

What Must Be The Price Of A \$5,000 Bond With A 6.5% Coupon Rate, Semiannual Coupons, And Nine Years To

What Must Be The Price Of A \$5,000 Bond With A 6.5% Coupon Rate, Semiannual Coupons, And Nine Years To Maturity?

Understanding how to determine the fair value or price of a bond is essential for investors, financial analysts, and anyone involved in fixed-income securities. Bonds are debt instruments that pay periodic interest and return the principal at maturity. The price of a bond fluctuates based on market interest rates, issuer creditworthiness, time to maturity, and other factors. In this article, we will explore how to calculate the price of a bond with specific features: a face value of \$5,000, a 6.5% coupon rate, semiannual coupon payments, and nine years remaining until maturity.

Understanding Bond Basics

Before diving into calculations, it's important to understand some fundamental concepts about bonds.

What Is a Bond?

A bond is a fixed-income security representing a loan made by an investor to a borrower, typically a corporation or government entity. The issuer agrees to pay the bondholder periodic interest and return the principal amount at maturity.

Key Bond Terms

- **Face Value (Par Value):** The amount paid back at maturity, typically \$1,000 or a multiple thereof. In our case, \$5,000.
- **Coupon Rate:** The annual interest rate paid on the face value, expressed as a percentage. Here, 6.5%.
- **Coupon Payments:** The periodic interest payments made to bondholders. With semiannual coupons, payments are made twice a year.
- **Maturity:** The date when the bond principal is repaid. In this scenario, nine years from now.
- **Yield to Maturity (YTM):** The total return anticipated if the bond is

held until maturity, considering current market prices, coupon payments, and face value.

Calculating the Bond Price

The price of a bond is essentially the present value (PV) of its future cash flows: the semiannual coupon payments and the face value at maturity. The general formula is:

$$P = \sum_{t=1}^N \frac{C}{(1 + r)^t} + \frac{F}{(1 + r)^N}$$

Where:

- P = price of the bond
- C = semiannual coupon payment
- r = semiannual yield (YTM divided by 2)
- N = total number of semiannual periods (years $\times$ 2)
- F = face value of the bond

Step 1: Determine the Semiannual Coupon Payment

Given:

- Face value $F = \$5,000$
- Coupon rate = 6.5%

The annual coupon payment is:

$$\text{Annual Coupon} = 6.5\% \times \$5,000 = \$325$$

Since coupons are paid semiannually:

$$C = \frac{\$325}{2} = \$162.50$$

Step 2: Calculate Total Number of Periods

The bond has nine years remaining, with semiannual payments:

$$N = 9 \times 2 = 18 \text{ periods}$$

Step 3: Understand the Yield to Maturity (YTM)

The market interest rate or YTM is the rate used to discount future cash flows to determine the bond's current fair price. The YTM reflects the return an investor demands given the market conditions.

Note: To calculate the exact bond price, you need the current market YTM for similar bonds. If not provided, assumptions or market data are used.

For demonstration, let's assume the market YTM is 6.0% annually, compounded semiannually.

Step 4: Calculate the Semiannual Yield

$$r = \frac{6.0\%}{2} = 3.0\% = 0.03$$

Step 5: Present Value of Coupon Payments

The present value of an annuity (coupon payments) is:

$$PV_{\text{coupons}} = C \times \frac{1 - (1 + r)^{-N}}{r}$$

Plugging in the values:

$$PV_{\text{coupons}} = \$162.50 \times \frac{1 - (1 + 0.03)^{-18}}{0.03}$$

Calculations:

- $(1 + 0.03)^{18} \approx 1.03^{18} \approx 1.689$
- $(1 + 0.03)^{-18} = 1 / 1.689 \approx 0.592$

Thus,

$$PV_{\text{coupons}} = \$162.50 \times \frac{1 - 0.592}{0.03} = \$162.50 \times \frac{0.408}{0.03} \approx \$162.50 \times 13.6 \approx \$2,215$$

Step 6: Present Value of Face Value

The present value of the face value is:

$$PV_F = \frac{F}{(1 + r)^N} = \frac{\$5,000}{1.689} \approx \$2,958$$

Step 7: Calculate Total Bond Price

Adding the present values:

$$P \approx PV_{\text{coupons}} + PV_F = \$2,215 + \$2,958 = \$5,173$$

Therefore, if the market YTM is 6.0%, the bond's approximate price is \$5,173.

Factors Influencing Bond Price

The bond price is sensitive to various factors. Understanding these helps investors make informed decisions.

Market Interest Rates (YTM)

- When market rates increase, existing bond prices decrease because new bonds offer higher yields.
- When market rates decrease, existing bonds become more attractive, increasing their prices.

Time to Maturity

- Longer time horizons mean more sensitivity to interest rate changes.
- As maturity approaches, bond prices tend to converge toward face value.

