What Is The Probability Of Drawing Two Yellow Marbles If The First One Is NOT Placed Back Into The Bag

What Is The Probability Of Drawing Two Yellow Marbles If The First One Is NOT Placed Back Into The Bag

Understanding probabilities is essential in various fields, from basic mathematics to real-world decision-making scenarios. One common question involves calculating the likelihood of specific outcomes when dealing with drawing items from a collection without replacement. A classic example is determining the probability of drawing two yellow marbles from a bag, especially when the first marble is not returned after being drawn. In this article, we will explore this problem in detail, breaking down the concepts, providing step-by-step calculations, and discussing related scenarios to deepen your understanding of probability in such contexts.

Fundamentals of Probability and Drawing Marbles

Before delving into the specific problem, it is important to understand some fundamental concepts related to probability and the process of drawing marbles from a bag.

Basic Probability Principles

Probability measures the likelihood of an event occurring and is expressed as a number between 0 and 1, or equivalently as a percentage between 0% and 100%. An event with a probability of 0 means it cannot happen, while a probability of 1 indicates certainty.

The formula for probability of an event A is:

$$
P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}
$$

In scenarios involving multiple steps or draws, the concept of compound probability comes into play, which involves calculating the probability of sequential events occurring.

Sampling Without Replacement

When drawing marbles from a bag, two main methods exist:

- Sampling With Replacement: After drawing a marble, it is returned to the bag, keeping the total number of marbles constant for subsequent draws.
- Sampling Without Replacement: The drawn marble is not returned, reducing the total number of marbles and possibly changing the composition of the remaining marbles.

The problem at hand involves sampling without replacement, which makes the second draw's probabilities dependent on the first.

Setting Up the Problem: Drawing Two Yellow Marbles

Let's define the parameters involved in the problem:

- N: Total number of marbles in the bag.
- Y: Number of yellow marbles in the bag.

Suppose you start with a bag containing a total of N marbles, of which Y are yellow. The goal is to find the probability that both of the first two marbles drawn (without replacement) are yellow.

Key points:

- The first marble is drawn, and if it is yellow, it is not replaced.
- The second marble is then drawn from the remaining marbles.
- We are interested in the probability that both marbles are yellow.

Example Scenario

To illustrate the calculations, consider a concrete example: Suppose the bag contains 10 marbles, of which 4 are yellow, and the remaining 6 are of other colors.

- $N = 10$
- $Y = 4$

The question: What is the probability that both marbles drawn are yellow, with no replacement?

Step-by-Step Calculation of the Probability

Calculating the probability of drawing two yellow marbles without replacement involves understanding the sequential nature of the draws and the changing composition of the bag after each draw.

Step 1: Probability of Drawing a Yellow Marble on the First Draw

The probability that the first marble is yellow is:

$$
P(\text{First yellow}) = \frac{Y}{N}
$$

In our example:

$$
P(\text{First yellow}) = \frac{4}{10} = 0.4
$$

Step 2: Probability of Drawing a Yellow Marble on the Second Draw, Given the First Was Yellow

If the first marble drawn was yellow, it is not replaced. Therefore, the remaining number of marbles is:

- Total remaining marbles: $N - 1 = 9$
- Remaining yellow marbles: $Y - 1 = 3$

Hence, the probability that the second marble is yellow, given the first was yellow, is:

$$
P(\text{Second yellow} \mid \text{First yellow}) = \frac{Y - 1}{N - 1}
$$

In the example:

$$
P(\text{Second yellow} \mid \text{First yellow}) = \frac{3}{9} = \frac{1}{3} \approx 0.333
$$

Step 3: Multiplying the Probabilities for the Sequential Event

The joint probability that both marbles are yellow is obtained by multiplying the probabilities:

$$
P(\text{Both yellow}) = P(\text{First yellow}) \times P(\text{Second yellow} \mid \text{First yellow})
$$

Using the example:

$$
P(\text{Both yellow}) = \frac{4}{10} \times \frac{3}{9} = \frac{4}{10} \times \frac{1}{3} = \frac{4}{30} = \frac{2}{15} \approx 0.133
$$

Thus, in this specific case, there is approximately a 13.3% chance of drawing two yellow marbles consecutively without replacement.

General Formula for Any Number of Marbles

The same logic applies to any initial quantities. The general probability P that both marbles are yellow in a draw of two marbles without replacement is:

$$
P = \frac{Y}{N} \times \frac{Y - 1}{N - 1}
$$

This formula accounts for the fact that the second draw's probability depends on the outcome of the first.

Extending to Different Numbers of Yellow Marbles

Suppose you are not given specific numbers but only the general case, with Y yellow marbles out of N, the probability remains:

$$
\boxed{
P = \frac{Y}{N} \times \frac{Y - 1}{N - 1}
}
$$

This formula is valid as long as $Y \geq 2$, since you need at least two yellow marbles to draw two yellow ones.

