Write Each Equation In Polar Coordinates. Express As A Function Of T . Assume That R > 0 (a) Y = 10

Understanding the Conversion of Cartesian Equations to Polar Coordinates

Write Each Equation In Polar Coordinates. Express As A Function Of T . Assume That R > 0 (a) Y = 10.

This statement introduces a fundamental problem in the realm of coordinate systems—transforming Cartesian equations into their polar equivalents. Such conversions are crucial in fields like physics, engineering, and mathematics, where problems often become more manageable or insightful when expressed in different coordinate systems. The goal here is to take the Cartesian equation $y = 10$, convert it into polar coordinates, and express it as a function of the parameter T, assuming that the radius r (or R) is positive.

Fundamentals of Polar Coordinates

Definition of Polar Coordinates

Polar coordinates represent points in the plane using a radius and an angle. Instead of the (x, y) Cartesian system, a point is described by:

- r (or R): The distance from the origin to the point.
- θ (theta): The angle measured counterclockwise from the positive x-axis to the line segment from the origin to the point.

These relationships are defined as:

- $x = r \cos \theta$
- $y = r \sin \theta$

Conversely, from Cartesian to polar:

- $r = \sqrt{x^2 + y^2}$
- $\theta = \text{arctangent}(y / x)$, with quadrant considerations.

Expressing Equations in Polar Coordinates

Any Cartesian equation can, in principle, be converted into a polar form by substituting x and y with their polar equivalents. Conversely, one can parametrize the equations as functions of θ or another parameter T, which is often used to represent an angle or a parameter along a curve.

Converting y = 10 into Polar Coordinates

Step 1: Recognize the Cartesian Equation

The given Cartesian equation:

$$
y = 10
$$

describes a horizontal line at $y = 10$ in the xy-plane.

Step 2: Express y in Terms of r and θ

Using the polar coordinate relationship:

$$
y = r \sin \theta
$$

the equation becomes:

$$
r \sin \theta = 10
$$

which can be rearranged to express r as a function of θ:

$$
r = \frac{10}{\sin \theta}
$$

Note that this expression is valid as long as $\sin \theta \neq 0$, i.e., for $\theta \neq 0, \pi, 2\pi, \ldots$.

Step 3: Express as a Function of T (Parameterization)

Here, T can be chosen as the parameter representing the angle θ itself, since the curve's points can be described by θ varying over an interval where the equation makes sense.

Therefore:

$$
r(T) = \frac{10}{\sin T}
$$

with the condition that $\sin T \neq 0$, i.e.,

$$
T \neq n\pi, \quad n \in \mathbb{Z}
$$

This parametrization gives the entire line y=10 (except at points where the line is undefined due to division by zero).

Interpreting the Polar Equation r(T) = 10 / sin T

Geometric Significance

- The equation $r = \frac{10}{\sin T}$ describes a straight line in the xy-plane.
- As T varies, r changes according to the sine of T, creating an infinite set of points along the line y=10.
- The restriction $\sin T \neq 0$ excludes angles where the line would be undefined in the polar form, corresponding to the points where the line crosses the x-axis (which it does not, since y=10 is always above it).

Special Cases and Limits

- When $T \to 0^+$, $\sin T \to 0^+$, so $r \to +\infty$.
- When $T \to \pi^-$, $\sin T \to 0^+$, so $r \to +\infty$, but with opposite signs depending on the quadrant.
- The curve extends infinitely, consistent with the nature of a line in Cartesian coordinates.

Alternative Parameterizations

Using θ as the Parameter

Instead of T, you can explicitly let θ be the parameter:

$$
r(\theta) = \frac{10}{\sin \theta}
$$

for $\theta \neq n \pi$.

Using a Different Parameter T

If T is an arbitrary parameter, you could define:

$$
T = \theta
$$

then:

$$
r(T) = \frac{10}{\sin T}
$$

This approach is particularly useful when plotting or analyzing the behavior of the curve as T varies over an interval.

Extending to Other Cartesian Equations

While y=10 is a straightforward example, the process applies broadly:

- For equations like $y = mx + c$, substitution yields $r \sin \theta = m r \cos \theta + c$, leading to a relation involving r and θ.
- For circles, parabolas, or ellipses, the substitution often results in more complex equations in r and θ, which can sometimes be simplified or parametrized.

Summary and Practical Applications

- Converting Cartesian equations to polar coordinates simplifies the analysis

of curves with symmetry around the origin.
- Expressing the equations as functions of a parameter T (or θ) allows for easy plotting, analysis, and understanding of the curve's behavior.
- The equation $y=10$ in polar form as $r = 10 / \sin T$ provides a clear parametric representation that is useful in physics (e.g., wavefronts), engineering (e.g., antenna patterns), and mathematics (e.g., conic sections).

