What Is The Equation Of The Streamline That Passes Through The Point (2, -1) When T= 2s. V= 2xyi + Y^2

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Understanding the behavior of fluid flow is essential in many engineering and physics applications. One key concept in fluid mechanics is the idea of streamlines, which represent the paths followed by fluid particles in a steady or unsteady flow. In this article, we will explore how to find the equation of the streamline passing through the point (2, -1) when the time T equals 2 seconds, given the velocity field V = 2xyi + y^2j. We will delve into the fundamental principles involved, step-by-step calculations, and the significance of the resulting equation.

Understanding the Velocity Field V = 2xyi + y^2j

Before deriving the streamline equation, it is crucial to interpret the provided velocity field accurately.

Components of the Velocity Field

- u (x-component): 2xy
- v (y-component): y^2

This implies that at any point (x, y), the velocity of the fluid particles is given by:
- $u = 2xy$
- $v = y^2$

Physical Interpretation

- The x-component of the velocity depends on both x and y, indicating a flow that varies with position.
- The y-component depends solely on y, suggesting the flow's vertical component scales with the y-coordinate.

Relevance of Streamline Equations in Fluid Mechanics

Streamline equations describe the trajectories that particles follow in a fluid flow at any given instant. They are essential for visualizing flow patterns, analyzing fluid transport, and solving engineering problems related to fluid behavior.

Mathematical Representation of Streamlines

- The equation of a streamline is derived from the velocity components as:

$$
\frac{dy}{dx} = \frac{v}{u}
$$

- This differential equation relates changes in y with respect to x along a streamline.

Calculating the Equation of the Streamline

Given the velocity components, we proceed to derive the streamline equation.

Step 1: Set Up the Differential Equation

Using the relation:

$$
\frac{dy}{dx} = \frac{v}{u} = \frac{y^2}{2xy}
$$

Simplify the expression:

$$
\frac{dy}{dx} = \frac{y^2}{2xy} = \frac{y}{2x}
$$

Step 2: Separate Variables

Rearranged as:

$$
\frac{dy}{y} = \frac{dx}{2x}
$$

This separation allows us to integrate both sides independently.

Step 3: Integrate Both Sides

Perform the integrations:

$$
\int \frac{1}{y} \, dy = \frac{1}{2} \int \frac{1}{x} \, dx
$$

which gives:

$$
\ln |y| = \frac{1}{2} \ln |x| + C
$$

where C is the constant of integration.

Step 4: Express the Solution

Exponentiating both sides:

$$
|y| = e^{C} \cdot |x|^{1/2}
$$

Let $k = e^{C}$, which is an arbitrary constant. Then, the general equation of the streamline is:

$$
y = \pm k \sqrt{x}
$$

Since the flow can be in either direction, the sign of k can be positive or negative, but for simplicity, we write:

$$
\boxed{ y = k \sqrt{x} }
$$

Note: The equation $y = k \sqrt{x}$ describes a family of streamlines depending on the constant k.

Determining the Specific Streamline Passing Through (2, -1) at T= 2s

To find the particular streamline passing through the point (2, -1), we substitute the point into the general equation.

Step 1: Substitute the Point into the Equation

$$
-1 = k \sqrt{2}
$$

Solve for k:

$$
k = \frac{-1}{\sqrt{2}}
$$

Step 2: Write the Specific Equation of the Streamline

The particular streamline passing through (2, -1) is:

$$
y = - \frac{1}{\sqrt{2}} \sqrt{x}
$$

or equivalently,

$$
\boxed{ y = - \frac{\sqrt{x}}{\sqrt{2}} }
$$

This is the explicit equation of the streamline passing through the specified point at the given time.

Accounting for the Time T= 2 Seconds

While the velocity field provided is time-independent, the mention of T= 2s suggests the flow is observed at that specific instant. In unsteady flows, streamlines can change with time, but here, since the velocity components are not explicitly time-dependent, the equation derived represents the flow pattern at T= 2s.

If the flow were time-dependent, additional steps involving the flow's temporal evolution would be necessary. However, given the data, the streamline equation remains valid for T= 2s.

Applications and Significance of the Derived Streamline Equation

Understanding the equation of a specific streamline enables engineers and scientists to:

- Visualize flow patterns around objects or within channels.
- Predict particle trajectories in fluid systems.
- Design efficient piping, ventilation, and fluid transport systems.
- Analyze pollutant dispersion or heat transfer in fluids.

The explicit form $y = - \frac{\sqrt{x}}{\sqrt{2}}$ provides insight into the flow structure near the point of interest, revealing how fluid particles move in the region surrounding (2, -1).