Alternative Approach: Using Combinatorics

Another way to compute this probability involves combinatorial calculations, especially when dealing with larger sets or multiple outcomes.

Number of Favorable Outcomes

- The number of ways to select two yellow marbles from the total yellow marbles:

$$
\binom{Y}{2}
$$

- The total number of ways to select any two marbles from the entire set:

$$
\binom{N}{2}
$$

Probability Calculation Using Combinatorics

The probability that both drawn marbles are yellow equals the ratio of favorable outcomes to total possible outcomes:

$$
P = \frac{\binom{Y}{2}}{\binom{N}{2}}
$$

Using the example with $N=10$ and $Y=4$:

$$
\binom{4}{2} = \frac{4 \times 3}{2 \times 1} = 6
$$

$$
\binom{10}{2} = \frac{10 \times 9}{2 \times 1} = 45
$$

Thus:

$$
P = \frac{6}{45} = \frac{2}{15} \approx 0.133
$$

which matches the previous calculation.

Comparison of Both Methods

Both the sequential probability method and the combinatorial approach yield the same result, confirming their consistency.

Method	Formula	Example Result (N=10, Y=4)
Sequential multiplication	$\frac{Y}{N} \times \frac{Y - 1}{N - 1}$	$\frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$
Combinatorial	$\frac{\binom{Y}{2}}{\binom{N}{2}}$	$\frac{6}{45} = \frac{2}{15}$

Factors Affecting the Probability

Several factors influence the probability of drawing two yellow marbles:

- Number of yellow marbles (Y): The higher the number of yellow marbles, the greater the probability.
- Total number of marbles (N): The larger the total number of marbles, the lower the probability, assuming fixed Y.
- Proportion of yellow marbles: The ratio Y/N indicates the likelihood of drawing a yellow marble in a single draw, influencing the two-draw probability.

Real-World Applications and Examples

Understanding this probability has practical applications in various fields.

1. Quality Control in Manufacturing

Suppose a batch contains a mix of defective and non-defective items. The probability of randomly selecting two defective items without replacement helps assess quality control processes.

2. Card Games and Gambling

Drawing specific cards or chips from a deck or pile involves similar probability calculations, especially when cards are

Frequently Asked Questions

What is the probability of drawing two yellow marbles without replacement from a bag containing yellow and other colored marbles?

The probability is calculated by multiplying the probability of drawing a yellow marble on the first draw by the probability of drawing another yellow marble on the second draw, given the first was yellow. Specifically, if there are Y yellow marbles and T total marbles, it is (Y/T) ((Y-1)/(T-1)).

How does removing the first yellow marble affect the probability of drawing a second yellow marble?

Removing the first yellow marble decreases the total number of marbles and the count of yellow marbles by one, which alters the probability for the second draw to (Y-1)/(T-1).

If there are 5 yellow marbles and 15 total marbles, what is the probability of drawing two yellow marbles consecutively without replacement?

The probability is (5/15) (4/14) = (1/3) (2/7) = 2/21 approximately 0.0952, or 9.52%.

Why is the probability of drawing two yellow marbles lower when the first is not replaced?

Because the total number of marbles and the number of yellow marbles decrease after the first draw, reducing the likelihood of drawing a second yellow marble in the subsequent draw.

How can one generalize the probability of drawing two specific-colored marbles without replacement?

The general formula is (number of desired color marbles / total marbles) multiplied by (remaining desired color marbles / remaining total marbles), i.e., (Y/T) ((Y-1)/(T-1)) for two identical desired outcomes.

Additional Resources

What Is The Probability Of Drawing Two Yellow Marbles If The First One Is NOT Placed Back Into The Bag is a classic problem in probability theory that highlights the concept of dependent events. This scenario involves understanding how the chance of drawing a yellow marble changes after the first draw, especially when the marble is not replaced, altering the composition of the remaining marbles. Such problems are fundamental in understanding real-world situations where the outcome of one event influences subsequent events, making them essential knowledge for students, educators, and anyone interested in probability.

Introduction to Probability and the Importance of Replacement

Probability is a measure of how likely an event is to occur, expressed as a number between 0 and 1 (or as a percentage between 0% and 100%). When dealing with drawing objects from a collection, such as marbles from a bag, the probability depends on the total number of objects and the number of objects that meet the desired condition.

Replacement plays a crucial role in probability calculations:
- With Replacement: After drawing an object, it is put back into the collection, keeping the total count and composition unchanged.
- Without Replacement: The object is not put back, which changes the composition and total number of objects remaining.

The problem at hand focuses on the without replacement case, which introduces dependency between the events—meaning the outcome of the first draw affects the second.

Setting Up the Problem

Imagine a bag containing a certain number of marbles, some of which are yellow, and others are of different colors. To analyze the probability of drawing two yellow marbles without replacement, we need to know:
- Total number of marbles in the bag (N)
- Number of yellow marbles in the bag (Y)

For example, suppose:
- Total marbles, N = 10
- Yellow marbles, Y = 4

The question then becomes: What is the probability of drawing two yellow marbles in succession if the first one is not replaced?

