What Is The Equation Of The Line That Passes Through The Point (-6,7) And Has A Slope Of -5? Write The

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Understanding the equation of a line is fundamental in algebra and coordinate geometry. When given specific information such as a point through which the line passes and its slope, you can precisely determine the line’s equation. In this article, we will explore how to find the equation of the line that passes through the point (-6, 7) and has a slope of -5. We will discuss the mathematical principles involved, provide step-by-step instructions, and include practical examples to enhance your understanding. Whether you're a student learning about linear equations or a professional applying these concepts, this comprehensive guide will clarify the process and help you master it.

Understanding the Basics of Line Equations

Before diving into the specific problem, it’s essential to understand some fundamental concepts related to lines in coordinate geometry.

What Is a Line in Coordinate Geometry?

A line in a 2D coordinate plane is a straight, one-dimensional figure extending infinitely in both directions. It can be described mathematically using an equation that relates the x-coordinate and y-coordinate of any point on the line.

Standard Form of a Line Equation

The most common form of a line's equation is the slope-intercept form:

$$ y = mx + b $$

where:
- m is the slope of the line,
- b is the y-intercept (the point where the line crosses the y-axis).

Point-Slope Form of a Line Equation

When you know a point (x_1, y_1) on the line and the slope m, the point-slope form is very useful:

$$ y - y_1 = m(x - x_1) $$

This form is often the easiest way to find the equation of a line given a point and slope.

Given Data and Objective

The problem provides:
- A point: $(-6, 7)$,
- Slope: $m = -5$.

Our goal is to find the equation of the line passing through this point with the given slope.

Step-by-Step Solution to Find the Equation of the Line

The process involves straightforward steps, leveraging the point-slope form.

Step 1: Write the point-slope form of the equation

Using the point $(-6, 7)$ and slope -5:

$$
y - 7 = -5(x - (-6))
$$

Simplify the double negative:

$$
y - 7 = -5(x + 6)
$$

Step 2: Distribute the slope across the parentheses

\[

$$
y - 7 = -5x - 30
$$

Step 3: Solve for y to get the slope-intercept form

Add 7 to both sides:

$$
y = -5x - 30 + 7
$$

Simplify:

$$
y = -5x - 23
$$

This is the slope-intercept form of the line.

Final Equation:

$$
\boxed{ y = -5x - 23 }
$$

Additional Forms of the Line Equation

While the slope-intercept form is often most convenient, other forms can also be useful.

Standard Form

Rearranging the slope-intercept form:

$$
y = -5x - 23
$$

Multiply both sides by 1 to write in standard form:

$$
5x + y = -23

\]

This form is helpful for certain algebraic operations and when analyzing the line’s intercepts.

Summary of Forms

- Slope-Intercept Form: \(y = -5x - 23 \)
- Standard Form: \(5x + y = -23 \)

Verifying the Equation

To ensure the correctness of the derived equation, verify that the point \((-6, 7)\) satisfies it.

Substitute \(x = -6 \):

\[
y = -5(-6) - 23 = 30 - 23 = 7
\]

Since the y-value matches the original point, the equation is correct.

Understanding the Significance of the Slope and Point

The slope \(-5\) indicates that the line decreases by 5 units vertically for every 1 unit increase horizontally. The point \((-6, 7)\) acts as a specific coordinate through which this line passes, anchoring its position in the plane.

Practical Applications of the Line Equation

Knowing how to find the equation of a line given a point and slope has numerous real-world applications:

- Physics: Modeling linear relationships like velocity and distance over time.
- Economics: Representing cost functions or revenue models.
- Engineering: Designing components with specific linear characteristics.
- Data Analysis: Fitting regression lines to data points.

Additional Tips for Finding Line Equations

- Always verify the point satisfies the derived equation.
- Convert between different forms for convenience depending on context.
- Practice with various points and slopes to become proficient.

