A Curve Has Equation Y=4x^3 -3x+3. Find The Coordinates Of The Two Stationary Points. Determine Whether

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Understanding the behavior of curves is fundamental in calculus, especially when analyzing the nature of their stationary points. In this article, we will explore how to find the stationary points of the curve defined by the equation $y = 4x^3 - 3x + 3$. We will determine the coordinates of these points and analyze whether they are maxima, minima, or points of inflection. This comprehensive guide aims to clarify each step involved, making it accessible for students and enthusiasts alike.

What Is a Stationary Point?

Before diving into calculations, it’s essential to understand what stationary points are.

Definition of Stationary Points

A stationary point on a curve occurs where the first derivative of the function equals zero. At these points, the slope of the tangent to the curve is horizontal, indicating potential locations for local maxima, minima, or points of inflection.

Importance of Stationary Points

Identifying stationary points helps in understanding the shape of the curve, optimizing functions, and analyzing real-world phenomena modeled by such equations.

Given Equation and Its Derivative

The curve in question is:

$$
y = 4x^3 - 3x + 3
$$

To find stationary points, the first step is to find the derivative of y with respect to x.

Calculating the First Derivative

Using basic differentiation rules:

$$
\frac{dy}{dx} = \frac{d}{dx}(4x^3) - \frac{d}{dx}(3x) + \frac{d}{dx}(3)
$$

$$
\frac{dy}{dx} = 12x^2 - 3
$$

The derivative $y' = 12x^2 - 3$ gives the slope of the tangent at any point x.

Finding the Stationary Points

Stationary points occur where $y' = 0$.

Setting the Derivative Equal to Zero

Solve for x:

$$
12x^2 - 3 = 0
$$

$$
12x^2 = 3
$$

$$
x^2 = \frac{3}{12} = \frac{1}{4}
$$

$$
x = \pm \frac{1}{2}
$$

Thus, the potential stationary points are at:

$$
x = \frac{1}{2} \quad \text{and} \quad x = -\frac{1}{2}
$$

Calculating Corresponding y-Coordinates

Substitute these x-values into the original equation to find the respective y-coordinates.

For $x = \frac{1}{2}$:

$$
y = 4 \left(\frac{1}{2} \right)^3 - 3 \left(\frac{1}{2} \right) + 3
$$

$$
= 4 \times \frac{1}{8} - \frac{3}{2} + 3
$$

$$
= \frac{4}{8} - \frac{3}{2} + 3
$$

$$
= \frac{1}{2} - \frac{3}{2} + 3
$$

$$
= -1 + 3 = 2
$$

For $x = -\frac{1}{2}$:

$$
y = 4 \left(-\frac{1}{2} \right)^3 - 3 \left(-\frac{1}{2} \right) + 3
$$

$$
= 4 \times -\frac{1}{8} + \frac{3}{2} + 3
$$

$$
= -\frac{4}{8} + \frac{3}{2} + 3
$$

$$
= -\frac{1}{2} + \frac{3}{2} + 3
$$

$$
= (-0.5 + 1.5) + 3 = 1 + 3 = 4
$$

Therefore, the stationary points are:

$$
\boxed{
\left(\frac{1}{2}, 2 \right) \quad \text{and} \quad \left(-\frac{1}{2}, 4 \right)
}
$$

Determining the Nature of the Stationary Points

Having identified the points, the next step is to classify whether each is a maximum, minimum, or a point of inflection.

Using the Second Derivative Test

Calculate the second derivative:

$$
\frac{d^2 y}{dx^2} = \frac{d}{dx} (12x^2 - 3) = 24x
$$

Evaluate at each stationary point:

- At $x = \frac{1}{2}$:

$$
\frac{d^2 y}{dx^2} = 24 \times \frac{1}{2} = 12 > 0
$$

A positive second derivative indicates a local minimum at $\left(\frac{1}{2}, 2 \right)$.

- At $x = -\frac{1}{2}$:

$$
\frac{d^2 y}{dx^2} = 24 \times -\frac{1}{2} = -12 < 0
$$

A negative second derivative indicates a local maximum at $\left(-\frac{1}{2}, 4 \right)$.

Summary of Findings

Based on the calculations:

- The curve has a stationary point at $\left(-\frac{1}{2}, 4 \right)$, which is a local maximum.
- The curve has a stationary point at $\left(\frac{1}{2}, 2 \right)$, which is a local minimum.

Implications and Applications of Stationary Point Analysis

Identifying stationary points is crucial in various fields:

Mathematics and Calculus

- Helps understand the shape of the curve.
- Aids in optimization problems where maxima or minima are sought.

Physics and Engineering

- Analyzing motion, where stationary points can indicate points of equilibrium.
- Designing systems that require optimization of parameters.

Economics and Business

- Maximizing profit or minimizing cost functions modeled by cubic equations.