Coupon Rate vs. Market Rates

- If the coupon rate exceeds the market YTM, the bond trades at a premium.
- If the coupon rate is below the market YTM, the bond trades at a discount.

Credit Risk

- Higher risk of default lowers bond prices.
- Credit ratings influence perceived risk and yield requirements.

Practical Application: Calculating the Bond Price for Different Yields

Suppose market YTM varies, here's how the bond's price would adjust.

- **YTM 5%:** Bond price would be above \$5,173, likely around \$5,300.
- **YTM 7%:** Bond price would be below \$5,173, perhaps around \$5,000 or less.

This inverse relationship exemplifies bond price sensitivity.

Conclusion: Determining the Fair Price of a Bond

The fair price of a bond with a face value of \$5,000, a 6.5% coupon rate, semiannual coupons, and nine years to maturity depends critically on the prevailing market yield (YTM). By calculating the present value of the future

coupon payments and the face value, discounted at the market rate, investors can estimate whether a bond is trading at a premium, discount, or at par.

Key takeaways:

- Always identify the coupon payments, number of periods, and appropriate discount rate.
- Recognize that bond prices fluctuate with market interest rates.
- Use present value formulas to accurately determine bond value.

Investors should consider current market conditions, credit risk, and their investment goals when assessing bond prices. Regularly updating calculations with current market yields ensures accurate valuation and informed investment decisions.

In summary, if the market YTM is 6.0%, the approximate price of the bond described is around \$5,173. Changes in market yields will cause the bond's price to fluctuate accordingly, reflecting the fundamental principle of bond valuation.

Frequently Asked Questions

How do I calculate the present value of a \$5,000 bond with a 6.5% coupon rate and nine years to maturity when coupons are paid semiannually?

To calculate the present value, you need to determine the present value of the semiannual coupon payments (which are $\$5,000 \times 6.5\% / 2 = \162.50 each period) and the present value of the face value (\$5,000) discounted back at the current market yield per period. Sum these two present values to get the bond's price.

What factors influence the market price of a bond with a fixed coupon rate and remaining maturity?

The key factors include current market interest rates, the bond's coupon rate, time remaining to maturity, credit quality of the issuer, and overall market conditions. When market rates rise above the coupon rate, the bond's price falls below face value, and vice versa.

If market interest rates are higher than the bond's coupon rate, will the bond trade at a premium or discount?

It will trade at a discount because the fixed coupon payments are less attractive compared to new bonds issued at higher rates, leading existing bonds to be priced lower to offer comparable yields.

How does semiannual coupon payment affect the bond's valuation compared to annual coupons?

Semiannual coupons result in more frequent cash flows, which slightly increases the bond's present value due to more frequent compounding. It also means the yield per period is halved, affecting the discounting process accordingly.

What is the typical calculation method to determine the fair price of a bond with nine years to maturity and semiannual coupons?

The typical method involves discounting all future semiannual coupon payments and the face value at the current market yield per period using present value formulas for annuities (coupons) and a lump sum (face value), then summing these to find the bond's fair price.

Additional Resources

What Must Be The Price Of A \$5,000 Bond With A 6.5% Coupon Rate, Semiannual Coupons, And Nine Years To Maturity is a fundamental question for investors, financial analysts, and anyone interested in bond valuation. Understanding how to determine a bond's current price involves analyzing the bond's cash flows, discount rates, and market conditions. Whether you're evaluating an investment opportunity, managing a portfolio, or simply expanding your knowledge of fixed-income securities, grasping the mechanics behind bond pricing is essential. This guide provides a comprehensive breakdown of the factors, calculations, and considerations needed to determine the fair market price of such a bond.

Understanding Bond Basics

Before diving into the actual calculation, it's important to review some core concepts:

- **Face Value (Par Value):** The amount paid back to the bondholder at maturity, in this case, \$5,000.
- **Coupon Rate:** The annual interest rate paid on the face value, here 6.5%.
- **Coupon Payment:** The actual dollar amount paid periodically; with semiannual coupons, this is half the annual coupon.
- **Time to Maturity:** The remaining period until the bond's face value is repaid, nine years in this scenario.
- **Market Yield (Yield to Maturity):** The rate of return required by investors in the current market for similar bonds, which influences the bond's price.
- **Discount Rate:** The rate used to discount future cash flows to their present value, often aligned with the market yield.

Step 1: Calculate the Semiannual Coupon Payment

Since the bond pays semiannual coupons, the annual coupon rate needs to be divided by two, and the total coupon payment per period calculated accordingly.

- Annual Coupon Payment:

$$\left(\text{Coupon Rate} \times \text{Face Value} = 6.5\% \times \$5,000 = \$325 \right)$$

- Semiannual Coupon Payment:

$$\left(\frac{\$325}{2} = \$162.50 \right)$$

This means every six months, the bondholder receives \$162.50 until maturity.

