What Is The Expected Return And Standard Deviation For The Minimum-variance Portfolio Of The Two Risky

What Is The Expected Return And Standard Deviation For The Minimum-variance Portfolio Of The Two Risky is a fundamental question in modern portfolio theory, especially when investors aim to optimize their investment mix between two risky assets. Understanding the expected return and standard deviation of this portfolio helps investors make informed decisions by balancing potential gains against associated risks. The minimum-variance portfolio represents the combination of assets that offers the lowest possible risk (variance or standard deviation) for a given level of expected return, making it a cornerstone concept for risk-averse investors seeking to minimize volatility while maintaining exposure to risky assets.

Introduction to Portfolio Risk and Return

Before diving into the specifics of the minimum-variance portfolio of two risky assets, it’s essential to understand the fundamental concepts of expected return and standard deviation within the context of investment portfolios.

Expected Return

Expected return is the weighted average of the returns of individual assets within a portfolio, based on their proportions. It provides an estimate of the potential profitability of an investment portfolio over a specified period, assuming historical or anticipated returns.

Mathematically, for a portfolio with two assets:
$$E(R_p) = w_1 \times E(R_1) + w_2 \times E(R_2)$$
where:
- $E(R_p)$ = expected return of the portfolio
- w_1, w_2 = weights of assets 1 and 2 in the portfolio (sum to 1)
- $E(R_1), E(R_2)$ = expected returns of assets 1 and 2

Standard Deviation (Risk)

Standard deviation measures the variability or volatility of returns, indicating how much the returns deviate from the expected return. A higher standard deviation means more risk, or higher uncertainty.

For a two-asset portfolio, the standard deviation depends not only on individual asset volatilities but also on the correlation between the assets.

Understanding the Minimum-Variance Portfolio

The minimum-variance portfolio is the most conservative combination of two risky assets, designed to minimize overall portfolio risk. It is particularly useful for investors who prioritize stability and wish to avoid excessive volatility.

Key Features of the Minimum-Variance Portfolio

- It is the point on the efficient frontier with the lowest possible risk.
- It may have an expected return lower than the individual assets' average returns.
- The optimal weights depend on asset volatilities and their correlation.

Why Focus on the Minimum-Variance Portfolio?

- To construct a diversified portfolio that minimizes risk.
- To understand the trade-off between risk and return.
- To serve as a benchmark for evaluating other portfolios.

Calculating the Expected Return and Standard Deviation of the Minimum-Variance Portfolio

To determine the expected return and standard deviation of the minimum-variance portfolio of two risky assets, investors need the following data:

- Expected returns of each asset ($E(R_1), E(R_2)$)
- Volatilities or standard deviations of each asset (σ_1, σ_2)
- Correlation coefficient between the assets (ρ)
- Variances (σ_1^2, σ_2^2)

Step 1: Compute the Covariance

The covariance between the two assets is:

$$\text{Cov}(R_1, R_2) = \rho \times \sigma_1 \times \sigma_2$$

Step 2: Determine the Weights of the Minimum-Variance Portfolio

The weights $w_1^{}$ and $w_2^{}$ that minimize the portfolio variance are given by:

$$w_1^{} = \frac{\sigma_2^2 - \rho \times \sigma_1 \times \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2 \times \rho \times \sigma_1 \times \sigma_2}$$

\[w_2^ = 1 - w_1^ \]

These weights ensure the portfolio has the lowest possible variance.

Step 3: Calculate the Expected Return of the Minimum-Variance Portfolio

Once the optimal weights are known:

\[E(R_{MV}) = w_1^ \times E(R_1) + w_2^ \times E(R_2) \]

Step 4: Calculate the Standard Deviation of the Minimum-Variance Portfolio

The portfolio’s standard deviation is:

\[\sigma_{MV} = \sqrt{w_1^{2} \times \sigma_1^2 + w_2^{2} \times \sigma_2^2 + 2 \times w_1^ \times w_2^ \times \text{Cov}(R_1, R_2)} \]

or equivalently,

\[\sigma_{MV} = \sqrt{w_1^{2} \times \sigma_1^2 + w_2^{2} \times \sigma_2^2 + 2 \times w_1^ \times w_2^ \times \rho \times \sigma_1 \times \sigma_2} \]

