

A Closed Cylindrical Can Is To Hold 1 Cubic Meter Of Liquid. Assuming There Is No Waste Or Overlap, How

A Closed Cylindrical Can Is To Hold 1 Cubic Meter Of Liquid. Assuming There Is No Waste Or Overlap, How can we determine the dimensions of such a can? This question involves understanding the relationship between volume, height, and radius of a cylinder, and applying mathematical formulas to find the optimal dimensions for a closed cylindrical container that can hold exactly one cubic meter of liquid. Whether you are a student, engineer, or DIY enthusiast, grasping these concepts is essential for designing efficient storage solutions or solving related mathematical problems.

Understanding the Basics of Cylindrical Volume

Before diving into calculations, it's important to understand the fundamental formula for the volume of a cylinder.

The Formula for Cylinder Volume

The volume (V) of a cylinder is calculated using the formula:

$$\bullet V = \pi \times r^2 \times h$$

Where:

- **V** is the volume of the cylinder,
- **r** is the radius of the circular base,
- **h** is the height of the cylinder,
- **π** (pi) is approximately 3.14159.

Given that the can must hold exactly 1 cubic meter of liquid, this becomes:

$$\bullet \pi \times r^2 \times h = 1$$

This equation forms the foundation for determining the dimensions of the can.

Calculating the Dimensions of the Can

Since the problem involves two variables, radius (r) and height (h), there are infinitely many solutions unless additional constraints are provided. Typically, the goal is to find the most practical or optimal dimensions based on specific criteria, such as minimizing material or maximizing stability.

Case 1: Fixing the Height and Solving for Radius

Suppose you decide on a specific height for the container; for example, you choose a height of 2 meters.

- Given: $h = 2$ meters
- Volume: $V = 1 \text{ m}^3$

Using the volume formula:

$$\pi \times r^2 \times 2 = 1$$

Solve for r:

$$r^2 = 1 / (2\pi)$$

Calculate:

$$r^2 \approx 1 / (2 \times 3.14159) \approx 1 / 6.28318 \approx 0.15915$$

Then:

$$r \approx \sqrt{0.15915} \approx 0.3989 \text{ meters}$$

Result:

- Radius ≈ 0.399 meters
- Diameter ≈ 0.798 meters

Implication: A can 2 meters tall with a radius of approximately 0.4 meters will hold exactly 1 cubic meter of liquid.

Case 2: Fixing the Radius and Solving for Height

Alternatively, suppose the can's radius is predetermined, say 0.5 meters.

- Given: $r = 0.5$ meters

Using the volume formula:

$$\pi \times (0.5)^2 \times h = 1$$

Calculate:

$$\pi \times 0.25 \times h = 1$$

Solve for h:

$$h = 1 / (\pi \times 0.25) \approx 1 / (0.7854) \approx 1.273 \text{ meters}$$

Result:

- Height ≈ 1.273 meters
- Diameter = 1 meter

Implication: A can with a radius of 0.5 meters needs to be approximately 1.273 meters tall to hold 1 cubic meter of liquid.

Optimizing the Can Dimensions for Practicality

While the previous calculations show various possible dimensions, in real-world applications, certain factors influence the optimal choice.

Factors to Consider in Dimension Selection

- **Material Costs:** Larger surface area increases material costs.
- **Stability:** A wider base (larger radius relative to height) may provide better stability.
- **Storage and Transportation:** Compact dimensions are often preferred for ease of handling.

Designing the Most Efficient Cylinder

Mathematically, for a fixed volume, the shape that minimizes surface area is the most efficient in terms of material use. For a cylinder, this is achieved when the height equals twice the radius ($h = 2r$), known as the "optimal" ratio.

Deriving the Optimal Dimensions

Given $V = \pi r^2 h$, and setting $h = 2r$:

$$V = \pi r^2 (2r) = 2\pi r^3$$

Solve for r :

$$r^3 = V / (2\pi)$$

Plugging in $V = 1$:

$$r^3 \approx 1 / (2 \times 3.14159) \approx 0.15915$$

Calculate:

$$r \approx \sqrt[3]{0.15915} \approx 0.541 \text{ meters}$$

Then,

$$h = 2r \approx 2 \times 0.541 \approx 1.082 \text{ meters}$$

Summary:

- Radius ≈ 0.541 meters
- Height ≈ 1.082 meters
- Diameter ≈ 1.082 meters

This configuration minimizes the material needed for a given volume, offering an efficient design.

Practical Applications and Considerations

Understanding the mathematical relationship between a cylinder's dimensions and its volume is essential across various industries and applications.

Industrial Storage Tanks

Manufacturers often design storage tanks with specific ratios to optimize material use and stability. Knowing how to calculate these dimensions helps in planning and cost estimation.

Packaging and Shipping

Designing cans or containers to hold precise volumes while minimizing material costs and maximizing ease of handling is critical in packaging industries.

DIY Projects and Home Use

For hobbyists or DIY enthusiasts, calculating dimensions can help in creating custom containers, ensuring they hold the desired volume without excess material.