Conclusion

In summary, for the curve $y = 4x^3 - 3x + 3$:

- The stationary points are located at $\left(-\frac{1}{2}, 4 \right)$ and $\left(\frac{1}{2}, 2 \right)$.
- The point $\left(-\frac{1}{2}, 4 \right)$ is a local maximum.
- The point $\left(\frac{1}{2}, 2 \right)$ is a local minimum.

Understanding how to find and classify stationary points enhances our grasp of curve behavior, which is essential in various scientific and mathematical contexts. Whether analyzing the shape of a graph or solving real-world problems, these skills are fundamental components of calculus.

Additional Tips:
- Always verify your derivative calculations carefully.
- Use the second derivative test to classify stationary points confidently.
- Remember that points where the second derivative is zero require further analysis, as the test is inconclusive.

By mastering these techniques, you can analyze complex functions and gain insights into their behavior, paving the way for more advanced studies in calculus and related disciplines.

Frequently Asked Questions

What is the first step to find the stationary points of the curve y=4x^3 - 3x + 3?

The first step is to differentiate the function y with respect to x to find its derivative y'.

How do you find the x-coordinates of the stationary points for the curve y=4x^3 - 3x + 3?

Set the derivative y' equal to zero and solve for x: $y' = 12x^2 - 3 = 0$.

What are the x-values of the stationary points for y=4x^3 - 3x + 3?

Solving $12x^2 - 3 = 0$ gives $x^2 = 1/4$, so $x = \pm 1/2$.

How do you find the y-coordinates of the stationary points for the given curve?

Substitute $x = \pm 1/2$ into the original equation $y=4x^3 - 3x + 3$ to find their corresponding y-values.

What are the coordinates of the stationary points for y=4x^3 - 3x + 3?

For $x=1/2$, $y=4(1/2)^3 - 3(1/2) + 3 = 4(1/8) - 1.5 + 3 = 0.5 - 1.5 + 3 = 2.0$.
For $x=-1/2$, $y=4(-1/2)^3 - 3(-1/2) + 3 = 4(-1/8) + 1.5 + 3 = -0.5 + 1.5 + 3 = 4.0$.
So, the stationary points are at (0.5, 2) and (-0.5, 4).

How can we determine whether each stationary point is a maximum, minimum, or point of inflection?

Calculate the second derivative y'' and evaluate it at each stationary point. If $y'' > 0$, it's a minimum; if $y'' < 0$, it's a maximum; if $y'' = 0$, further analysis is needed to classify the point.

What is the second derivative of y=4x^3 - 3x + 3, and what does it tell us about the stationary points?

The second derivative is $y'' = 24x$. At $x=0.5$, $y''=12 > 0$, so the point (0.5, 2) is a local minimum. At $x=-0.5$, $y''=-12 < 0$, so the point (-0.5, 4) is a local maximum.

Additional Resources

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Introduction: Exploring the Geometry of Cubic Functions

The study of cubic functions holds a vital place in the realm of mathematics, offering insights into complex behaviors such as multiple turning points, inflection points, and intricate curvature patterns. Among these, stationary points—points where the slope of the tangent to the curve is zero—are particularly significant, revealing the locations of local maxima, minima, or saddle points.

In this comprehensive analysis, we turn our focus to the specific cubic function:

$Y = 4x^3 - 3x + 3$

Our goal is to meticulously determine the coordinates of the two stationary points of this curve and analyze their nature—whether they are maxima, minima, or saddle points. This investigation combines differential calculus with algebraic techniques, providing a detailed understanding of the curve's critical features.

Understanding the Function: Basic Properties and Behavior

Before diving into the calculus-based exploration, it's essential to grasp the fundamental properties of the given cubic function.

The general form:

$Y = ax^3 + bx + c$

where, in our case,

- $a = 4$ (coefficient of x^3),
- $b = -3$ (coefficient of x),
- $c = 3$ (constant term).

Key characteristics:

- Degree and end behavior: Since the leading coefficient ($a=4$) is positive, the cubic function will tend to positive infinity as x approaches positive infinity and negative infinity as x approaches negative infinity.
- Symmetry: Cubic functions are generally asymmetric; however, the specific coefficients influence the shape and nature of the turning points.
- Critical points: Occur where the derivative of the function equals zero, indicating potential local extrema or saddle points.

Calculating the Derivative: The Path to Stationary Points

The first critical step involves finding the derivative of the function with respect to x, which tells us the slope of the tangent at any point on the curve.