Practical Example: Calculating the Minimum-Variance Portfolio

Suppose an investor considers two risky assets with the following characteristics:
- Asset 1: Expected return \(E(R_1) = 8\% \), volatility \(\sigma_1 = 15\% \)
- Asset 2: Expected return \(E(R_2) = 12\% \), volatility \(\sigma_2 = 20\% \)
- Correlation coefficient \(\rho = 0.3 \)

Step 1: Covariance Calculation

\[\text{Cov}(R_1, R_2) = 0.3 \times 0.15 \times 0.20 = 0.009 \]

Step 2: Optimal Weights

\[w_1^ = \frac{(0.20)^2 - 0.3 \times 0.15 \times 0.20}{0.15^2 + 0.20^2 - 2 \times 0.3 \times 0.15 \times 0.20} \]
\[w_1^ = \frac{0.04 - 0.009}{0.0225 + 0.04 - 2 \times 0.3 \times 0.15 \times 0.20} \]
\[w_1^ = \frac{0.031}{0.0625 - 0.018} = \frac{0.031}{0.0445} \approx 0.6966 \]

Thus, $w_2^{} = 1 - 0.6966 = 0.3034$.

Step 3: Expected Return of the Minimum-Variance Portfolio

$$E(R_{MV}) = 0.6966 \times 8\% + 0.3034 \times 12\%$$
$$E(R_{MV}) = 5.57\% + 3.64\% = 9.21\%$$

Step 4: Standard Deviation of the Minimum-Variance Portfolio

$$\sigma_{MV} = \sqrt{0.6966^2 \times 0.0225 + 0.3034^2 \times 0.04 + 2 \times 0.6966 \times 0.3034 \times 0.009}$$
$$\sigma_{MV} = \sqrt{0.0109 + 0.0037 + 0.0038}$$
$$\sigma_{MV} = \sqrt{0.0184} \approx 0.1358 \text{ or } 13.58\%$$

Result: The minimum-variance portfolio has an expected return of approximately 9.21% with a standard deviation of roughly 13.58%.

Implications for Investors

Understanding the expected return and standard deviation of the minimum-variance portfolio provides valuable insights into risk management:

Key Points:
- The minimum-variance portfolio offers the lowest possible risk for a given pair of risky assets.
- It serves as a foundational point on the efficient frontier, from which investors can evaluate risk-return trade-offs.
- Adjusting asset weights allows investors to move along the efficient frontier, balancing higher returns against higher risks.
- The correlation between assets significantly impacts the portfolio's risk; diversification benefits are maximized when assets are less correlated.

Limitations and Considerations

While the minimum-variance portfolio offers an optimal risk-reduction strategy, several limitations should be considered:

- Estimation Errors: Expected returns, volatilities, and correlations are based on historical data and may not accurately predict future performance.
- Market Changes: Asset relationships can change over time, affecting the portfolio’s risk profile.
- Focus on Variance: Standard deviation treats upside and downside volatility equally, which may not align with investor preferences.

- Single-Period

Frequently Asked Questions

What is the minimum-variance portfolio in a two-risky asset setting?

The minimum-variance portfolio is the portfolio that offers the lowest possible risk (standard deviation) for a given set of two risky assets, achieved by optimally allocating weights to minimize overall portfolio variance.

How do you calculate the expected return of the minimum-variance portfolio with two risky assets?

The expected return is calculated as a weighted average of the individual asset expected returns, using the optimal weights that minimize variance: $E(R_p) = w_1 E(R_1) + w_2 E(R_2)$.

What determines the weights of assets in the minimum-variance portfolio?

The weights are determined by solving for the combination that minimizes the portfolio variance, which depends on each asset's variance, covariance, and the expected returns.

How is the standard deviation of the minimum-variance portfolio computed?

It is computed using the formula: $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1,R_2)}$, where w_1 and w_2 are optimal weights, σ_1 and σ_2 are asset standard deviations, and $Cov(R_1,R_2)$ is the covariance.

Can the expected return of the minimum-variance portfolio be higher than that of individual assets?

Yes, if the assets are combined in a way that reduces overall risk, the portfolio's expected return can be higher relative to the risk-adjusted return, but it generally depends on the assets' returns and correlations.