Conclusion

Designing a closed cylindrical can to hold exactly 1 cubic meter of liquid involves understanding the fundamental relationship between volume, radius, and height. Depending on the constraints and practical considerations, various dimensions are possible. Fixing one dimension and solving for the other allows for tailored designs, while optimizing for material efficiency leads to a specific ratio where height equals twice the radius. By applying the volume formula and mathematical reasoning, engineers, designers, and hobbyists can create efficient, cost-effective containers suited to their specific needs.

In summary:

- The volume formula for a cylinder is $V = \pi r^2 h$.
- To find dimensions, decide on fixing either radius or height, then solve accordingly.
- The most material-efficient design occurs when height equals twice the radius.
- Practical considerations influence the final choice of dimensions.

Understanding these principles ensures effective and efficient design of cylindrical containers for various applications, ensuring they meet volume requirements without unnecessary waste or overlap.

Frequently Asked Questions

What is the radius of a closed cylindrical can that holds 1 cubic meter of liquid?

The radius can be found using the volume formula $V = \pi r^2 h$. Since the volume is 1 m^3 , you can express h in terms of r : $h = 1 / (\pi r^2)$. Without additional constraints, the radius can vary, but common practical sizes can be calculated based on specific height or radius preferences.

How do you determine the height of the cylinder if the radius is known?

If the radius r is known, the height h can be calculated using $h = 1 / (\pi r^2)$, since the volume $V = \pi r^2 h = 1 \text{ m}^3$.

What is the optimal ratio of height to radius for a cylindrical can holding 1 cubic meter of liquid?

There isn't a universally optimal ratio; it depends on manufacturing and storage considerations. However, for minimal surface area (which reduces material cost), the optimal ratio occurs when height equals the diameter, i.e., $h = 2r$, which can be used as a practical guideline.

How does changing the radius affect the height of the can for a fixed volume?

Increasing the radius decreases the height, since $h = 1 / (\pi r^2)$. Conversely, decreasing the radius increases the height, maintaining the constant volume of 1 m^3 .

Are there standard sizes for such cylindrical cans in the industry?

Yes, industries often use standard sizes based on common ratios like height-to-radius or height-to-diameter to optimize manufacturing and usability, but the exact dimensions depend on the intended application and design constraints.

What considerations should be taken into account when designing a can to hold exactly 1 cubic meter of liquid?

Design considerations include material strength, ease of handling, manufacturing costs, stability, and the intended use. Ensuring the dimensions are practical for production and storage, and that the can's proportions prevent tipping or structural issues, are essential factors.

Additional Resources

Cylindrical Can Design and Capacity Optimization: An Expert Analysis

In the realm of packaging and storage, the design of a container plays a pivotal role in maximizing efficiency, minimizing waste, and ensuring the safe containment of liquids. When tasked with creating a closed cylindrical can that holds exactly 1 cubic meter of liquid, engineers and designers must consider various factors, from geometric calculations to material constraints. This article provides an in-depth exploration of how to determine the optimal dimensions of such a can, ensuring precision and practicality in its design.

Understanding the Basics: Geometry of a Cylinder

Before delving into calculations, it is essential to understand the fundamental geometric principles that govern the volume of a cylinder.

The Cylinder Formula

The volume (V) of a cylinder is given by the formula:

$$V = \pi r^2 h$$

where:

- (r) is the radius of the base,
- (h) is the height of the cylinder,
- (π) is Pi (~ 3.1416).

Knowing this, the goal is to determine suitable values of (r) and (h) such that the volume is exactly 1 cubic meter:

$$\pi r^2 h = 1 \text{ m}^3$$

Design Objectives and Constraints

In designing the can, several key objectives and constraints must be considered:

- Exact Capacity: The can must hold precisely 1 m³ of liquid, with no waste or overlaps.

- Material Efficiency: Minimize material usage for cost-effectiveness.
- Structural Integrity: The shape should support handling and stacking, avoiding deformation.
- Manufacturing Practicality: Dimensions should be feasible for manufacturing processes.

Determining Dimensions: The Mathematical Approach

The core of the problem involves choosing values for r and h that satisfy the volume requirement. Since the volume formula involves two variables, there are infinitely many solutions. However, from a practical standpoint, certain dimensions are more desirable depending on application needs.

2.1. General Equation

Expressing height in terms of radius:

$$h = \frac{1}{\pi r^2}$$

This inverse relationship indicates that increasing the radius decreases the necessary height, and vice versa.

2.2. Practical Range of Dimensions

To determine feasible dimensions, consider typical manufacturing constraints:

- Minimum radius: Based on material thickness and handling, say at least 0.1 meters.
- Maximum radius: Limited by transportation or storage constraints, perhaps up to 1 meter.

Optimizing for Practical Design

Different design priorities influence the choice of dimensions:

- Tall and Narrow Can: Small radius, large height.
- Short and Wide Can: Large radius, small height.
- Balanced Design: Moderate radius and height for stability and ease of manufacture.