Conclusion

Transforming Cartesian equations into polar coordinates is a fundamental skill that enhances our understanding of geometric and algebraic properties of curves. The process involves substituting x and y with their polar equivalents, solving for r, and choosing an appropriate parameter—often the angle θ or a custom parameter T—to describe the curve succinctly. In the specific case of the line y=10, the polar form $r = 10 / \sin T$ elegantly captures the entire set of points along the line, excluding points where the sine of T is zero. This approach underscores the power of coordinate transformations in simplifying complex problems and revealing the underlying symmetry of geometric figures.

Frequently Asked Questions

How do you convert the equation y = 10 into polar coordinates as a function of t?

Since $y = r \sin \theta$, setting $y = 10$ gives $r \sin \theta = 10$, so $r = 10 / \sin \theta$, which can be expressed as $r(t) = 10 / \sin t$ assuming t represents θ.

What is the polar form of the line y = 10 when expressed as a function of t?

The polar form is $r(t) = 10 / \sin t$, valid for t where $\sin t \neq 0$.

How do we interpret the equation y = 10 in polar coordinates with respect to t?

It represents all points where the radius r is 10 divided by sin t, meaning r depends on the angle t, with the condition that $t \neq 0$ or π, where $\sin t = 0$.

Is y = 10 a curve or a line in polar coordinates when expressed as r(t) = 10 / sin t?

It represents a straight line $y = 10$ in Cartesian coordinates, but in polar coordinates, it's expressed as a curve with r depending on t via $r(t) = 10 /$

sin t.

For y = 10, what are the domain restrictions for t in the polar form r(t) = 10 / sin t?

t must be in (0, π) or (π, 2π), excluding t = 0 and t = π where sin t = 0 and r becomes undefined.

How can you graph y = 10 in polar coordinates using r(t) = 10 / sin t?

Plot points for various t values where sin t ≠ 0, calculating r = 10 / sin t, which traces a straight line parallel to the x-axis at y = 10 in Cartesian coordinates.

What is the significance of assuming R > 0 in the context of converting y = 10 to polar coordinates?

Assuming R > 0 indicates considering only the positive radius values, which aligns with the standard polar coordinate convention and helps specify the direction of the curve.

Can the equation y = 10 be expressed as a function of t in the form r(t) = ... for all t?

Yes, as long as t ≠ 0 or π, with r(t) = 10 / sin t; at these points, the function is undefined due to division by zero.

How does the equation y = 10 relate to the polar equation r = 10 / sin t in terms of geometric interpretation?

It represents a horizontal line in Cartesian coordinates, and in polar form, it corresponds to all points where the radius r is 10 divided by sin t, forming a straight line in the plane.

What steps are involved in converting y = 10 into a polar coordinate function of t?

First, recognize y = r sin t, then solve for r: r = y / sin t. Substituting y = 10 yields r(t) = 10 / sin t, giving the polar form as a function of t.

Additional Resources

Write Each Equation In Polar Coordinates. Express As A Function Of T. Assume That R > 0 (a) Y = 10

Introduction

In the realm of mathematics, especially in the fields of calculus and analytical geometry, the transition between coordinate systems holds significant importance. Among these, the conversion of equations from Cartesian coordinates to polar coordinates offers profound insights into the geometric nature of curves and their behaviors. This process becomes particularly valuable when analyzing curves with symmetry about the origin or when dealing with problems involving angular motion or radial symmetry.

The specific problem at hand involves rewriting a simple Cartesian equation, $Y = 10$, into its equivalent form in polar coordinates, with the goal of expressing it as a function of the parameter t—often an angular parameter such as θ or t, depending on the context. Since the problem states $Y = 10$ and assumes $R > 0$, it invites a detailed exploration of how this horizontal line translates into polar form, and how to parametrize it as a function of a variable t.

This article will systematically dissect this transformation process, starting with foundational concepts of polar coordinates, followed by the detailed conversion of the given Cartesian equation, and culminating in a comprehensive parametrization as a function of t. The goal is to not only provide the necessary formulas but also to deepen understanding through thorough explanations, geometric interpretations, and applications.

Understanding Polar Coordinates

The Basics of Polar Coordinates

In Cartesian coordinates, points are described by their x and y coordinates. Conversely, in polar coordinates, a point in the plane is represented by a pair (r, θ), where:

- r is the radial distance from the origin to the point.
- θ (theta) is the angle measured counterclockwise from the positive x-axis to the line segment connecting the origin to the point.

The relationships between Cartesian and polar coordinates are given by the formulas:

\[
x = r \cos \theta

\]

$$
y = r \sin \theta
$$

Similarly, if given x and y, the conversion to polar coordinates involves:

$$
r = \sqrt{x^2 + y^2}
$$

$$
\theta = \arctan \left(\frac{y}{x} \right)
$$

Significance of $R > 0$

The assumption $R > 0$ signifies that we are considering only points in the plane that are away from the origin with a positive radius. This preserves the standard interpretation of radius as a positive distance. When converting equations, this assumption simplifies the analysis and avoids ambiguities associated with negative radii or points on the negative x-axis.

