Write A System Of Equations To Describe The Situation Below, Solve Using Substitution, And Fill In The

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Introduction: Understanding the Importance of Systems of Equations

In mathematics, systems of equations are powerful tools used to solve problems involving multiple variables. They allow us to model real-world situations where more than one unknown interacts simultaneously. Whether it's calculating costs, determining quantities, or analyzing relationships, systems of equations help translate complex scenarios into manageable mathematical forms.

This article focuses on how to construct a system of equations based on a given problem, solve it using the substitution method, and interpret the results. Mastering this process enhances problem-solving skills and deepens understanding of algebraic concepts.

Scenario Description: A Real-World Example

Imagine you are managing a small business that sells two types of products: Product A and Product B. Your goal is to determine the number of units of each product you need to sell to meet certain revenue targets, given the prices per unit.

Here's the detailed situation:

- The total revenue from selling both products is $1,200.
- The revenue from Product A is $50 per unit.
- The revenue from Product B is $30 per unit.
- The total number of units sold for both products is 40 units.
- You need to find out how many units of Product A and Product B were sold.

This scenario involves two unknowns: the number of units sold for each product. We will now translate this scenario into a system of equations.

Step 1: Defining Variables

The first step in modeling the problem is to assign variables to the unknowns:

1. **x** = the number of units of Product A sold
2. **y** = the number of units of Product B sold

These variables will be used to formulate the equations.

Step 2: Creating the System of Equations

Using the information from the scenario, we can write equations based on the total units and total revenue.

Equation 1: Total Units Sold

Since the total units sold for both products is 40, we have:

```
x + y = 40
```

This is our first equation.

Equation 2: Total Revenue

Total revenue is calculated as the sum of revenue from each product:

```
50x + 30y = 1200
```

This is our second equation.

Thus, the system of equations describing the scenario is:

```
x + y = 40
50x + 30y = 1200
```

Step 3: Solving the System Using Substitution

The substitution method involves solving one of the equations for one variable and then substituting that expression into the other equation. Let's proceed step-by-step.

Step 3.1: Solve for One Variable

From the first equation:

```
x + y = 40
```

Solve for x:

```
x = 40 - y
```

Step 3.2: Substitute into the Second Equation

Replace x in the second equation:

```
50(40 - y) + 30y = 1200
```

Now, simplify and solve for y.

Step 4: Solving for the Variables

```
50(40 - y) + 30y = 1200
```

Distribute 50:

```
2000 - 50y + 30y = 1200
```

Combine like terms:

```
2000 - 20y = 1200
```

Subtract 2000 from both sides:

```
-20y = 1200 - 2000
-20y = -800
```

Divide both sides by -20:

```
y = \(\frac{-800}{-20}\) = 40
```

Now that y is known, substitute back into the expression for x:

```
x = 40 - y = 40 - 40 = 0
```

Step 5: Interpreting the Solution

The solution indicates:

- x = 0 units of Product A sold
- y = 40 units of Product B sold

This means all 40 units sold are of Product B, and none of Product A.

Verification of the Solution

Verify that the solution satisfies both equations:

1. Units:

```
x + y = 0 + 40 = 40 ✓
```

2. Revenue:

```
50(0) + 30(40) = 0 + 1200 = 1200 ✓
```

Both equations are satisfied, confirming the correctness of the solution.

Additional Considerations and Variations

While this example is straightforward, real-world problems can be more complex. Here are some considerations:

1. **Constraints:** Sometimes, variables must be non-negative integers (e.g., units can't be negative or fractional). Ensure your solutions make sense in context.

2. **Multiple solutions:** Some systems may have infinite solutions or none, depending on the equations.

3. **Alternative methods:** Besides substitution, methods such as addition/subtraction (elimination) or graphing can be used.

Practice Problem for Reinforcement

Try solving this similar scenario:

- A bakery sells small and large loaves of bread.
- The total number of loaves sold is 50.
- The total revenue from both types is $250.
- Each small loaf sells for $3, and each large loaf sells for $5.

Formulate the system of equations, solve using substitution, and interpret the results.