Summary

In this article, we examined how to determine the equation of the streamline passing through a specific point in a fluid flow characterized by the velocity field $V = 2xyi + y^2j$. By deriving the differential equation $dy/dx = y/(2x)$ and integrating, we obtained the general family of streamlines $y = k \sqrt{x}$. Substituting the point (2, -1), we identified the particular streamline as:

$$
\boxed{
y = - \frac{\sqrt{x}}{\sqrt{2}}
}
$$

This process illustrates core principles in fluid mechanics, including the relationship between velocity fields and streamlines, the importance of differential equations in flow analysis, and how specific flow paths can be characterized mathematically.

Remember: While the flow pattern described here is based on idealized conditions, real-world flows may involve additional complexities such as viscosity, turbulence, and time-dependent behavior, which require more advanced analysis techniques.

Additional Resources for Further Learning:
- Fluid Mechanics by Frank M. White
- Introduction to Fluid Mechanics by Robert W. Fox, Alan T. McDonald, Philip J. Pritchard
- Online simulations of streamlines and flow visualization tools

Keywords: streamline equation, velocity field, fluid flow, differential equations, flow analysis, fluid mechanics, flow visualization, flow pattern

Frequently Asked Questions

What is the velocity field given in the problem?

The velocity field is given as $V = 2xyi + y^2j$.

What is the general form of the equation of a streamline in a velocity field?

The equation of a streamline is found by solving the differential equation $dy/dx = v_y / v_x$.

How do you set up the differential equation for the streamline passing through (2, -1)?

Since $V = 2xyi + y^2j$, the differential equation is $dy/dx = y^2 / (2xy)$, which simplifies to $dy/dx = y / (2x)$.

What is the simplified differential equation for the streamline in this case?

The simplified differential equation is $dy/dx = y / (2x)$.

How do you solve the differential equation to find the streamline equation?

Separate variables: $dy/y = dx / (2x)$, then integrate both sides to obtain $\ln|y| = (1/2) \ln|x| + C$.

What is the general equation of the streamline after integration?

The general solution is $\ln|y| = (1/2) \ln|x| + C$, or equivalently $y = K \sqrt{x}$, where $K = e^C$.

How do you find the specific constant K using the point (2, -1)?

Substitute $x=2$ and $y=-1$ into $y = K \sqrt{x}$: $-1 = K \sqrt{2}$, so $K = -1 / \sqrt{2}$.

What is the equation of the streamline passing through (2, -1)?

The specific streamline equation is $y = - (1/\sqrt{2}) \sqrt{x}$.

How do you evaluate the streamline at T=2s, or is time relevant in this context?

Since the streamline equation is derived from the velocity field and initial point, time T=2s does not directly affect the streamline equation; it describes the path in space, not the particle's position at a specific time.

Additional Resources

Understanding the Equation of the Streamline Passing Through the Point (2, -1) at T = 2s for the Velocity Field V = 2xyi + y^2

Introduction to Streamlines and Their Significance in Fluid Dynamics

A fundamental concept in fluid mechanics, streamlines are imaginary lines that represent the trajectory followed by fluid particles in steady flow. They serve as a valuable visualization tool, illustrating the direction of flow at any given instant, and are essential for analyzing complex fluid behaviors.

Key points about streamlines include:

- They are tangent to the velocity vector at every point.
- In steady flows, streamlines coincide with pathlines and streaklines.
- The shape and orientation of streamlines reveal flow characteristics such as acceleration, vortices, or stagnation points.

Understanding the mathematical equation governing streamlines allows engineers and scientists to predict flow patterns, optimize designs, and analyze phenomena such as mixing, diffusion, or boundary layer development.

Given Data and Problem Statement

The problem involves a velocity field:

$$
\mathbf{V} = 2xy\, \mathbf{i} + y^2\, \mathbf{j}
$$

and asks us to find the equation of the streamline passing through the point $(2, -1)$ when

$T= 2\,s$.

Important details:

- The velocity components are:
- $u = 2xy$
- $v = y^2$
- The point of interest is:
- $(x, y) = (2, -1)$
- The problem specifies a time $T= 2\,s$, which suggests a temporal context, but since streamlines are generally instantaneous features, the focus is on the velocity field at that instant, unless the velocity field explicitly depends on time. Here, it appears to be steady (no explicit t dependence).

Note: The mention of $T= 2\,s$ might be included to reinforce the specific instant or to confirm the velocity field is valid at that time, assuming steady flow.

Analyzing the Velocity Field

Before deriving the streamline equation, it's essential to understand the nature of the velocity field:

$$
\mathbf{V} = u\, \mathbf{i} + v\, \mathbf{j} = 2xy\, \mathbf{i} + y^2\, \mathbf{j}
$$

Characteristics:

- The x-component $u= 2xy$ varies with both x and y.
- The y-component $v= y^2$ depends only on y.
- The flow is not necessarily divergence-free, but checking divergence can give insights:

$$
\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial (2xy)}{\partial x} + \frac{\partial (y^2)}{\partial y} = 2y + 2y = 4y
$$

Since divergence depends on y, the flow isn't incompressible everywhere unless $y=0$. This characteristic might influence the structure of streamlines but does not prevent us from deriving their equations.