Let's explore some specific examples to illustrate the options.

Sample Calculations and Design Variations

3.1. Example 1: Wide and Short Can

Assume a radius of 0.5 meters:

$$h = \frac{1}{\pi \times (0.5)^2} = \frac{1}{3.1416 \times 0.25} \approx \frac{1}{0.7854} \approx 1.273 \text{ meters}$$

Dimensions:

- Radius: 0.5 m
- Height: approximately 1.273 m

Analysis:

- Pros: Stable base, easier to manufacture with standard sheet metal sizes.
- Cons: Tall height may complicate handling.

3.2. Example 2: Narrow and Tall Can

Assume a radius of 0.2 meters:

$$h = \frac{1}{\pi \times (0.2)^2} = \frac{1}{3.1416 \times 0.04} \approx \frac{1}{0.1257} \approx 7.958 \text{ meters}$$

Dimensions:

- Radius: 0.2 m
- Height: approximately 7.958 m

Analysis:

- Pros: Very narrow base, easier stacking.
- Cons: Excessive height makes manufacturing and handling impractical.

3.3. Example 3: Balanced Dimensions

Assuming a radius of 0.3 meters:

$$h = \frac{1}{\pi \times (0.3)^2} = \frac{1}{3.1416 \times 0.09} \approx \frac{1}{0.2827} \approx 3.536 \text{ meters}$$

\]

Dimensions:

- Radius: 0.3 m
- Height: approximately 3.536 m

Analysis:

- Pros: Reasonable height for manufacturing.
- Cons: Still somewhat tall; may require reinforcement.

Design Considerations Beyond Dimensions

While the calculations provide theoretical dimensions, practical design involves additional considerations:

Material Selection

- Steel, aluminum, or plastic, each with different strength and weight properties.
- Material thickness influences the minimum feasible radius and height.

Manufacturing Constraints

- Standard sheet sizes limit the maximum radius.
- Forming processes may influence the achievable dimensions without defects.

Handling and Transportation

- Taller cans are more prone to tipping.
- Wider cans may be more stable but harder to transport in limited spaces.

Structural Integrity

- Thinner walls require reinforcing, especially for large volumes.
- The height-to-radius ratio affects the can's stability and resistance to deformation.

Alternative Approaches: Optimizing for Cost and Material Use

Designers often seek to minimize material usage while maintaining capacity. This involves calculating the surface area of the cylinder:

$$A = 2\pi r^2 + 2\pi r h$$

where:

- $(2\pi r^2)$ is the area of the top and bottom disks,
- $(2\pi r h)$ is the lateral surface area.

Minimizing the surface area for a given volume leads to an optimal ratio:

$$h = 2r$$

which balances the lateral surface and base areas for efficiency.

4.1. Applying the Optimal Ratio

Set $(h = 2r)$, then:

$$V = \pi r^2 h = \pi r^2 \times 2r = 2\pi r^3$$

Solve for (r) :

$$r^3 = \frac{V}{2\pi} = \frac{1}{2\pi} \approx \frac{1}{6.2832} \approx 0.1592$$

$$r = \sqrt[3]{0.1592} \approx 0.538 \text{ meters}$$

and

$$h = 2r \approx 1.076 \text{ meters}$$

Design Summary:

- Radius: approximately 0.538 m
- Height: approximately 1.076 m

This configuration offers a balanced design, minimizing material use while maintaining the exact volume.

Real-World Application and Final Recommendations

When designing a cylindrical can for practical use, the mathematical calculations provide essential guidance. However, real-world constraints often necessitate adjustments:

- **Manufacturability:** Choose dimensions that align with standard manufacturing equipment.
- **Handling:** Ensure the height and width are manageable for handling and stacking.
- **Cost:** Balance between minimal material use and structural integrity.

Based on the analysis, a good compromise is the balanced dimensions derived from the optimal ratio:

- **Radius:** approximately 0.54 meters
- **Height:** approximately 1.08 meters

This configuration ensures the can holds exactly 1 m³ of liquid, with dimensions suitable for manufacturing, handling, and transportation.

Conclusion

Designing a closed cylindrical can with a capacity of exactly one cubic meter involves a confluence of geometric calculations, practical constraints, and application-specific considerations. By understanding the fundamental formula $(V = \pi r^2 h)$, analyzing various dimension scenarios, and applying principles like the optimal ratio $(h = 2r)$, engineers can craft designs that are both functional and efficient.

The key takeaway is that while mathematics provides the foundation, successful product design hinges on balancing theoretical dimensions with real-world requirements—culminating in a container that is not only precise in capacity but also optimized for manufacturing, handling, and cost.

In summary:

- Exact volume requires precise calculation of radius and height.
- Multiple configurations exist; choice depends on practical needs.
- Applying geometric optimization can minimize material use.
- Final design should balance capacity, manufacturability, and handling considerations.

By adopting these principles, manufacturers and designers can develop cylindrical containers that meet exact specifications while adhering to practical constraints, ensuring efficiency and reliability in storage and transportation.

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