Conclusion

Finding the equation of a line passing through a specific point with a given slope is a fundamental skill in algebra. By applying the point-slope form and simplifying to slope-intercept or standard form, you can accurately describe any such line. For the given point $(-6, 7)$ and slope -5, the equation is $y = -5x - 23$. Mastery of this process enables you to analyze and interpret linear relationships across various disciplines, making it an essential tool in both academic and professional settings.

FAQs

1. **Can the equation be written in other forms?** Yes, besides slope-intercept and standard forms, you can also express the line in point-slope form or general form as needed.

2. **What if the slope is zero?** A slope of zero indicates a horizontal line, with the equation $y = c$, where c is the y-coordinate of the point.

3. **How do I find the line if I only have two points?** Use the two points to first calculate the slope, then apply the point-slope form with either point.

By understanding these principles and methods, you are well-equipped to find the equation of any line given a point and slope, a fundamental skill in algebra and analytical geometry.

Frequently Asked Questions

What is the equation of the line passing through the point (-6, 7) with a slope of -5?

Using point-slope form: $y - 7 = -5(x + 6)$. Simplifying, $y = -5x - 30 + 7$, so the equation is $y = -5x - 23$.

How do you find the equation of a line given a point and a slope?

Use the point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point and m is the slope. Then, simplify to slope-intercept form if needed.

What is the slope-intercept form of the line passing through (-6, 7) with slope -5?

The slope-intercept form is $y = -5x - 23$.

Can the equation of a line be written in standard form for this problem?

Yes, the standard form is $5x + y = -16$, obtained by rearranging the slope-intercept form.

What is the importance of knowing both the point and the slope when writing a line's equation?

Knowing both allows you to precisely determine the line's equation using the point-slope formula, ensuring accuracy in its representation.

How does the slope affect the orientation of the line passing through (-6, 7)?

A slope of -5 indicates the line descends steeply from left to right, tilting downward.

What are some real-life applications of finding a line's equation given a point and slope?

Examples include modeling trends in data, predicting future values, or designing objects with specific inclinations.

Is the point (-6, 7) on the line y = -5x - 23?

Yes, substituting $x = -6$ into the equation: $y = -5(-6) - 23 = 30 - 23 = 7$, which matches the point (-6, 7).

How can I verify my line equation passes through a specific point?

Substitute the point's coordinates into the equation; if both sides are equal, the point lies on the line.

Additional Resources

What Is The Equation Of The Line That Passes Through The Point (-6,7) And Has A Slope Of -5?

Understanding the equation of a line is a fundamental concept in algebra and analytic geometry, serving as a cornerstone for more advanced mathematical topics. When given specific information such as a point through which the line passes and its slope, one can derive the precise equation that describes that line. In this comprehensive exploration, we will delve into the methods of finding the equation of a line, interpret the significance of the given point and slope, and provide a step-by-step process to arrive at the final equation. This article aims to serve as an informative guide for students, educators, and enthusiasts seeking a deeper understanding of linear equations and their applications.

Introduction to the Equation of a Line

Before we explore the specific problem, it is essential to understand what an equation of a line represents and its standard forms.

The Concept of a Line

In the coordinate plane, a line is a collection of points extending infinitely in both directions, characterized by a constant slope. The slope indicates the steepness and direction of the line, describing how much the y-coordinate changes relative to the change in the x-coordinate.

Standard Forms of a Line Equation

Several forms are used to represent lines algebraically, including:

- Slope-Intercept Form: \(y = mx + b \)
- Where \(m \) is the slope, and \(b \) is the y-intercept.
- Point-Slope Form: \(y - y_1 = m(x - x_1) \)
- Useful when a point \((x_1, y_1) \) and the slope \(m \) are known.
- General Form: \(Ax + By + C = 0 \)

For problems involving a specific point and slope, the point-slope form is especially convenient.

The Significance of the Given Data

In our case, the problem provides two critical pieces of information:

- A point through which the line passes: (-6, 7)
- The slope of the line: -5

Understanding these allows us to construct the line's equation systematically.

Step-by-Step Derivation of the Equation

Let's now explore the process of deriving the line's equation step by step.