Step-by-Step Breakdown of the Calculation

1. Probability of Drawing a Yellow Marble on the First Draw

The probability of success on the first try is straightforward:
- P(First is Yellow) = Number of Yellow Marbles / Total Marbles = Y / N

Using our example:
- P(First Yellow) = 4 / 10 = 0.4 (or 40%)

2. Adjusting for the Second Draw

If the first marble is yellow and is not replaced, the total number of marbles decreases by one, and the number of yellow marbles decreases by one:
- New total, $N' = N - 1$
- New number of yellow marbles, $Y' = Y - 1$

The probability of drawing a yellow marble on the second draw, given that the first was yellow, is:
- P(Second is Yellow | First was Yellow) = $Y' / N' = (Y - 1) / (N - 1)$

Using our example:
- P(Second Yellow | First Yellow) = $(4 - 1) / (10 - 1) = 3 / 9 = 1 / 3 \approx 0.333...$

3. Calculating the Overall Probability

Since these are dependent events, we multiply the probabilities:
- P(Both are Yellow) = P(First Yellow) P(Second Yellow | First Yellow)

In our example:
- $P = (4 / 10) (3 / 9) = (2 / 5) (1 / 3) = 2 / 15 \approx 0.1333$ (or 13.33%)

General Formula and Variations

General Formula

For any initial counts, the probability of drawing two yellow marbles without replacement can be expressed as:

$$P(\text{both yellow}) = (Y / N) ((Y - 1) / (N - 1))$$

This formula reflects the dependency between the two events.

Variations Based on Different Counts

Case 1: More Yellow Marbles

Suppose:
- $N = 12$
- $Y = 5$

Then:
- P(First Yellow) = $5 / 12 \approx 0.4167$
- P(Second Yellow | First Yellow) = $4 / 11 \approx 0.3636$
- Overall probability = $(5 / 12) (4 / 11) \approx 0.1515$ or 15.15%

Case 2: Fewer Yellow Marbles

Suppose:

- N = 8
- Y = 2

Then:
- P(First Yellow) = 2 / 8 = 0.25
- P(Second Yellow | First Yellow) = 1 / 7 ≈ 0.1429
- Overall probability ≈ 0.25 0.1429 ≈ 0.0357 or 3.57%

Visualizing the Probabilities

Understanding the probability steps can be aided by visual aids:
- Tree diagrams illustrating each branch of outcomes
- Venn diagrams showing overlap
- Probability tables summarizing initial counts and resulting probabilities

These tools help clarify how the probabilities change after each event and reinforce the concept of dependency.

Extending the Problem

The same approach can be extended to more complex scenarios:
- Drawing more than two marbles
- Different colors and multiple desired outcomes
- Probabilities with replacement versus without replacement

For example, finding the probability of drawing exactly two yellow marbles in a series of three draws without replacement involves calculating combinations and considering different sequences.

Practical Applications of This Concept

Understanding the probability of drawing two yellow marbles if the first one is not placed back into the bag has real-world relevance:
- Quality control: selecting items from a batch without replacement
- Card games: drawing specific cards without returning them
- Genetics: calculating probabilities of traits when drawing alleles
- Inventory management: estimating chances of selecting specific items

In each case, the concept of dependent probabilities is crucial for accurate modeling and decision-making.

Summary and Key Takeaways

- The probability of drawing two yellow marbles without replacement depends on the initial counts.
- The events are dependent, meaning the outcome of the first affects the second.
- The formula for the probability is:

P(both yellow) = (Y / N) ((Y - 1) / (N - 1))

- Replacing the marble after the first draw would make the events independent, simplifying calculations to:

P(both yellow with replacement) = (Y / N)^2

- Always adjust counts after each draw to reflect the changing composition of the collection.

Final Thoughts

The problem of What Is The Probability Of Drawing Two Yellow Marbles If The First One Is NOT Placed Back Into The Bag is an excellent illustration of dependent events in probability. By carefully understanding how the counts change after each draw and applying the multiplication rule, you can accurately compute probabilities in real-world scenarios. Mastery of these concepts not only enhances problem-solving skills but also deepens understanding of how probability models reflect real-world dependencies.

Remember: Always consider whether the event involves replacement or not, as this fundamentally alters the calculation approach and outcomes.

What Is The Probability Of Drawing Two Yellow Marbles If The First One Is Not Placed Back Into The Bag

What Is The Probability Of Drawing Two Yellow Marbles If The First One Is Not Placed Back Into The Bag

Related Articles

- You Want To Purchase A New Motorcycle That Costs $26,800. The Most You Can Pay Each Month Is $530 Over
- 4.1 According To Generally Recognised Accounting Practice (GRAP) 1.96 List Six Items That Should Appear
- What Class Of Organic Compound Is Formed When Cyclopentanone Reacts With Ethylamine In The Presence Of

Back to Home