Why is the standard deviation of the minimum-variance portfolio important?

It measures the portfolio's risk and helps investors understand the least possible volatility they can expect when investing in the two risky assets, aiding in risk management.

How does correlation between two risky assets affect the minimum-variance portfolio's return and risk?

Lower or negative correlation reduces the portfolio's overall risk, leading to a lower standard deviation, while the expected return depends on asset weights and their individual returns.

Is the minimum-variance portfolio always on the efficient frontier?

Yes, the minimum-variance portfolio is always on the efficient frontier, representing the portfolio with the lowest risk for its expected return.

Can the expected return of the minimum-variance portfolio be negative?

Yes, if both assets have low or negative expected returns, or if their combination results in a portfolio with an overall negative expected return, the minimum-variance portfolio can also have a negative expected return.

Additional Resources

What Is The Expected Return And Standard Deviation For The Minimum-variance Portfolio Of The Two Risky Assets?

In the dynamic arena of investment management, understanding how to optimize a portfolio's risk and return is crucial for both individual investors and institutional fund managers. Among the myriad strategies employed, constructing a minimum-variance portfolio—the portfolio that offers the lowest possible risk for a given set of assets—stands out as a fundamental concept. When dealing with two risky assets, this approach becomes a manageable yet insightful exercise, revealing key principles of diversification, risk mitigation, and expected performance. So, what exactly are the expected return and standard deviation of this optimal portfolio? This article aims to unpack these concepts with clarity and depth, guiding you through the underlying theory, the mathematical formulations, and practical implications.

Understanding the Foundations: Risk, Return, and Portfolio Optimization

Before diving into the specifics of the minimum-variance portfolio, it’s essential to grasp the foundational ideas of investment risk and return.

Expected Return

The expected return of an asset—or a portfolio—is a probabilistic measure of the average return an investor anticipates over a certain period. It is calculated as the weighted average of the individual asset returns, where the weights correspond to the proportion of the total investment allocated to each asset.

Mathematically, for two assets, A and B:
- Expected return of the portfolio, $E(R_p)$, is:

$$
E(R_p) = w_A \times E(R_A) + w_B \times E(R_B)
$$

where:
- w_A and w_B are the weights of assets A and B, respectively, with $w_A + w_B = 1$.
- $E(R_A)$ and $E(R_B)$ are the expected returns of the assets.

Risk and Standard Deviation

Risk in investments is primarily quantified by the variability or volatility of returns, which is expressed as the standard deviation (σ). A higher standard deviation indicates greater volatility and, typically, higher risk.

For a two-asset portfolio, the standard deviation is not just a weighted average of the individual asset risks; it also depends on the correlation or covariance between the assets.

The standard deviation σ_p of the portfolio is given by:

$$
\sigma_p = \sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \text{Cov}(A, B)}
$$

where:
- σ_A and σ_B are the standard deviations of assets A and B.
- $\text{Cov}(A, B)$ is the covariance between the assets.

This formula highlights the importance of diversification: by combining assets with less-than-perfectly correlated returns, an investor can reduce the overall portfolio risk.

The Minimum-Variance Portfolio: Concept and Construction

The minimum-variance portfolio (MVP) is the composition of assets that yields the lowest possible risk (standard deviation), given their expected returns and covariances. It does not necessarily maximize return; instead, it seeks the optimal balance where risk is minimized for the given assets.

Mathematical Formulation

To identify this portfolio, we formulate an optimization problem:

- Objective: Minimize \(\sigma_p \) with respect to the weights \(w_A \) and \(w_B \).
- Constraints:
- \(w_A + w_B = 1 \) (full investment, no leverage)
- Optionally, \(w_A, w_B \geq 0 \) if short-selling is not allowed.

The key is to find the weights \(w_A^ \) and \(w_B^ \) that solve:

\[
\min_{w_A, w_B} \quad \sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \text{Cov}(A, B)
\]

subject to

\[
w_A + w_B = 1
\]

Using calculus or matrix algebra, the optimal weights are derived as:

\[
w_A^ = \frac{\sigma_B^2 - \text{Cov}(A, B)}{\sigma_A^2 + \sigma_B^2 - 2 \text{Cov}(A, B)}
\]

and

\[
w_B^ = 1 - w_A^
\]

Once these weights are obtained, the properties of the MVP—its expected return and standard deviation—can be directly calculated.