1. Recall the Point-Slope Form

The point-slope form of a line's equation is:

$$
y - y_1 = m(x - x_1)
$$

where:

- (x_1, y_1) is a point on the line.
- m is the slope.

In our problem:

- $(x_1, y_1) = (-6, 7)$
- $m = -5$

2. Substitute the Known Values

Plugging these into the point-slope form:

$$
y - 7 = -5(x - (-6))
$$

which simplifies to:

$$
y - 7 = -5(x + 6)
$$

3. Expand and Simplify

Distribute the slope:

$$
y - 7 = -5x - 30
$$

Add 7 to both sides to isolate y:

$$
y = -5x - 30 + 7
$$

$$
y = -5x - 23
$$

This is the slope-intercept form of the line.

Final Equation of the Line: Analysis and Interpretation

The derived equation:

$$
\boxed{ y = -5x - 23 }
$$

fully describes the line passing through the point (-6, 7) with a slope of -5. Let's analyze its components and implications.

1. Slope ($m = -5$)

- The negative slope indicates the line descends as x increases.
- For every unit increase in x, y decreases by 5 units.
- The steepness of the line is significant; a slope of -5 suggests a relatively steep decline.

2. Y-Intercept ($b = -23$)

- The y-intercept is the point where the line crosses the y-axis ($x = 0$).
- Here, the line crosses the y-axis at $(0, -23)$.
- This point is not on the given point but is consistent with the line's slope and the algebraic form.

3. The Line's Behavior and Graphical Representation

- The line slopes downward from left to right.
- It crosses the y-axis at -23, well below the point (-6, 7), which is in the second quadrant.

Validating the Equation

It is crucial to verify that the derived line passes through the given point (-6, 7). Substituting $x = -6$ into the equation:

$$
y = -5(-6) - 23 = 30 - 23 = 7

\]

which matches the given \(y \)-coordinate. Therefore, the equation is consistent, confirming its correctness.

Broader Context and Applications

Understanding how to derive the equation of a line from a point and slope has significant practical applications:

- Engineering and Physics: Calculating trajectories, designing structures, or analyzing motion.
- Economics: Modeling relationships between variables, such as cost and quantity.
- Data Analysis: Fitting linear models to data points.
- Computer Graphics: Rendering lines and shapes based on algebraic equations.

Furthermore, mastering these foundational techniques prepares students for more complex topics, including systems of equations, inequalities, and calculus.

Variations and Related Concepts

While the problem focuses on a specific point and slope, several related concepts are important:

1. Equation of a Line Passing Through Two Points

Given two points, the line's slope and equation can be found without prior knowledge of the slope, using the two-point form.

2. Parallel and Perpendicular Lines

- Lines with the same slope are parallel.
- Lines with slopes that are negative reciprocals are perpendicular.

3. Standard and General Forms

Transforming the slope-intercept form into the general form for various applications, such as plotting or solving systems.

Summary and Key Takeaways

- The equation of a line passing through a point \((x_1, y_1) \) with slope \(m \) can be directly written in point-slope form.
- Substituting the known point and slope into the point-slope form yields the slope-intercept form.
- The specific line passing through (-6, 7) with slope -5 is:

\[

\boxed{ y = -5x - 23 }
\]

- Verification confirms the line indeed passes through the given point.
- Understanding these concepts enhances problem-solving skills in algebra and beyond.

Final Reflections

The process of deriving the line's equation from its key features exemplifies the power of algebraic manipulation and the elegance of mathematical reasoning. Recognizing the relationship between points, slopes, and equations allows for precise modeling of linear relationships across numerous fields. Whether in academic contexts or real-world applications, mastering these foundational skills is essential for analytical thinking and effective communication of mathematical ideas.

In conclusion, the equation of the line that passes through the point (-6, 7) and has a slope of -5 is \(y = -5x - 23 \). This straightforward yet profound result encapsulates the core principles of linear equations and serves as a stepping stone toward more advanced mathematical understanding.

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What Is The Equation Of The Line That Passes Through The Point 6 7 And Has A Slope Of 5 Write The

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