Mathematical Formulation of Streamlines

The general differential equation for a streamline is:

$$
\frac{dy}{dx} = \frac{v}{u}
$$

where u and v are the velocity components at a point (x, y).

Applying to our velocity components:

$$
\frac{dy}{dx} = \frac{v}{u} = \frac{y^2}{2xy}
$$

Simplify the right-hand side:

$$
\frac{dy}{dx} = \frac{y^2}{2xy} = \frac{y}{2x}
$$

This differential equation relates x and y along the streamline.

Deriving the Equation of the Streamline

Step 1: Write the differential equation

$$
\frac{dy}{dx} = \frac{y}{2x}
$$

Step 2: Separate variables

$$
\frac{dy}{y} = \frac{dx}{2x}
$$

Step 3: Integrate both sides

$$
\int \frac{1}{y}\, dy = \int \frac{1}{2x}\, dx
$$

which gives:

$$
\ln |y| = \frac{1}{2} \ln |x| + C
$$

where C is the constant of integration.

Step 4: Exponentiate to solve for the relation

$$
|y| = e^{C} \cdot |x|^{1/2}
$$

or equivalently:

$$
y = K \sqrt{x}
$$

where $K = \pm e^{C}$ depends on the initial condition.

Applying Initial Conditions to Find the Specific Streamline

The point $(2, -1)$ lies on the streamline, so:

$$
-1 = K \sqrt{2}
$$

which implies:

$$
K = \frac{-1}{\sqrt{2}}
$$

Note on the sign:

Since K is negative, the streamline passing through $(2, -1)$ is:

$$
y = - \frac{1}{\sqrt{2}} \sqrt{x}
$$

Domain considerations:

- $\sqrt{x}$ is real for $x \ge 0$.

- The specific point $(2, -1)$ has $x=2 > 0$, so valid.
- The negative sign accounts for the negative y value at $x=2$.

Final streamline equation:

$$
\boxed{
y = - \frac{1}{\sqrt{2}} \sqrt{x}
}
$$

This is the equation of the streamline passing through $(2, -1)$.

Understanding the Physical Significance of the Derived Equation

The derived equation reveals several key insights:

- The streamline is a square-root curve in the x-y plane.
- It extends into the region where y is negative, consistent with the initial point.
- The shape indicates how the flow behaves: as x increases, y becomes more negative but at a decreasing rate (due to the square root).

Flow behavior:

- Near the point $(2, -1)$, the flow is directed along this curve.
- The flow's nature can be further analyzed by examining the velocity components at various points on this streamline.

Verification at the Point (2, -1)

To ensure the validity of the streamline equation:

- Substitute $x=2$:

$$
y = - \frac{1}{\sqrt{2}} \sqrt{2} = -1
$$

which matches the given point, confirming the correctness.

Velocity Components at the Point (2, -1)

Calculating the velocity components at $(2, -1)$:

$$
u = 2xy = 2 \times 2 \times (-1) = -4
$$
$$
v = y^2 = (-1)^2 = 1
$$

The velocity vector at this point:

$$
\mathbf{V} = -4\, \mathbf{i} + 1\, \mathbf{j}
$$

indicates:

- The flow is directed primarily leftward (negative x-direction).
- Slight upward component (positive y-direction).

This is consistent with the flow pattern implied by the streamline equation.

Implications of the Time $T= 2\,s$

Given that the velocity field appears steady (no explicit t dependence), the mention of $T= 2\,s$ might be to specify the instant at which the velocity field is evaluated or the particle's position along the streamline at that time.

In a dynamic scenario where particles move along streamlines:

- The position of a particle at $T= 2\,s$ would be determined by integrating the velocity over time.
- However, since the velocity field is steady, the streamline equation remains valid at that instant.

If the flow were unsteady, additional information would be needed to relate particle position to time. Since it isn't specified, the simplest interpretation is that the flow is steady at $T= 2\,s$, and the derived streamline equation applies directly.

Summary of the Derived Streamline Equation

- The equation of the streamline passing through $(2, -1)$ is:

$$
\boxed{
y = - \frac{1}{\sqrt{2}} \sqrt{x}
}
$$

- This curve accurately passes through the specified point and aligns with the velocity field's direction.

- The shape indicates a flow pattern that extends into the negative y-region, with the flow direction consistent with computed velocity components.

Additional Considerations and Advanced Insights

Flow topology:

- The velocity components

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