Conclusion: Mastering Systems of Equations

Understanding how to model real-world scenarios with systems of equations is an essential skill in algebra. By carefully defining variables, translating problem statements into equations, and applying methods such as substitution, you can efficiently find solutions that inform decision-making and problem-solving in various contexts.

Remember, practice is key. Work through different problems, verify solutions, and explore alternative methods to deepen your understanding. With these skills, you'll be well-equipped to handle complex problems involving multiple variables with confidence.

Summary of Key Steps

1. **Define variables** representing unknown quantities.
2. **Translate the problem** into a system of equations based on given information.
3. **Solve one equation** for one variable.
4. **Substitute** into the other equation and solve for the remaining variable.
5. **Check your solution** in both original equations.
6. **Interpret the results** in the context of the problem.

By mastering this process, you will be able to effectively approach a wide range of algebraic problems and real-world situations that require solving systems of equations.

Frequently Asked Questions

What is the first step in writing a system of equations to model a real-world situation involving two variables?

Identify the key quantities involved and translate the relationships described in the problem into algebraic equations using those variables.

How do you set up a system of equations to describe a situation where one variable depends on the other?

Express one variable in terms of the other or write an equation based on the given relationships, then create a second equation representing the other condition or total.

What is the substitution method in solving systems of equations?

The substitution method involves solving one equation for one variable and then substituting that expression into the other equation to find the value of the remaining variable.

Can you give an example of a situation suitable for solving with substitution?

Yes, for example, if a store sells pencils and erasers, and you know the total cost and the relationship between the quantities, you can set up equations and use substitution to find the number of each item purchased.

How do you interpret the solution of a system of equations in a real-world context?

The solution provides the specific values of the variables that satisfy all conditions of the problem, such as quantities or costs, giving a meaningful answer to the original situation.

What does 'fill in the' refer to in the context of solving systems of equations, and how is it relevant?

It likely refers to completing missing information or calculating the final values after solving the system, such as filling in the quantities, costs, or other details in a table or statement based on the solution.

Why is it important to verify your solution after solving a system of equations in a real-world problem?

Verifying ensures that the solution makes sense within the context, satisfies all original conditions, and accurately models the situation, preventing errors from incorrect assumptions or calculations.

Additional Resources

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Introduction

In the realm of algebra, systems of equations serve as essential tools for modeling and solving real-world problems involving multiple variables. Whether tracking financial transactions, analyzing physical phenomena, or solving puzzles, the ability to translate a scenario into mathematical language and then find the solution is a vital skill. This article aims to thoroughly explore the process of constructing a system of equations from a given situation, solving it using the substitution method, and interpreting the solutions accurately.

We will delve into each step with clarity, starting from understanding the problem context, formulating the equations, applying the substitution technique, and finally filling in the missing information. Through this comprehensive analysis, readers will gain insights into both the conceptual framework and practical execution of solving systems of equations, equipping them with the tools necessary for tackling similar problems in academics, work, or daily life.

Understanding the Situation

Before jumping into mathematical formulations, it is crucial to comprehend the scenario fully. Typically, such problems involve two or more unknown quantities that are related through certain conditions or constraints.

Clarifying the Context

Suppose we are presented with a problem involving two quantities—such as the number of products sold, the amount of money earned, or the quantities of ingredients used—whose interdependence we need to analyze. The first step is to identify what is given:

- What are the known quantities?
- What are the unknown quantities?
- How are these quantities related?
- Are there any constraints or conditions specified?

Example Scenario

To illustrate, imagine a scenario involving two friends, Alice and Bob, who are selling handcrafted jewelry. The problem may specify:

- The total number of jewelry pieces sold by both friends combined.
- The total earnings from their sales.
- The prices per piece for each person, which may differ.

From these details, we can derive relationships between the variables—such as the number of pieces sold by each person and the total earnings.

Formulating the System of Equations

Once the scenario is understood, the next step is translating it into algebraic expressions. This involves defining variables, establishing relationships, and writing equations that encapsulate the problem's conditions.

Defining Variables

Choose variables that represent the unknown quantities. For example:

- Let x = number of pieces sold by Alice.
- Let y = number of pieces sold by Bob.

