When The Function F(x) Is Divided By X+1, The Quotient Is X^(2)-7x-6 And The Remainder Is -3. Find The

When The Function F(x) Is Divided By X+1, The Quotient Is X^(2)-7x-6 And The Remainder Is -3. Find The

Understanding polynomial division is fundamental in algebra, especially when dealing with polynomial functions and their properties. When a polynomial $F(x)$ is divided by a linear divisor such as $x + 1$, the division process yields a quotient and a remainder, which can be used to reconstruct the original polynomial or to analyze its properties. This article explores the process of finding the original polynomial $F(x)$ given the quotient and remainder upon division, emphasizing the importance of polynomial division in algebraic problem-solving, and providing a step-by-step methodology to arrive at the solution.

Introduction to Polynomial Division

Polynomial division is akin to the long division process used with numbers but applied to polynomials. When dividing a polynomial $F(x)$ by a divisor $D(x)$, the division yields a quotient polynomial $Q(x)$ and a remainder R such that:

$$
F(x) = D(x) \times Q(x) + R
$$

where:

- $D(x)$ is the divisor polynomial,
- $Q(x)$ is the quotient polynomial,
- R is the remainder, which is either a constant (if the divisor is linear) or a polynomial of degree less than $D(x)$.

In our specific problem, the divisor is $x + 1$, which is linear, and the quotient and remainder are given:

$$
Q(x) = x^2 - 7x - 6, \quad R = -3
$$

Our goal is to find the original polynomial $F(x)$.

Understanding the Given Data

Before embarking on the solution, it is crucial to interpret the given information correctly.

Given:

- Divisor: $x + 1$
- Quotient: $Q(x) = x^2 - 7x - 6$
- Remainder: $R = -3$

To Find:

- The original polynomial $F(x)$

Applying the Polynomial Division Formula

The fundamental relationship in polynomial division states:

$$
F(x) = D(x) \times Q(x) + R
$$

Substituting the known values:

$$
F(x) = (x + 1) \times (x^2 - 7x - 6) + (-3)
$$

Our task reduces to expanding this expression to find $F(x)$.

Step-by-Step Calculation of $F(x)$

Step 1: Multiply $(x + 1)$ by $Q(x)$

Using polynomial multiplication (distributive property):

$$
(x + 1)(x^2 - 7x - 6) = x \times (x^2 - 7x - 6) + 1 \times (x^2 - 7x - 6)
$$

Step 2: Expand the terms

- $x \times (x^2 - 7x - 6) = x^3 - 7x^2 - 6x$
- $1 \times (x^2 - 7x - 6) = x^2 - 7x - 6$

Step 3: Combine the expanded terms

$$
x^3 - 7x^2 - 6x + x^2 - 7x - 6
$$

which simplifies to:

$$
x^3 - 6x^2 - 13x - 6
$$

Step 4: Add the remainder -3

$$
F(x) = x^3 - 6x^2 - 13x - 6 + (-3)
$$

$$
F(x) = x^3 - 6x^2 - 13x - 9
$$

Final Expression for $F(x)$

The polynomial $F(x)$ is:

$$
\boxed{F(x) = x^3 - 6x^2 - 13x - 9}
$$

This is the original polynomial function that, when divided by $x + 1$, yields the quotient $x^2 - 7x - 6$ and the remainder -3.

Verifying the Solution

To ensure our solution is accurate, we can perform the division process in reverse or substitute specific values into $F(x)$ to verify the division.

Verification by Polynomial Division

1. Divide $F(x)$ by $x + 1$ using synthetic division or polynomial long division.
2. Confirm that the quotient matches $x^2 - 7x - 6$.
3. Confirm that the remainder is -3.

Step 1: Polynomial long division

Dividing $F(x) = x^3 - 6x^2 - 13x - 9$ by $x + 1$:

- The leading term x^3 divided by x gives x^2.
- Multiply $x + 1$ by x^2: $x^3 + x^2$.
- Subtract: $(x^3 - 6x^2) - (x^3 + x^2) = -7x^2$.
- Bring down the remaining terms: $-7x^2 - 13x$.

Repeat:

- Divide $-7x^2$ by x: $-7x$.
- Multiply $x + 1$ by $-7x$: $-7x^2 - 7x$.

- Subtract: $(-7x^2 - 13x) - (-7x^2 - 7x) = -6x$.
- Bring down -9.

Next:

- Divide $-6x$ by x: -6.
- Multiply $x + 1$ by -6: $-6x - 6$.
- Subtract: $(-6x - 9) - (-6x - 6) = -3$.

The remainder is -3, which matches our earlier calculation, and the quotient is $x^2 - 7x - 6$. This confirms the correctness of our derived polynomial $F(x)$.