Converting the Cartesian Equation $Y = 10$ to Polar Coordinates

Geometric Interpretation

The equation $Y = 10$ in Cartesian coordinates represents a horizontal line parallel to the x-axis, intersecting the y-axis at $y = 10$. Geometrically, this line extends infinitely in both directions along the x-axis, maintaining a constant y-value of 10.

In the context of polar coordinates, this line manifests as a collection of points where the y-coordinate is fixed at 10, but the x-coordinate varies accordingly. To express this line in polar form, we substitute the relationships:

$$
x = r \cos \theta
$$

$$
y = r \sin \theta
$$

Given that $Y = 10$, we substitute y:

$$
r \sin \theta = 10
$$

This yields the polar equation of the horizontal line:

$$
r = \frac{10}{\sin \theta}
$$

This formula is valid for all θ such that $\sin \theta \neq 0$, i.e., θ ≠ 0, π, 2π, etc., where the line would be undefined in the polar form because r would tend toward infinity.

Expressing as a Function of T: Parametrization

While the polar equation $r = \frac{10}{\sin \theta}$ describes the line, to express this as a function of a parameter t, we typically consider t as an angular parameter, such as θ, or define a parametric form directly.

Choosing the Parameter t

In many contexts, t is used as an arbitrary parameter that can represent an angle or any real variable. For the purposes of this problem, we will treat t as the angle θ, which makes sense because the polar form inherently involves an angular component.

Thus, the parametrization becomes:

$$
r(t) = \frac{10}{\sin t}
$$
$$
\theta(t) = t
$$

with t spanning the domain where $\sin t \neq 0$, i.e., $t \neq n\pi$, for integers n.

Complete Parametric Equations

The parametric equations for the points on the line in terms of t are:

$$
x(t) = r(t) \cos t = \frac{10}{\sin t} \cos t
$$
$$
y(t) = r(t) \sin t = 10
$$

Since y is constant at 10, the x-coordinate varies with t, corresponding to the horizontal line at y=10.

Analytical Insights and Domain Considerations

Validity of the Parametrization

The function $r(t) = \frac{10}{\sin t}$ is undefined at points where $\sin t = 0$, i.e., at $t = n\pi$. These points correspond to the angles where the line approaches infinity, which physically indicates that the point is approaching the line at infinity in the polar plane.

For practical purposes, the domain of t can be specified as:

$$
t \in \mathbb{R} \setminus \{ n\pi \mid n \in \mathbb{Z} \}
$$

Within each interval between these points, the function $r(t)$ is continuous and provides a valid parametrization of the line.

Behavior at Specific Angles

- At $t = \frac{\pi}{2}$, $\sin t = 1$, so:

$$
r = \frac{10}{1} = 10
$$

and the point in Cartesian coordinates is:

$$
x = 10 \cos \frac{\pi}{2} = 0, \quad y = 10
$$

which is the point directly above the origin at (0,10).

- As $t \to 0^+$, $\sin t \to 0^+$, so $r \to +\infty$. The line extends infinitely in one direction.

Applications and Broader Context

Significance of the Polar Representation

Expressing the line Y = 10 in polar coordinates and as a function of t has several applications:

- In calculus, it facilitates the computation of line integrals, contour integrals, or area calculations involving the line.
- In physics, especially in mechanics and electromagnetism, it allows for

modeling phenomena with radial or angular symmetry.
- In computer graphics, parametric equations enable plotting and rendering of curves and lines efficiently.

Extending to Other Equations

The methodology used here can be generalized to convert other Cartesian equations into polar form and parametrize them as functions of an angular parameter:

- Lines with arbitrary slope and intercept.
- Circles, ellipses, hyperbolas, and other conic sections.
- Complex curves with symmetry about the origin.

Summary and Final Remarks

The transformation of the Cartesian equation Y = 10 into polar coordinates illustrates the elegance and utility of coordinate system conversions. By recognizing that Y = 10 corresponds to the polar equation:

$$
r = \frac{10}{\sin \theta}
$$

we gain a new perspective on the line as a set of points parameterized by the angle θ.

Expressing this as a function of t (chosen as θ), the parametric equations:

$$
x(t) = \frac{10}{\sin t} \cos t, \quad y(t) = 10
$$

provide a complete, analytical description suitable for computation, visualization, and further mathematical analysis.

This process highlights the interplay between geometric intuition and algebraic manipulation, reinforcing the importance of understanding multiple coordinate systems in advanced mathematics.

References

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Final Thoughts

Mastering the conversion of equations from Cartesian to polar coordinates and
parametrization as functions of a variable t enhances one's ability to
analyze and interpret complex geometrical figures. It bridges the gap between
algebraic representation and geometric intuition, a core skill in both pure
and applied mathematics.
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