in fundamental principles of coordinate geometry. By understanding the relationship between slopes, especially the negative reciprocal property, we can systematically formulate such lines using point-slope and slope-intercept forms. This process not only enhances comprehension of geometric relationships but also equips practitioners with a versatile tool for solving diverse real-world problems. Whether dealing with simple algebraic lines or complex geometric constructs, mastering this concept is essential for advancing in mathematics, engineering, and related fields.

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