Derivative calculation:

Given,

$Y = 4x^3 - 3x + 3$

the derivative (dy/dx) is:

$dy/dx = d/dx [4x^3 - 3x + 3] = 12x^2 - 3$

This derivative function is key to locating stationary points because these are points where the slope is zero:

$dy/dx = 0$

Equation for stationary points:

$12x^2 - 3 = 0$

Finding the Coordinates of the Stationary Points

Step 1: Solve for x

Set the derivative to zero:

$12x^2 - 3 = 0$

Rearranged:

$12x^2 = 3$

$x^2 = 3 / 12 = 1 / 4$

Taking square roots:

$x = \pm\sqrt{(1/4)} = \pm 1/2$

Step 2: Find the corresponding y-coordinates

For each x-value, substitute back into the original function:

- For $x = 1/2$:

$Y = 4(1/2)^3 - 3(1/2) + 3$

Calculate step-by-step:

$(1/2)^3 = 1/8$

$Y = 4(1/8) - 3(1/2) + 3 = (4/8) - (3/2) + 3 = (1/2) - (3/2) + 3$

Simplify:

$(1/2) - (3/2) = -1$

Then,

$Y = -1 + 3 = 2$

- For $x = -1/2$:

$Y = 4(-1/2)^3 - 3(-1/2) + 3$

Calculate step-by-step:

$(-1/2)^3 = -1/8$

$Y = 4(-1/8) - 3(-1/2) + 3 = (-4/8) + (3/2) + 3 = (-1/2) + (3/2) + 3$

Simplify:

$(-1/2) + (3/2) = 1$

Then,

$Y = 1 + 3 = 4$

Therefore, the two stationary points are:

- At $(x, y) = (1/2, 2)$

- At $(x, y) = (-1/2, 4)$

Determining the Nature of Stationary Points: Maxima, Minima, or Saddle?

Having identified the x-coordinates and corresponding y-values, the next step involves classifying these stationary points.

Method: Second Derivative Test

- Compute the second derivative of the function.
- Evaluate the second derivative at each stationary point.
- Use the sign of the second derivative to determine the nature:

- If second derivative > 0: Local minimum.
- If second derivative < 0: Local maximum.
- If second derivative = 0: Test inconclusive; further analysis needed.

Calculating the second derivative:

$dy/dx = 12x^2 - 3$

Differentiate again:

$d^2y/dx^2 = d/dx [12x^2 - 3] = 24x$

Evaluate at $x = 1/2$:

$d^2y/dx^2 = 24(1/2) = 12 > 0$

Conclusion: At $(1/2, 2)$, the second derivative is positive, indicating a local minimum.

Evaluate at $x = -1/2$:

$d^2y/dx^2 = 24(-1/2) = -12 < 0$

Conclusion: At $(-1/2, 4)$, the second derivative is negative, indicating a local maximum.

Summary of Stationary Points and Their Characteristics

Stationary Point	Coordinates	Nature
Point 1	(0.5, 2)	Local Minimum
Point 2	(-0.5, 4)	Local Maximum

This classification reveals a classic "hill and valley" pattern characteristic of cubic functions with two stationary points.

Graphical Interpretation and Significance

Visualizing the curve based on these findings:

- The curve reaches a local maximum at (-0.5, 4), representing the highest point in its immediate vicinity.
- It then descends, crossing the y-axis somewhere between the maximum and minimum.
- The local minimum at (0.5, 2) marks the lowest point in that region.
- Beyond these points, the cubic's end behaviors dominate, with the graph tending towards positive infinity as $x \to \infty$ and negative infinity as $x \to -\infty$.

Understanding these critical points provides insight into the function's shape, aiding in applications ranging from physics to engineering, where the behavior of such functions influences real-world phenomena.

Analytical Significance and Broader Implications

The explicit determination of stationary points for the cubic function $Y=4x^3 - 3x + 3$ exemplifies the power of calculus in understanding complex curves. These points are not just mathematical curiosities but serve as foundational elements in various disciplines:

1. Optimization Problems: Identifying maxima and minima is crucial for maximizing efficiency or minimizing costs in practical scenarios.
2. Curve Sketching: Knowledge of stationary points is vital for accurate graph plotting, which in turn influences data interpretation.
3. Physical Systems Modeling: Cubic functions often model systems with nonlinear behaviors, such as motion under specific forces or economic models.

Furthermore, the methodology employed—finding derivatives, solving critical points, and classifying them—serves as a template applicable across diverse mathematical contexts.

Conclusion: The Critical Role of Calculus in Deciphering Curves

In dissecting the cubic function $Y=4x^3 - 3x + 3$, we have pinpointed its two stationary points at (0.5, 2) and (-0.5, 4), identified as a local minimum and maximum respectively. This analysis underscores the integral role of calculus in revealing the nuanced features of complex functions, transforming abstract equations into meaningful geometric insights.

Such detailed exploration not only deepens our understanding of specific functions but also exemplifies the analytical rigor necessary for advancing mathematical literacy. Whether in academic pursuits or practical applications, mastering these techniques equips individuals with the tools to interpret, analyze, and leverage the intricate behaviors of mathematical curves.

References & Further Reading:

- Stewart, J. (2015). Calculus: Concepts and Contexts. Brooks Cole.
- Thomas, G. B., & Finney, R. L. (2000). Calculus and Analytic Geometry. Pearson Education.
- Khan Academy. (n.d.). Derivative and its Applications. [Online resource]

This detailed examination illustrates the profound relationship between algebraic functions and their graphical representations, emphasizing the importance of calculus in uncovering the hidden features of mathematical curves.

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