Establishing Relationships and Constraints

Based on the problem, identify the equations:

1. Total items sold: If the total number of jewelry pieces sold is known, say, 40, then:

$$
x + y = 40
$$

2. Total earnings: If Alice earns \$15 per piece and Bob earns \$20 per piece, and total earnings are \$700, then:

$$
15x + 20y = 700
$$

These two equations form the system that models the situation.

Summary

In this example, the system of equations is:

$$
\begin{cases}
x + y = 40 \\
15x + 20y = 700
\end{cases}
$$

This system will be solved to find the specific values of x and y, revealing how many pieces each person sold.

Solving the System Using Substitution

The substitution method involves solving one of the equations for one variable and then substituting that expression into the other equation. This method simplifies the system to a single-variable equation, which can then be solved directly.

Step 1: Solve One Equation for a Variable

From the first equation:

$$
x + y = 40
$$

Solve for y:

$$
y = 40 - x
$$

Step 2: Substitute into the Second Equation

Replace y in the second equation:

$$
15x + 20(40 - x) = 700
$$

Step 3: Simplify and Solve for x

Distribute:

$$
15x + 20 \times 40 - 20x = 700
$$

$$
15x + 800 - 20x = 700
$$

Combine like terms:

$$
(15x - 20x) + 800 = 700
$$

$$
-5x + 800 = 700
$$

Subtract 800 from both sides:

$$
-5x = -100
$$

Divide both sides by -5:

$$
x = 20
$$

Step 4: Find y

Recall:

$$
y = 40 - x = 40 - 20 = 20
$$

Result

The solution indicates:

- Alice sold 20 pieces.

- Bob sold 20 pieces.

Verifying the Solution

Verification is a crucial step to ensure the solution satisfies all conditions.

- Check total items:

$$
x + y = 20 + 20 = 40 \quad \checkmark
$$

- Check total earnings:

$$
15 \times 20 + 20 \times 20 = 300 + 400 = 700 \quad \checkmark
$$

Since both conditions are satisfied, the solution is valid.

Filling in the Missing Information

Often, problems may require filling in missing data or interpreting the solution in context.

Practical Interpretation

In this example, the interpretation is straightforward:

- Both Alice and Bob sold 20 pieces each.
- Total earnings were \$700, aligning with the initial conditions.

Extending the Problem

Suppose the problem asked for additional insights, such as:

- How much money did each person earn?
- If the prices per piece changed, how would the solution adapt?
- What is the impact if total sales or earnings change?

These extensions involve adjusting the equations accordingly and re-solving, illustrating the flexibility of the algebraic approach.

Analytical Insights and Broader Implications

The Power of Systematic Approach

The systematic process—defining variables, translating conditions into equations, choosing an appropriate solving method, and verifying—ensures clarity and accuracy. The substitution method, in particular, is effective when one equation is easily solved for one variable.

Real-World Applications

Systems of equations and their solutions permeate various fields:

- Economics: Modeling supply and demand.
- Physics: Calculating forces and velocities.
- Business: Analyzing profit and cost functions.
- Environmental Science: Balancing ecological equations.

Limitations and Considerations

While powerful, systems of equations can become complex with more variables or nonlinear relationships. In such cases, alternative methods like elimination, graphing, or matrix techniques (e.g., Gaussian elimination) may be more suitable.

Conclusion

Constructing and solving systems of equations is a foundational skill in algebra that empowers us to analyze complex situations systematically. By translating real-world scenarios into mathematical models, applying the substitution method effectively, and interpreting solutions accurately, we gain valuable insights into the relationships between variables.

This article has provided a comprehensive guide—from understanding the initial problem, formulating equations, solving through substitution, to filling in missing data—equipping readers with a robust framework for tackling similar problems. Whether in academic settings or real-life applications, mastering these techniques enhances problem-solving efficiency and analytical thinking, essential skills in an increasingly data-driven world.

References

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Note: For specific problems, always ensure to carefully read the scenario, clearly define your variables, and double-check your solutions to confirm they satisfy all given conditions.

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