Implications and Applications in Algebra

Understanding how to reconstruct a polynomial from its division quotient and remainder is crucial in multiple algebraic contexts. Some key applications include:

- Factorization: Recognizing factors of polynomials by division.
- Polynomial Remainder Theorem: Quickly evaluating polynomial remainders when divided by linear factors.
- Synthetic Division: Simplifying polynomial division for higher-degree polynomials.
- Roots and Zeroes: Finding roots by setting the polynomial equal to zero and analyzing its factors.

Additional Tips for Solving Similar Problems

When faced with problems similar to this one, consider the following strategies:

1. Identify the divisor, quotient, and remainder clearly.
2. Use the division formula $F(x) = D(x) \times Q(x) + R$.
3. Perform polynomial multiplication carefully, aligning like terms.
4. Simplify step-by-step to avoid errors.
5. Verify your result by performing the reverse division or plugging in values.

Conclusion

In this comprehensive exploration, we've demonstrated how to find the original polynomial $F(x)$ given the quotient and remainder when dividing by a linear divisor. By applying polynomial multiplication and the division formula, we derived that:

$$
\boxed{F(x) = x^3 - 6x^2 - 13x - 9}
$$

This process not only helps solve specific algebraic problems but also enhances understanding of polynomial behaviors, factorization, and the fundamental theorem of algebra. Mastery of polynomial division techniques is essential for students and professionals working with algebraic expressions, equations, and higher-level mathematics.

Keywords: polynomial division, quotient, remainder, algebra, polynomial factorization, polynomial theorem, algebraic problem-solving, reconstruct polynomial

Frequently Asked Questions

What is the original function F(x) given that dividing by (x+1) yields a quotient of $x^2 - 7x - 6$ and a remainder of -3?

$F(x) = (x + 1)(x^2 - 7x - 6) - 3$, which simplifies to $F(x) = x^3 - 6x^2 - 13x - 3$.

How do you find the original function F(x) if the quotient and remainder after division by (x+1) are given?

Use the relation F(x) = (x+1) quotient + remainder. Substitute the given quotient and remainder to find F(x).

What is the process to verify the original function F(x) after dividing by (x+1)?

Multiply the quotient by (x+1), then add the remainder. If the result matches F(x), the division is verified.

What is the value of F(-1) based on the given division details?

Since the division remainder is -3 when dividing by (x+1), which is zero at x = -1, F(-1) = -3.

Can you find the degree of the original polynomial F(x) from the given quotient?

Yes. Since the quotient is degree 2 and dividing by a linear polynomial reduces degree by 1, F(x) is degree 3.

What is the significance of the remainder being -3 in the division process?

The remainder indicates the leftover after division, confirming that F(x) evaluated at x = -1 equals -3.

How does the factor (x + 1) relate to the roots of F(x)?

Since dividing by (x+1) results in a remainder, (x+1) is not necessarily a factor of F(x), unless the remainder is zero.

Is the original function F(x) uniquely determined by the given quotient and remainder?

Yes. The division process yields a specific F(x) = x^3 - 6x^2 - 13x - 3.

What are the steps to derive F(x) from the given division information?

Multiply the quotient (x^2 - 7x - 6) by (x+1), then add the remainder (-3): F(x) = (x+1)(x^2 - 7x - 6) - 3.

What is the simplified form of the polynomial F(x) based on the division details?

F(x) = x^3 - 6x^2 - 13x - 3.

Additional Resources

When The Function F(x) Is Divided By X+1, The Quotient Is X^(2)-7x-6 And The Remainder Is -3. Find The

---

Introduction

In the realm of algebra, polynomial division is a fundamental concept that bridges the gap between simple algebraic expressions and more complex functions. Whether you're a student tackling calculus or a professional solving engineering problems, understanding how to interpret division of polynomials is crucial. Today, we explore a specific problem involving polynomial division, given the quotient and remainder, and aim to find the original function \(F(x)\).

This problem encapsulates core principles of polynomial algebra—particularly the polynomial division algorithm—and demonstrates how to reverse-engineer a polynomial when certain division parameters are provided. We'll dissect the problem, explore the underlying principles, and methodically arrive at the solution, enhancing both comprehension and problem-solving skills.

Understanding Polynomial Division: The Foundation

Before diving into the specific problem, let's establish a clear understanding of polynomial division.

What Is Polynomial Division?

Polynomial division resembles long division with numbers, but it applies to algebraic expressions. If you have a polynomial $F(x)$ divided by a divisor $D(x)$, the division yields a quotient $Q(x)$ and a remainder $R(x)$:

$$
F(x) = D(x) \times Q(x) + R(x)
$$

- $D(x)$: The divisor polynomial (in our case, $x + 1$)
- $Q(x)$: The quotient polynomial
- $R(x)$: The remainder polynomial or constant

The degree of the remainder polynomial is always less than the degree of the divisor polynomial.