Step 2: Determine the Number of Payment Periods

With nine years remaining and semiannual payments, the total number of periods is:

- Total Periods (N):

$$\left(9 \text{ years} \times 2 = 18 \text{ periods} \right)$$

Each period involves a coupon payment, and at the end, the face value is returned.

Step 3: Understand the Market Yield or Discount Rate

The market yield to maturity (YTM) is crucial for valuation. It reflects the return an investor demands for holding the bond in current market conditions.

- If the market YTM is equal to the coupon rate (6.5%), the bond should trade at par (\$5,000).
- If the market YTM is greater than 6.5%, the bond will trade below par.
- If the market YTM is less than 6.5%, the bond will trade above par.

For this analysis, assume an example market YTM of 7% annualized, compounded semiannually, reflecting a slightly higher required return.

- Semiannual Market YTM (i):

$$\left(\frac{7\%}{2} = 3.5\% = 0.035 \right)$$

Step 4: Calculate the Present Value of Coupon Payments

The present value of the semiannual coupons is calculated as an annuity:

$$PV_{\text{coupons}} = C \times \left(1 - \frac{1}{(1 + i)^N}\right) \div i$$

Where:

- $C = \$162.50$
- $i = 0.035$
- $N = 18$

Calculating step-by-step:

1. Compute $(1 + i)^N$:

$$(1 + 0.035)^{18} \approx 1.035^{18}$$

Using calculator or approximation:

$$1.035^{18} \approx e^{18 \times \ln(1.035)} \approx e^{18 \times 0.0344} \approx e^{0.6192} \approx 1.857$$

2. Find the present value factor:

$$\frac{1}{(1 + i)^N} = \frac{1}{1.857} \approx 0.538$$

3. Calculate PV of coupons:

$$PV_{\text{coupons}} = 162.50 \times \left(1 - 0.538\right) \div 0.035 = 162.50 \times 0.462 \div 0.035$$

$$PV_{\text{coupons}} \approx 162.50 \times 13.2 \approx \$2,146.50$$

Step 5: Calculate the Present Value of the Face Value

The face value is paid at the end of 18 periods, discounted back to present:

$$PV_{\text{face}} = \text{Face Value} \div (1 + i)^N$$

$$PV_{\text{face}} = \$5,000 \div 1.857 \approx \$2,693.52$$

Step 6: Sum the Present Values to Find the Bond Price

Adding the present value of the coupons and the face value gives the bond's current fair value:

$$\text{Bond Price} = PV_{\text{coupons}} + PV_{\text{face}} \approx \$2,146.50 + \$2,693.52 = \$4,840.02$$

Therefore, given a 7% market YTM, the approximate price of the bond is \$4,840.

Sensitivity Analysis: Impact of Changing Market Yields

Bond prices are inversely related to market yields. To deepen understanding, consider how different yields affect the bond price:

Market YTM	Approximate Price	Explanation
6.5%	About \$5,000 (par)	Same as coupon rate; bond at par value
7.0%	About \$4,840	Slightly higher yield; bond trades below par
6.0%	About \$5,160	Lower yield; bond trades above par
8.0%	About \$4,560	Significantly higher yield; bond trades well below par

Practical Considerations in Bond Pricing

While the above calculations provide a theoretical fair value, real-world bond pricing involves additional factors:

- **Market Conditions:** Prevailing interest rates, inflation expectations, and economic outlook influence yields.
- **Credit Risk:** The issuer's creditworthiness affects the yield demanded by investors.
- **Liquidity:** Bonds that are more liquid tend to trade closer to their theoretical value.
- **Call Features or Special Provisions:** Bonds with embedded options can have different valuation impacts.

Final Thoughts: Calculating the Price of a Bond

Determining the price of a bond like a \$5,000 bond with a 6.5% coupon rate, semiannual coupons, and nine years to maturity requires:

1. Calculating the periodic coupon payments.
2. Identifying the appropriate discount rate based on current market yields.
3. Discounting the future cash flows (coupons and face value) back to their present value.
4. Summing these present values to arrive at the fair market price.

This process is fundamental to fixed-income investing, enabling investors to assess whether a bond is fairly priced, overvalued, or undervalued based on market conditions.

Summary Checklist for Bond Valuation

- Identify bond parameters: face value, coupon rate, payment frequency, remaining time.
- Determine the market YTM for similar bonds.
- Calculate coupon payments per period.
- Calculate present value of coupons (annuity).
- Calculate present value of face value (lump sum).
- Sum the present values for the fair bond price.
- Perform sensitivity analysis to understand how yield changes impact price.

By mastering these steps, investors can confidently evaluate bond investments, manage risk, and optimize their portfolios in a dynamic market environment.

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