Calculating the Expected Return of the Minimum-Variance Portfolio

The expected return for the MVP is straightforward once the optimal weights are known.

\[
E(R_{MVP}) = w_A^ E(R_A) + w_B^ E(R_B)
\]

It is important to note that the expected return of the MVP depends on the individual asset returns and their optimal weightings. Typically, since the MVP is constructed to minimize risk rather than maximize return, its expected return might be lower than that of a more aggressive, higher-risk portfolio.

Practical Implication:

- Investors seeking risk minimization should pay close attention to the trade-off between risk and return.
- The MVP may serve as a "safe" baseline portfolio, especially in volatile markets.

Calculating the Standard Deviation of the Minimum-Variance Portfolio

The standard deviation of the MVP is obtained by substituting the optimal weights into the risk formula:

\[
\sigma_{MVP} = \sqrt{(w_A^)^2 \sigma_A^2 + (w_B^)^2 \sigma_B^2 + 2 w_A^ w_B^ \text{Cov}(A, B)}
\]

This value represents the lowest achievable volatility considering the assets' individual risks and their covariance.

Key Insights:
- The standard deviation of the MVP is always less than or equal to the individual asset risks.
- The degree of risk reduction hinges on the correlation between the assets: the more negatively correlated, the greater the potential risk reduction.

Real-World Example: An Illustrative Calculation

To concretize these concepts, consider two assets with the following characteristics:

- Asset A:
- Expected return, \(E(R_A) = 8\% \)
- Standard deviation, \(\sigma_A = 10\% \)
- Asset B:
- Expected return, \(E(R_B) = 12\% \)
- Standard deviation, \(\sigma_B = 15\% \)
- Covariance: \(\text{Cov}(A, B) = 0.012 \)

Suppose an investor wants to find the MVP:

1. Calculate the optimal weight of Asset A:

\[
w_A^ = \frac{\sigma_B^2 - \text{Cov}(A, B)}{\sigma_A^2 + \sigma_B^2 - 2 \text{Cov}(A, B)} = \frac{0.0225 - 0.012}{0.01 + 0.0225 - 2 \times 0.012} = \frac{0.0105}{0.0225 - 0.024} = \frac{0.0105}{-0.0015} \approx -7
\]

2. Compute Asset B’s weight:

\[
w_B^ = 1 - w_A^ \approx 1 - (-7) = 8
\]

This indicates a highly leveraged position on Asset B with a short position on Asset A, which may not be feasible under typical constraints. If short-selling is not allowed, the calculation would need adjustment.

3. Expected return of the MVP:

\[
E(R_{MVP}) = w_A^ \times 8\% + w_B^ \times 12\%
\]

4. Standard deviation:

\[
\sigma_{MVP} = \sqrt{(w_A^)^2 \times 10^2 + (w_B^)^2 \times 15^2 + 2 \times w_A^ \times w_B^ \times 12}
\]

This example demonstrates the importance of constraints and real-world limitations, as the theoretical MVP might suggest impractical weights, especially when assets have high covariance or certain correlations.

Implications for Investors and Portfolio Managers

Understanding the expected return and standard deviation of the minimum-variance portfolio offers practical insights:

- Risk Management: The MVP serves as a benchmark for the lowest achievable risk, enabling investors to gauge whether their current portfolios are efficiently diversified.
- Portfolio Construction: When combining two risky assets, the MVP provides a guidance point for the optimal balance, especially when combined with other assets or investment strategies.
- Trade-offs: While the MVP minimizes risk, it often does so at the expense of return. Investors must decide whether the risk reduction justifies the potential lower return.
- Diversification Strategy: The key to minimizing risk lies in the correlation between assets. Assets with low or negative correlations contribute more effectively to risk reduction.

Limitations and Considerations:
- Real-world constraints such as transaction costs, taxes, and short-selling restrictions can alter the theoretical weights.
- Estimation errors in expected returns and covariances can lead to suboptimal portfolios.
- The MVP is

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