The Polynomial Division Algorithm

The polynomial division algorithm states:

> For any polynomials $F(x)$ and $D(x)$ with $D(x) \neq 0$, there exist unique polynomials $Q(x)$ and $R(x)$ such that:
>
> $$
> F(x) = D(x) \times Q(x) + R(x)
> $$
>
> where $\deg(R) < \deg(D)$.

In our problem, $D(x) = x + 1$, which is a first-degree polynomial. Therefore, the remainder $R(x)$ must be a constant (degree zero).

Dissecting the Given Data

The problem provides:

- The quotient: $Q(x) = x^2 - 7x - 6$
- The remainder: $R = -3$
- The divisor: $x + 1$

Our goal is to find the original function $F(x)$.

Applying the Division Algorithm

Using the division relation:

$$
F(x) = (x + 1) \times Q(x) + R
$$

Substitute $Q(x)$ and R:

$$
F(x) = (x + 1)(x^2 - 7x - 6) + (-3)
$$

Our task reduces to expanding this expression and simplifying to find $F(x)$.

Step-by-Step Solution

Step 1: Expand the Product $(x + 1)(x^2 - 7x - 6)$

Apply distributive property (FOIL):

$$
(x + 1)(x^2 - 7x - 6) = x \times (x^2 - 7x - 6) + 1 \times (x^2 - 7x - 6)
$$

Calculate each term:

- $x \times x^2 = x^3$
- $x \times (-7x) = -7x^2$
- $x \times (-6) = -6x$
- $1 \times x^2 = x^2$
- $1 \times (-7x) = -7x$
- $1 \times (-6) = -6$

Now, combine all:

$$
x^3 - 7x^2 - 6x + x^2 - 7x - 6
$$

Simplify like terms:

- $-7x^2 + x^2 = -6x^2$
- $-6x - 7x = -13x$

So, the expanded form is:

$$
x^3 - 6x^2 - 13x - 6
$$

Step 2: Add the Remainder (-3)

Recall:

$$
F(x) = \text{expanded product} + R
$$

Thus:

$$
F(x) = x^3 - 6x^2 - 13x - 6 + (-3) = x^3 - 6x^2 - 13x - 9
$$

Final Expression:

$$
\boxed{
F(x) = x^3 - 6x^2 - 13x - 9
}
$$

This is the original polynomial function $F(x)$ that, when divided by $x + 1$, yields the quotient $x^2 - 7x - 6$ and the remainder -3.

Verification: Confirming the Solution

To ensure accuracy, let's verify the division:

$$
F(x) \div (x + 1)
$$

Using polynomial division or synthetic division:

- Synthetic division with root -1:

Set up coefficients of $F(x)$:

$$
1 \quad -6 \quad -13 \quad -9
$$

Perform synthetic division:

1. Bring down the leading coefficient (1).

2. Multiply -1 (divisor root) by 1: -1; add to -6: $-6 + (-1) = -7$.

3. Multiply -1 by -7: 7; add to -13: $-13 + 7 = -6$.

4. Multiply -1 by -6: 6; add to -9: $-9 + 6 = -3$.

Resulting bottom row:

$$
1 \quad -7 \quad -6 \quad -3
$$

- The quotient coefficients are: $1, -7, -6$, forming $x^2 -7x -6$.

- The remainder is -3.

This matches the given quotient and remainder, confirming the correctness of the derived $F(x)$.

Implications and Applications

Understanding such polynomial problems is fundamental in various mathematical and engineering fields:

- Algebraic Factorization: Knowing how to reconstruct a polynomial from division data aids in factorization tasks.
- Root Finding: Since division by $x+1$ relates to evaluating $F(-1)$, understanding remainders connects to the Remainder Theorem.
- Polynomial Interpolation and Approximation: Reconstructing functions from partial division info supports data modeling.
- Control Systems & Signal Processing: Polynomial manipulations underpin system stability analyses.

Conclusion

The problem of finding $F(x)$ given the quotient and remainder upon division by a linear divisor exemplifies core algebraic principles. By applying the polynomial division algorithm, expanding the product, and adding the remainder, we derived:

$$
\boxed{
F(x) = x^3 - 6x^2 - 13x - 9
}
$$

This process underscores the importance of systematic algebraic operations and verification in ensuring accuracy. Mastery of polynomial division not only aids in solving such problems but also forms the backbone of more advanced mathematical concepts across science and engineering.

When The Function F X Is Divided By X 1 The Quotient Is X 2 7x 6 And The Remainder Is 3 Find The

When The Function F X Is Divided By X 1 The Quotient Is X 2 7x 6 And The Remainder Is 3 Find The

Related Articles

- Underline The Adjectives, Including Articles, In Each Of The Following Sentences.The Number At The End
- A Bag Contains 6 Blue Marbles, 8 Green Marbles, 9 Red Marbles, And 17 Yellow Marbles. What Is The Ratio
- What Law Of Psychophysics States That Two Stimuli Must Differ By A Constant Minimum Percentage (rather

Back to Home